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Effect of random density irregularities on nonlinear evolution of Langmuir waves in interplanetary medium

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Abstract. The evolution of nonlinear Langmuir waves in the interplanetary medium is investigated by appropriately accounting for the random density irregularities of the medium. A pair of modified Zakharov equations, which describe these waves, is solved numerically as an initial value problem for large scale ($\gg 10^2$ km) initial perturbations. For an ion acoustic-Langmuir solitary wave, the random irregularities damp the Langmuir wave by way of scattering and let the ion density perturbation radiate away in a few days. However an initial solitary or shock-like Langmuir wave excites the ion density perturbations within a fraction of a second, and then itself gets damped. These effects will strongly decelerate the collapse of large scale Langmuir waves. The possibility of detecting these processes, by means of interplanetary scintillation, is discussed.

Keywords. Langmuir waves; interplanetary medium; sun; radio emission; random density irregularities; modified Zakharov equations.

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1. Introduction

Type III radio bursts are associated with the ejection of high intensity electron streams into the interplanetary medium from the sun. These streams travel to very large distances such as 1 a.u. and beyond (1974 June/July issue of Space Science Reviews) and excite Langmuir waves through beam-plasma instability. The high frequency Langmuir waves, when their amplitudes become large enough, couple themselves to the low frequency ion acoustic oscillations through the ponderomotive force. In turn, the density fluctuations associated with the ion acoustic waves trap the Langmuir waves in them. This nonlinear interaction between the Langmuir and the ion acoustic waves influences the propagation characteristics of both of them (Zakharov 1972, Thornhill and ter Haar 1978). The time development of such coupled Langmuir-ion acoustic waves has earlier been studied quite extensively (Bardwell and Goldman 1976; Bardwell 1976; Nicholson *et al* 1978) especially with regard to their existence in the interplanetary medium around 0·5–1 a.u. from the sun. At such distances, the nonlinear effects were found to be quite important. Some of these earlier works had included other effects like wave-particle interactions, OTSI (oscillating two-stream instability), presence of magnetic fields etc. One of their most important conclusions was to show the possibility of

The authors felicitate Prof. D S Kothari on his eightieth birthday and dedicate this paper to him on this occasion.

Langmuir collapse (Nicholson *et al* 1978) whereby Langmuir waves with large widths shrink in size to very small widths of the order of a few Debye lengths. The collapsing Langmuir waves by means of the ponderomotive force give rise to pockets of extremely large density fluctuations having the same scale sizes. These studies, however, did not take into account the random density irregularities so commonly present in the interplanetary medium. Although some attempts to consider the effect of such background random density irregularities on the Langmuir waves in the interplanetary plasma have recently been made (Goldman and Du Bois 1982; Du Bois and Pesme 1984). However the latter studies were confined to the quasilinear regime of weak turbulence in which the strong coupling of the Langmuir waves to the ion acoustic waves through the ponderomotive force was ignored. We feel that such an approach to the Langmuir-ion acoustic interactions is rather incomplete especially since the Langmuir waves generated by the electron beam-plasma instability in the interplanetary plasma after a type III burst can very often grow to large amplitudes at which the ponderomotive force is not negligible. In the present work, we seek to study the strongly coupled Langmuir and ion acoustic waves (scale lengths $\gg 10^2$ km) in the context of the interplanetary medium in the presence of the back-ground random density irregularities. The existence of such density irregularities, with sizes $\sim 10^2$ km around 0·5 to 1·0 a.u. from the sun, has by now been very well established by the interplanetary scintillations studies (Readhead *et al* 1978; Gapper *et al* 1982) and satellite observations (Cronyn 1972; Neugebauer 1975).

In § 2, we derive a pair of modified Zakharov equations to describe this process. In § 3, we numerically solve these equations as an initial value problem with various initial conditions. Taking the initial perturbation to be a Langmuir-ion acoustic solitary wave, our computations for the solar wind parameters around 0·5 a.u., show that the Langmuir wave amplitude damps within a fraction of a second leaving behind the ion density perturbation. This is due to the scattering of the initially coherent Langmuir wave by the random density irregularities. Then the ion density perturbation that is left behind radiates away extremely slowly that is, in a couple of days. When this occurs, the originally smooth large size density perturbations breaks up into two or three smaller ones. Whereas an initial shock-like or a solitary type Langmuir wave with no initial density perturbation excites the density perturbation within a fraction of a second while itself undergoes damping. Later on, the density perturbation radiates away. In the case of shock waves, the density perturbations are excited at the leading and trailing edges of the shocks since the ponderomotive force is maximum at these places. Further evolution is same as in the previous cases.

IPS observations similar to the one carried out by Gapper *et al* (1982) are the most ideal to detect the excitation and propagation of density perturbations after a type III burst in the interplanetary medium. The breaking up of the large scale size density fluctuations will correspond to a decrease in the power spectrum at these scale sizes as the regions of density perturbations move outwards.

2. The modified Zakharov equations

We take the interplanetary medium to be a plasma with hot electrons and cold ions containing isotropic random density irregularities with an average scale size say L ($\sim 10^2$ km). In the absence of magnetic field, two of the most common oscillations such a plasma can sustain are the high frequency Langmuir oscillations ($\omega \sim \omega_p = (4\pi n_0 e^2/m_e)^{1/2}$) and the low frequency ion acoustic oscillations ($\omega \sim \omega_{pi} = (4\pi n_0 e^2/m_i)^{1/2}$). The equations describing the electron fluid dynamics are,

$$\frac{\partial n_e}{\partial t} + \frac{\partial}{\partial x}(n_e v_e) = 0, \tag{1}$$

$$\frac{\partial v_e}{\partial t} + v_e \frac{\partial v_e}{\partial x} = -\frac{e}{m_e} E - \frac{\gamma_e T_e}{m_e n_e} \frac{\partial n_e}{\partial x}, \tag{2}$$

and

$$\partial E/\partial x = 4\pi e (n_i - n_e). \tag{3}$$

In (1)–(3), n_e and n_i are the electron and ion densities, v_e is the electron fluid velocity, T_e is the electron temperature and γ_e is the ratio of specific heats of the electrons. The electrons, since they are light take part in both the Langmuir and ion acoustic oscillations whereas the heavy ions take part only in ion acoustic oscillations. Accordingly, we can expand the variables as (Tamoikin and Fainshtein, 1973):

$$n_e = n_0 + \delta n(x) + \langle n_s \rangle + \langle n_f \rangle + n'_e, \tag{4}$$

$$n_i = n_0 + \delta n(x) + \langle n_i \rangle + n'_i, \tag{5}$$

$$v_e = \langle v_s \rangle + \langle v_f \rangle + v', \tag{6}$$

and

$$E = \langle E_s \rangle + \langle E_f \rangle + E', \tag{7}$$

where the angular brackets denote averaging over the random density irregularities represented by $\delta n(x)$. Moreover n_0 is the mean value of ion and electron densities and the prime denotes deviations of the quantities from their average values due to the fluctuations. The subscripts s and f refer to slow and fast oscillations. In the following treatment, the scale sizes of the random irregularities are considered to be much smaller than those of the slowly oscillating quantities. On substituting (4)–(7) in (1)–(3) and neglecting the nonlinear terms (Thornhill and ter Haar 1978) in the fast oscillations, after eliminating $\langle n_f \rangle$ and $\langle v_f \rangle$ and making use of the inequality

$$\frac{\partial}{\partial t}(\langle n_s \rangle, \langle n_i \rangle; \langle v_s \rangle) \ll \frac{\partial}{\partial t}(\langle n_f \rangle; \langle v_f \rangle), \tag{8}$$

we get,

$$\frac{\partial^2 \varepsilon}{\partial t^2} + \omega_p^2 \left(1 + \frac{\delta n + \langle n_s \rangle}{n_0}\right) \varepsilon - \frac{\gamma_e T_e}{m_e} \frac{\partial^2 \varepsilon}{\partial x^2} = 0. \tag{9}$$

Here $\varepsilon = \langle E_f \rangle + E'$. We define ε as,

$$\varepsilon = \tfrac{1}{2}[\varepsilon_0(x,t) \exp(-i\omega_p t) + \text{c.c}]. \tag{10}$$

$\varepsilon_0(x,t)$ is a slowly varying function of space and time. Putting (10) in (9) and ignoring the second order time variations of ε_0, we obtain,

$$\frac{2i}{\omega_p} \frac{\partial \varepsilon_0}{\partial t} + 3\gamma_e \lambda_D^2 \frac{\partial^2 \varepsilon_0}{\partial x^2} - \frac{(\langle n_s \rangle + \delta n)}{n_0} \varepsilon_0 = 0. \tag{11}$$

Further we split ε_0 as, $\varepsilon_0 = \langle \varepsilon_0 \rangle + \varepsilon_0'$ where $\langle \varepsilon_0 \rangle$ is the average and ε_0' is the deviation from the mean value. Averaging (11) over the random irregularities, we obtain

$$\left(\frac{2i}{\omega_p}\frac{\partial}{\partial t} + 3\gamma_e\lambda_D^2\frac{\partial^2}{\partial x^2} - \frac{\langle n_s \rangle}{n_0}\right)\langle \varepsilon_0 \rangle = \frac{1}{n_0}\langle \delta n \varepsilon_0' \rangle. \tag{12}$$

Subtracting (12) from (11), we have

$$\frac{2i}{\omega_p}\frac{\partial \varepsilon_0'}{\partial t} + 3\gamma_e\lambda_D^2\frac{\partial^2 \varepsilon_0'}{\partial x^2} - \frac{\langle n_s \rangle}{n_0}\varepsilon_0' = \frac{\delta n}{n_0}\langle \varepsilon_0 \rangle + \left[\frac{\delta n}{n_0}\varepsilon_0' - \frac{\langle \delta n \varepsilon_0' \rangle}{n_0}\right]. \tag{13}$$

Since the fluctuations $(\delta n/n_0)$ and ε_0' are of the same order, we can neglect the second order term $(\delta n\varepsilon_0' - \langle \delta n \varepsilon_0' \rangle)/n_0$ in (13) (Tamoikin and Fainshtein, 1973). We then Fourier transform (13), solve for ε_0' and take its inverse Fourier transform to obtain,

$$\varepsilon_0' = \frac{i}{2n_0}\iint \delta n(x',t')\langle \varepsilon_0(x',t') \rangle \times \exp\left\{i\left[k(x'-x) - \frac{\omega_p}{2}\left(3k^2\gamma_e\lambda_D^2 + \frac{\langle n_s \rangle}{n_0}\right)(t'-t)\right]\right\}\mathrm{d}x'\mathrm{d}t'\mathrm{d}k. \tag{14}$$

On multiplying with $(\delta n/n_0)$ and taking the average over the random irregularities, (14) gives,

$$A \equiv \frac{\langle \varepsilon_0' \delta n \rangle}{n_0} = \frac{i\langle \delta n^2 \rangle}{4n_0^2}\iint g(\bar{\chi})\exp\left\{i\left[\frac{\omega_p}{2}\left(3k^2\gamma_e\lambda_D^2 + \frac{\langle n_s \rangle}{n_0}\right) - k\xi\right]\right\} \times \langle \varepsilon_0(x-\xi,\, t-\tau) \rangle\, \mathrm{d}\bar{\chi}\, \mathrm{d}\tau\, \mathrm{d}k, \tag{15}$$

where $g(\bar{\chi})$ is the correlation function for the random irregularities and ξ is the x-component of the vector $\bar{\chi}$. Substituting (15) in (12), we obtain,

$$\frac{2i}{\omega_p}\frac{\partial \langle \varepsilon_0 \rangle}{\partial t} + 3\gamma_e\lambda_D^2\frac{\partial^2 \langle \varepsilon_0 \rangle}{\partial x^2} + iA\langle \varepsilon_0 \rangle = \frac{\langle n_s \rangle \langle \varepsilon_0 \rangle}{n_0}. \tag{16}$$

The term $iA\langle \varepsilon_0 \rangle$ represents damping of the Langmuir wave due to scattering by the irregularities.

The low frequency ion acoustic waves, on the other hand, are described by the equations,

$$\frac{\partial n_i}{\partial t} + \frac{\partial}{\partial x}(n_i v_i) = 0, \tag{17}$$

$$\frac{\partial v_i}{\partial t} + v_i\frac{\partial v_i}{\partial x} = -\frac{e}{m_i}\frac{\partial \phi}{\partial x}, \tag{18}$$

and

$$\frac{\partial^2 \phi}{\partial x^2} = 4\pi e(\tilde{n}_e - n_i), \tag{19}$$

where ϕ is defined by, $-\partial\phi/\partial x = \langle E_s \rangle + E'$ and $\tilde{n}_e (= n_0 + \delta n(x) + \langle n_s \rangle + n_e')$, is the low frequency electron fluctuations. To find $\tilde{n}_e$ we average (2) over the fast oscillations, and neglect the electron inertia (Mohan and Buti 1982); this gives

$$\tilde{n}_e = n_0 + \delta n(x) + \frac{1}{T_e}\left(e\phi - \frac{e^2|\varepsilon|^2}{4m_e\omega_p^2}\right). \tag{20}$$

We now use the method of reductive perturbations (Taniuti 1974; Mohan and Buti 1980, 1982) to equations (17)–(19) and introduce the following stretched variables:

$$\xi = \epsilon^{1/2}(x - C_s t), \quad \tau = \epsilon^{3/2}t, \tag{21}$$

where $C_s = (T_e/m_i)^{1/2}$ is the ion acoustic speed. The quantities n_i, v_i and ϕ are expanded as,

$$n_i = n_0 + \epsilon n^{(1)} + \epsilon^2 n^{(2)} + \dots, \tag{22}$$

$$v_i = \epsilon v^{(1)} + \epsilon^2 v^{(2)} + \dots, \tag{23}$$

and

$$\phi = \epsilon\phi^{(1)} + \epsilon^2\phi^{(2)} + \dots. \tag{24}$$

The smallness parameter ε is chosen in such a way that $\tilde{n}_i (= \langle n_i \rangle + n_i' = \varepsilon n^{(1)})$ and $|\varepsilon|$ occurring in (20) are of the same order (Nishikawa *et al* 1974). This way we can take into account the second order ion nonlinearities.

From the second order equations in ε, after eliminating $n^{(2)}$, $v^{(2)}$ and $\phi^{(2)}$ and on using the first order solutions, namely

$$n^{(1)} = (n_0/C_s)v^{(1)} = (n_0 e/T_e)\phi^{(1)}, \tag{25}$$

we get,

$$\frac{\partial \tilde{n}_i}{\partial t} + \frac{C_s}{n_0}\tilde{n}_i\frac{\partial \tilde{n}_i}{\partial x} + \frac{C_s T_e}{8\pi n_0 e^2}\frac{\partial^3 \tilde{n}_i}{\partial x^3} = -\frac{C_s n_0 e^2}{8T_e m_e \omega_p^2}\frac{\partial}{\partial x}|\varepsilon|^2. \tag{26}$$

Following the procedure used in the case of (13) (Tamoikin and Fainshtein 1973), we average (26) over the random inhomogeneities to obtain,

$$\frac{\partial}{\partial t}\left(\frac{\langle n_s \rangle}{n_0}\right) + C_s\left(1 + \frac{\langle n_s \rangle}{n_0}\right)\frac{\partial}{\partial x}\left(\frac{\langle n_s \rangle}{n_0}\right) + \frac{C_s\lambda_D^2}{2}\frac{\partial^3}{\partial x^3}\left(\frac{\langle n_s \rangle}{n_0}\right)$$
$$- \frac{m_i C_s^2}{2en_0}\left(\frac{\partial B}{\partial x} + D\right) = -\frac{C_s n_0 e^2}{8T_e m_e \omega_p^2}\frac{\partial}{\partial x}|\langle \varepsilon_0 \rangle|^2, \tag{27}$$

where

$$B = -e\frac{\langle \delta n^2 \rangle}{2\pi}\left\{\int \frac{g(\vec{\chi})}{S(\omega)\chi}\exp[i(\omega\tau - k_0\chi)]\right.$$
$$\times \frac{\langle n_s(x-\xi, t-\tau)\rangle}{n_0}\left[1 + \frac{\xi C_s}{\omega\chi^2}(k_0\chi - i)\right]\mathrm{d}\chi\,\mathrm{d}\omega\,\mathrm{d}\tau \tag{28}$$

and

$$D = \frac{eT_e\langle \delta n^2 \rangle}{2\pi m_i}\frac{\partial}{\partial x}\int \frac{g(\vec{\chi})}{\omega S(\omega)\chi^3}\exp[i(\omega\tau - k_0\chi)]$$
$$\times \frac{\langle n_s(x-\xi, t-\tau)\rangle}{n_0}\left\{\xi(k_0\rho - i) + \frac{C_s}{\omega}\left[k_0^2\xi^2\right.\right.$$
$$\left.\left. + (1 + ik_0\chi)\left(1 - \frac{3\xi^2}{\chi}\right)\right\}\mathrm{d}\chi\,\mathrm{d}\omega\,\mathrm{d}\tau \tag{29}$$

with $$k_0 = \frac{\omega_{p_i}}{C_s}[S(\omega)]^{1/2} \quad \text{and} \quad S(\omega) = \left(\frac{\omega_{pi}^2}{\omega^2} - 1\right).$$

The term on the right side of (29) represents the ponderomotive force (Gaponov and Miller 1958) which couples the ion acoustic perturbation to the electric field of the Langmuir wave.

We expand $\langle \varepsilon_0(x-\xi, t-\tau)\rangle$ and $\langle n_s(x-\xi, t-\tau)\rangle$ occurring in (15), (28) and (29) about x and t and retain the first three terms in the expansion. Then after carrying out the integrations, (16) and (27) reduce to

$$\frac{2i}{\omega_p}\frac{\partial}{\partial t}\langle \varepsilon_0\rangle + 3\gamma_e\lambda_D^2\frac{\partial^2\langle \varepsilon_0\rangle}{\partial x^2} = \frac{\langle n_s\rangle}{n_0}\langle \varepsilon_0\rangle - \frac{i}{4}\frac{\overline{L}}{\sqrt{3}\lambda_D}\frac{\langle \delta n^2\rangle}{n_0^2}\langle \varepsilon_0\rangle \tag{30}$$

and

$$\frac{\partial}{\partial t}\left(\frac{\langle n_s\rangle}{n_0}\right) + C_s\left(1+\frac{\langle n_s\rangle}{n_0}\right)\frac{\partial}{\partial x}\left(\frac{\langle n_s\rangle}{n_0}\right) + \frac{C_s\lambda_D^2}{2}\frac{\partial^3}{\partial x^3}\left(\frac{\langle n_s\rangle}{n_0}\right)$$
$$+\alpha_1\frac{\partial^3}{\partial x\partial t^2}\left(\frac{\langle n_s\rangle}{n_0}\right) - \alpha_2\frac{\partial^3}{\partial x^2\partial t}\left(\frac{\langle n_s\rangle}{n_0}\right) + \delta\frac{\partial^2}{\partial x\partial t}\left(\frac{\langle n_s\rangle}{n_0}\right)$$
$$= -\frac{C_s n_0 e^2}{8T_e m_e\omega_p^2}\frac{\partial}{\partial x}|\langle \varepsilon_0\rangle|^2, \tag{31}$$

where $$\alpha_1 = \frac{C_s}{3\omega_{pi}^2}\frac{\langle \delta n^2\rangle}{n_0^2}\frac{\overline{L^2}}{\lambda_D^2}, \quad \alpha_2 = C_s\alpha_1$$

and $$\delta = \frac{1}{3}\overline{L}\frac{\langle \delta n^2\rangle}{n_0^2}, \text{ with } \overline{L} = \int_0^\infty g(\bar{\chi})\,d\chi$$

and $\overline{L^2} = \int_0^\infty g(\bar{\chi})\chi\, d\chi$. The quantities $\overline{L}$ and $\overline{L^2}$ characterize the integral scales of the random density irregularities. Equations (30) and (31) are the modified Zakharov equations. On using the normalizations,

$$x \to x(\sqrt{3}\lambda_D)^{-1}, \quad t \to t\,C_s/(\sqrt{3}\,\lambda_D),$$
$$\langle n_s\rangle \to \frac{\langle n_s\rangle}{n_0} = N, \quad \langle \varepsilon_0\rangle = \langle \varepsilon_0\rangle/(16\pi n_0 T_e)^{1/2} = E,$$

with $\gamma_e = 3$, they reduce to,

$$2i\mu\frac{\partial E}{\partial t} + \frac{\partial^2 E}{\partial x^2} = NE - iaE, \tag{32}$$

and

$$\frac{\partial N}{\partial t} + \frac{\partial N}{\partial x} + N\frac{\partial N}{\partial x} + \frac{1}{6}\frac{\partial^3 N}{\partial x^3} + p\frac{\partial^3 N}{\partial x^2\partial t}$$
$$+q\frac{\partial^3 N}{\partial x\partial t^2} + r\frac{\partial^2 N}{\partial x\partial t} = -\frac{1}{2}\frac{\partial}{\partial x}|E|^2, \tag{33}$$

where

$$\mu = \left(\frac{m_e}{3m_i}\right)^{1/2}, \quad a = \frac{i\bar{L}}{4\sqrt{3}\lambda_D}\left(\frac{\langle\delta n^2\rangle}{n_0^2}\right),$$

$$p = \frac{C_s\alpha_1}{3\lambda_D^2}, \quad q = -\frac{\alpha_2}{3\lambda_D^2} \quad \text{and} \quad r = \frac{\delta}{\sqrt{3}\lambda_D}.$$

3. Discussion

Since analytical solution of Zakharov equations is not possible, we have solved (32) and (33) numerically as an initial value problem for various kinds of initial perturbations. The electric field equation (32) is discretized using the Crank-Nicholson (Smith 1969) scheme as,

$$2i\mu(E_l^{j+1} - E_l^j)\frac{1}{T} + (E_{l+1}^{j+1} - 2E_l^{j+1} + E_{l-1}^{j+1} + E_{l+1}^j - 2E_l^j + E_{l-1}^j)\frac{1}{2H^2}$$
$$+\frac{ia}{2}(E_l^j + E_l^{j+1}) - \frac{1}{2}(N_l^{j+1}\, E_l^{j+1} + N_l^j E_l^j) = 0, \tag{34}$$

where l and j refer to the spatial points and temporal levels and H and T are the corresponding step sizes. For the ion density equation (33) the following Zabusky-Kruskal method (Zabusky and Kruskal 1965; Appert and Vaclavik 1977) is used:

$$(N_l^{j+1} - N_l^{j-1})\frac{1}{2T} + (N_{l+1}^j - N_{l-1}^j)\frac{1}{2H}$$
$$+ (N_{l+1}^j + N_l^j + N_{l-1}^j)(N_{l+1}^j - N_{l-1}^j)\frac{1}{6H}$$
$$+ (N_{l+2}^j - 2N_{l+1}^j + 2N_{l-1}^j - N_{l-2}^j)\frac{1}{12H^3}$$
$$+ p\{(N_{l+1}^j - 2N_l^j + N_{l-1}^j) - N_{l+1}^{j-1} - 2N_l^{j-1} + N_{l-1}^{j-1})\}\frac{1}{H^2T}$$
$$+ q\{(N_{l+1}^{j-2} - 2N_{l+1}^{j-1} + N_{l+1}^j) - (N_{l-1}^{j-2} - 2N_{l-1}^{j-1} + N_{l-1}^j)\}\frac{1}{HT^2}$$
$$+ r\{(N_{l-1}^{j-2} - N_{l-1}^j) - (N_{l+1}^{j-2} - N_{l+1}^j)\}\frac{1}{4HT}$$
$$+ (|E|_{l+1}^j + |E|_l^j + |E|_{l-1}^j)(|E|_{l+1}^j - |E|_{l-1}^j)\frac{1}{6H} = 0. \tag{35}$$

The plasma parameters used for the present calculations (Forslund 1970; Smith and Sime 1979; Readhead *et al* 1978) are $n_0 = 5\,\mathrm{cm}^{-3}$, $\lambda_D = 5 \times 10^2$ cm, $\omega_{pi} = 3 \times 10^3\,\mathrm{sec}^{-1}$, $\bar{L} = 10^7$ cm, $\langle\delta n^2\rangle^{1/2}/n_0 = 10^{-4}$. These are typical of the interplanetary medium around 0·5 a.u.

In the absence of irregularities i.e., $a = p = q = r = 0$, (32) and (33) have the following solitary wave solution (Zakharov 1972; Nishikawa *et al* 1974):

$$E = F \operatorname{sech}(X/W) \tanh(X/W) \exp\{i[\mu M X + (\mu M^2 + \Omega)t]\}$$

and

$$N = -G \operatorname{sech}^2 (X/W), \tag{36}$$

where

$$X = (x - Mt), \quad F = (6\sqrt{3}/5)(1-M), \quad W = (10/3)^{1/2}(1-M)^{1/2},$$

$$G = -(9/5)(1-M), \quad \text{and} \quad \Omega = (1 - \mu^2 W^2 M^2)/(2\mu W^2), \tag{37}$$

with M as the Mach number. This wave moves with a constant velocity without any change in its shape.

However, since the interplanetary medium contains the random irregularities, we know that the quantities a, p, q and r are nonzero and so an initial wave given by (36) will

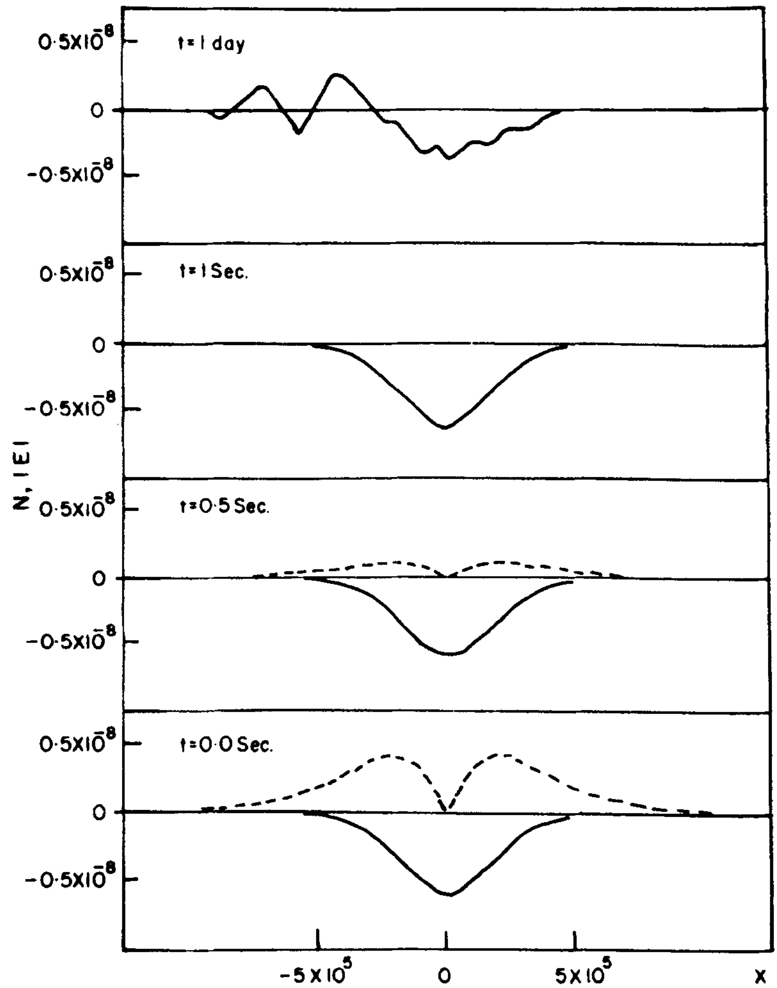

Figure 1. Time development of Langmuir-ion acoustic solitary wave. Full lines represent ion density perturbation and dotted lines represent the envelope of the Langmuir field, $|E|$. At $t = 0{\cdot}0$ we have taken $(1-M) = 0{\cdot}4 \times 10^{-8}$.

not remain stationary. This is shown in figure 1. Here we see that at first the random irregularities cause the Langmuir wave to damp. This is because the irregularities scatter the initially coherent Langmuir wave by introducing random phase fluctuations. On the other hand, the low frequency ion density perturbation is affected very little by the irregularities. After the Langmuir wave damps, the ion density perturbation spreads out by way of radiation since there is no more ponderomotive force to hold it. In this process it breaks up into a few smaller ones within two or three days.

Figure 2 shows the process of excitation of an ion density perturbation by a localized large amplitude Langmuir wave. This wave has no accompanying ion density perturbation to start with. Here we begin with a localized Langmuir wave with an arbitrary amplitude F' given by,

$$E = F' \operatorname{sech}(X/W), \tag{38}$$

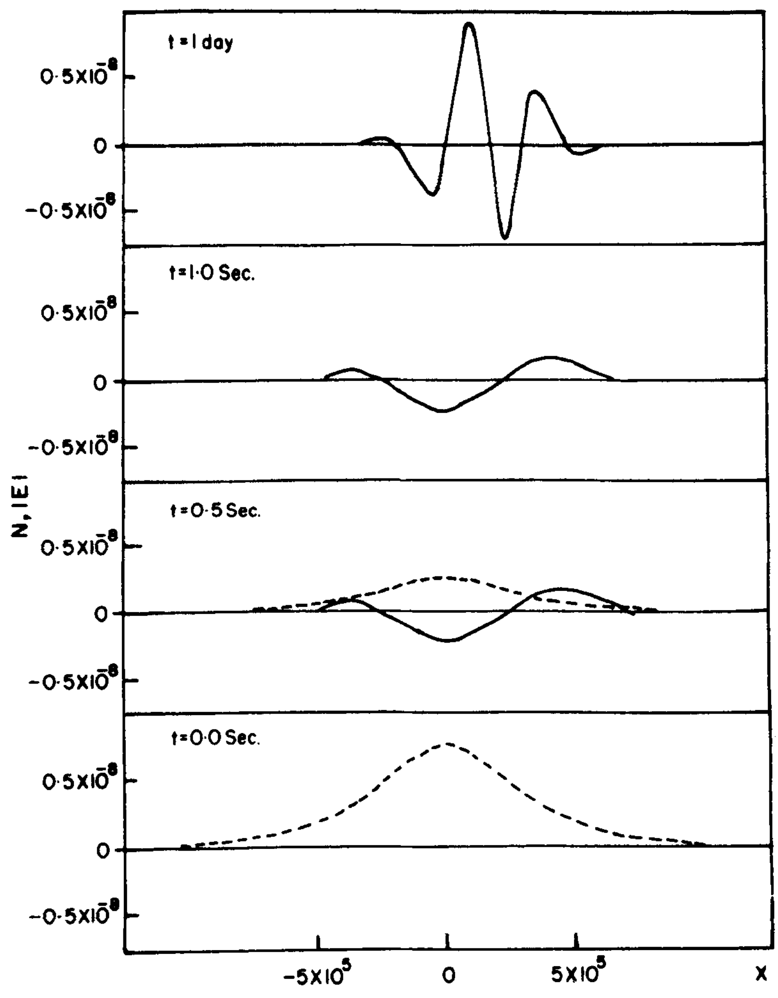

Figure 2. Time development of a localized Langmuir wave with zero initial ion density perturbation. The values of the quantities used at $t = 0{\cdot}0$ are $(1-M) = 0{\cdot}4 \times 10^{-8}$, $F' = 0{\cdot}75 \times 10^{-8}$.

where X and W are defined by (37). We observe that the ponderomotive force of the Langmuir wave excites the density perturbation which attains its maximum within a fraction of a second; at the same time, Langmuir wave itself undergoes damping. Later on, as in the previous case the density perturbation radiates away.

On certain occasions depending upon the nature of the electron stream, the beam-plasma instability in the interplanetary medium will be able to produce shock-like Langmuir waves. In the early stages, they would not have any associated density perturbations. The leading and the trailing edges of such shocks can respectively be represented by,

$$E = F'[1 - \tanh(X/W)], \tag{39}$$

and

$$E = F'[1 + \tanh(X/W)], \tag{40}$$

where X, W and F' are defined by (37) and (38). Such shock waves are more likely to be

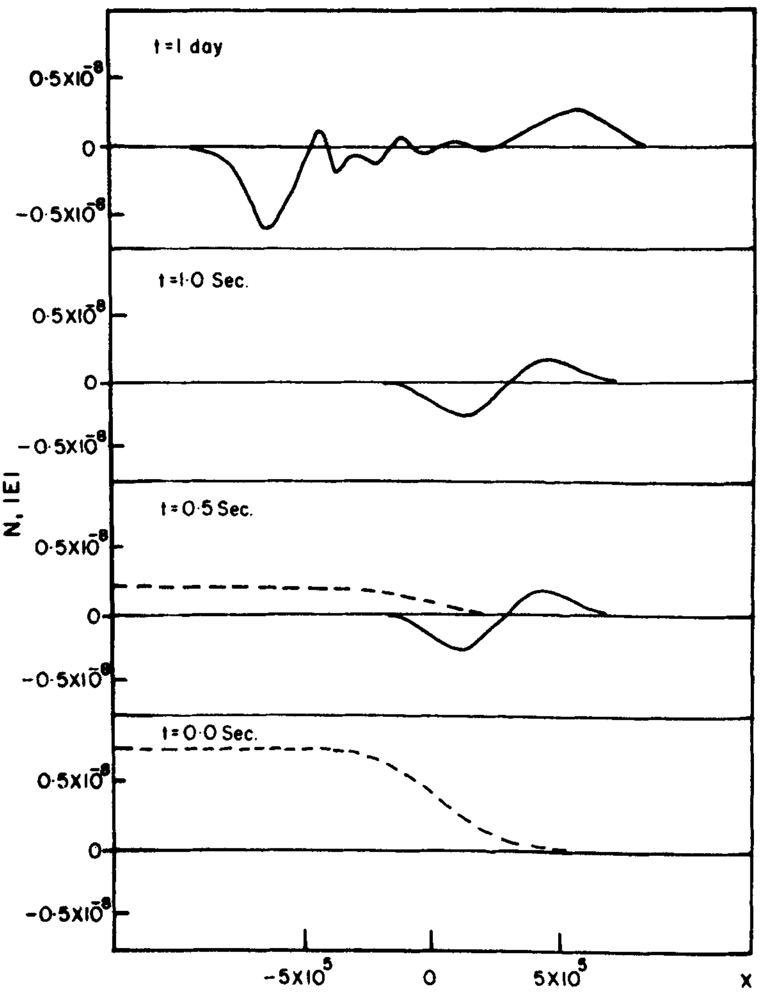

Figure 3. Time development of the leading edge of a Langmuir shock wave. The various quantities at $t = 0{\cdot}0$ are same as those in figure 2.

present in type-II solar bursts (Weiss 1963; Kundu 1965). The time evolution of the shock waves given by (39) and (40) are depicted in figures 3 and 4 respectively. Here we see that the ion density perturbations are excited only in a narrow region around the leading and the trailing edges. This is because the ponderomotive force represented by the right side of (33) would be vanishingly small at all other places. Further evolution of the ion density perturbation in these two cases is same as those described in the previous cases.

4. Conclusions

The results presented here have important consequences on the density fluctuations propagating, across the interplanetary medium, away from the sun. The modulational instability and collapse (Thornhill and ter Haar 1978; Buti 1977, Nicholson *et al* 1978)

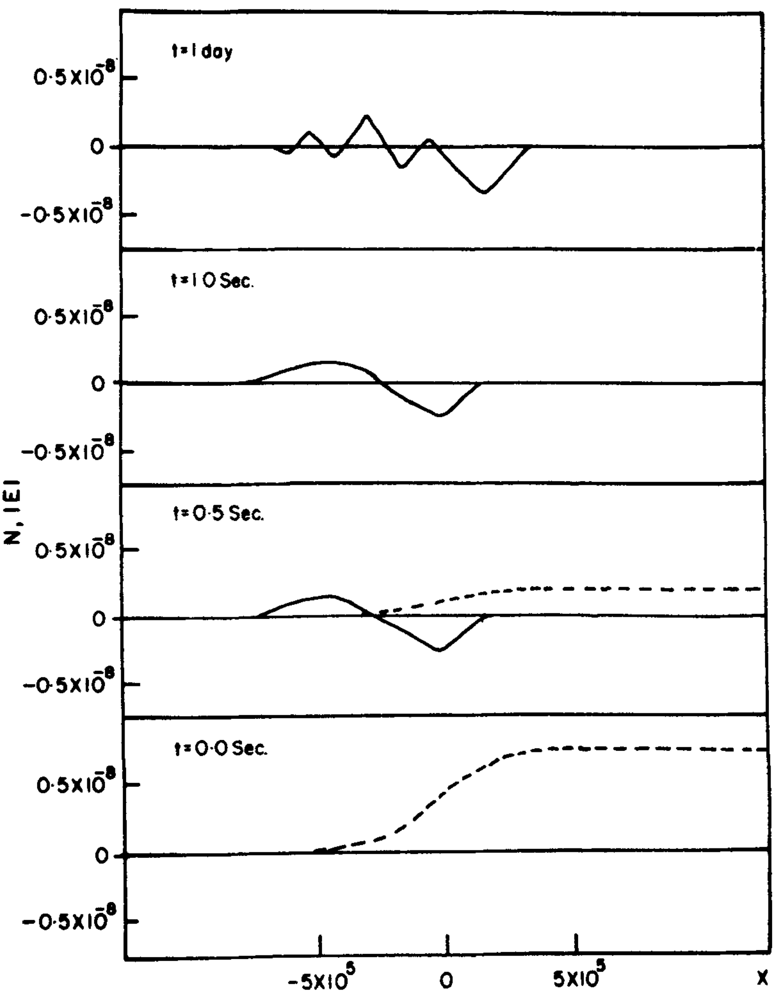

Figure 4. The time development of the trailing edge of a Langmuir shock. The various quantities at $t = 0{\cdot}0$ are same as those in figure 2.

of a long wave length ($\gg 10^2$ km) Langmuir wave in interplanetary plasma will be seriously affected by the process we have discussed here. The breaking up of an ion density perturbation will spread the energy of the wave over a larger area instead of concentrating in a small region as in the case of collapse. Consequently, we see that the random irregularities of the interplanetary medium help to avoid the collapse of very long wave length Langmuir waves. We are justified in drawing these conclusions even though our calculations are based on one-dimensional model whereas the collapse is a three-dimensional phenomena, simply because we know that the necessary condition for the occurrence of collapse is satisfied only if the one-dimensional system is modulationally unstable (Buti 1977).

Long time simultaneous IPS observations of many radio sources all over the sky are the most ideal for obtaining an accurate picture of the development of density fluctuations after a type III radio burst. Similar observations have been reported by Gapper *et al* (1982) where they could detect the regions of density disturbances moving away from the sun over a few days although none of them has been identified with a type III burst.

In order to see the breaking up of large scale fluctuations within these regions, one has to observe the changes occurring in the power spectrum of density fluctuations corresponding to these scale sizes. For the large scale sizes considered here, two frequency IPS observations (Shishov 1975; Gapper and Hewish 1981) within the regions of density disturbances are essential. The breaking up will correspond to a shift in the intensity of power spectrum towards smaller scale sizes. This shift will be observable when the power spectrum within the region of disturbance is followed for a few days.

To check our model, we are planning to make two-frequency IPS observations at the IPS observatory of the Physical Research Laboratory to follow the changes occurring in the power spectrum of the density fluctuations in the interplanetary medium after type III bursts.

We are also planning to take into account the presence of the interplanetary magnetic field to find its effect on the process of breaking up as a result of Langmuir waves propagating in an arbitrary direction with respect to the magnetic field.

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