Pramāṇa, Vol. 26, No. 5, May 1986, pp. 409–417. © Printed in India.

Stochastic model of nuclear levelwidth

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MS received 13 November 1985; revised 28 February 1986

Abstract. We investigate here the origin of nuclear levelwidth as an effect of the coupling of particle mode and the surface vibration mode of the nucleus. This interaction is taken to be stochastic in nature, characterized by a single correlation time t_0, the random nature of the interaction originating from the partition of the total hamiltonian into those of the two modes. The Redfield equation of motion for the density matrix for the particle mode is solved. The solution of the Redfield equation shows that the occupation number in any particle state decays with a time constant depending on the correlation time t_0 and the quantum-mechanical matrix elements of the interaction hamiltonian. The inverse of this decay time will give the width of this level. Numerical calculations have been done for ${}^{207}_{82}Pb_{125}$.

Keywords. Nuclear levelwidth; stochastic method; density matrix; particle-vibration coupling.

PACS No. 21·10

1. Introduction

Measurement of nuclear levelwidth (Malmskog *et al* 1970) plays a significant role in getting information on different modes of excitations of the nuclei. Levelwidth measurements (Sann *et al* 1970; Hardy 1981) are also instrumental in comparing the validity of different nuclear models. One such model is the description of the nucleus in terms of independent modes of excitations (Bohr and Mottelson 1973). Studies of giant resonances in photo-absorption cross-section and inelastic scattering experiments have unveiled a rich variety of vibrations of different types (Bertsch *et al* 1983). At low excitations the only important degree of freedom is surface vibrations. The lifetimes of the elementary excitations are manifested in the width of the strength function (Bohr and Mottelson 1969, 1975) and are studied through transfer reaction data. The width of the strength function involves an analysis of the coupling of the single particle motion with the collective modes of vibration (Siemssen *et al* 1981). As a result of this coupling, the occupation number i.e. average number of particle in a particular particle state m decays with a time constant T_m, characteristic of that state, and we call $\hbar/T_m$ the levelwidth of that state.

We present here a stochastic method of calculating the levelwidth of the strength function using the statistical density matrix formalism. We first note that in the absence of an interaction with an external "bath" the energy level spectrum of the system of nucleons in the nucleus consists of pure delta functions and the average number of particle in any energy level remains constant in time. If there is a random interaction between the particle modes and the "bath" then the individual particle levels become broadened and the average number of particle in a level decays with time. This random interaction is, in general, a fluctuating time-dependent quantity. The physical origin of the fluctuations can be quite different. Fluctuations may be imposed from the outside by random boundary conditions or may indicate a lack of knowledge about the exact state of the system, either because of quantum noise or because of the impossibility of handling a huge number of variables (Graham 1973). In the stochastic model the dynamical observables of interest are treated by exact equations of motion in which the effects of microscopic motions of other variables are included in the guise of a random force (Fox 1978). In a complete description involving coordinates of all particles and their interactions there is no dissipation and hence no levelwidth. Dissipation in the form of random part enters because of the partitioning of the hamiltonian in those of different modes and their interactions (van Vliet 1978). Van Hove (1957) gave several examples including that of electron-phonon interaction. In the method of statistical mechanics this is equivalent to (Fano 1957) projecting the Liouville operator and viewing the motion within a subspace of the whole Liouville space.

We have assumed that the individual nucleons of the nucleus move in some self-consistent field with sharp energy levels. The surface vibrations of the nucleus provide the external "bath". The interaction between the particle modes and the surface vibrations of the nucleus has been assumed to be of stochastic nature: (i) the statistical average of the interaction hamiltonian itself being zero, and (ii) the statistical average of the product of interaction at two different time depending on their time difference and decaying with relaxation time t_0. With these assumptions about the stochastic nature of interaction, the equation of motion for the density matrix of the nucleonic particle mode has been solved and the rate equation for the statistical average of the particle correlation function has been set up. These equations have the usual form of decay equations and the decay times for the particle occupation numbers give the levelwidths for the corresponding strength functions.

In the optical potential model the lifetime of the quasiparticle is directly related to the imaginary part of the optical potential. Jeukenne *et al* (1976) used Bruckner's theory of nuclear matter to evaluate this imaginary part and connected the experimental data on cross-sections with the single-particle spreading width. Our calculation is based on the coupling of the particle mode with the surface vibrational mode. This is the case of weak coupling and is valid for nuclei with spherical core (Bohr and Mottelson 1975). In the other case, that of strong coupling, the collective mode with which the particle mode is coupled is the rotational mode. This happens for nuclei with non-zero equilibrium deformation and the present analysis is not applicable there. Considering the importance of contributions of particle-vibration coupling to static nuclear properties like nuclear moments it is reasonable to assume (Bohr and Mottelson 1975) that this coupling is a major source for the origin of levelwidth in these nuclei.

We mention here that application of stochastic method to processes like reaction and decay is not new. Mantzouranis (1981) applied stochastic method to calculate the decay time of a compound nucleus in terms of optical model potential in overlapping resonances. The basic idea there was that the matrix elements of the interaction between the compound levels and the channels are random variables with a Gaussian distribution. Gross (1979) used Fokker-Planck formalism to incorporate fluctuations to account for widths of deep inelastic collisions of heavy ions, and Mukamel *et al* (1981) used statistical description in terms of rate equations to describe stochastic effects in these calculations.

2. The model

The particle hamiltonian is written in the form

$$H_0 = \sum_i \varepsilon_i a_i^+ a_i, \tag{1}$$

where ε_i's are the single particle energies and a_i^+ and a_i are the corresponding Fermion creation and annihilation operators satisfying anticommutation relations. The interaction of the particle mode with the surface vibrational mode is of the form

$$H'(t) = \sum_{\lambda\mu ij} G_{ij}^{\lambda\mu}(t)\, a_i^+ a_j (c_{\lambda\mu} + c_{\lambda\mu}^+), \tag{2}$$

where $c_{\lambda\mu}^+$ and $c_{\lambda\mu}$ are the creation and annihilation operators for the $\lambda\mu$-th vibrational mode (satisfying boson commutation relations and commuting with a_i's and a_i^+'s) and $G_{ij}^{\lambda\mu}$'s are the particle-vibration coupling constants. These latter quantities can be calculated in different models (Mottelson 1967). In case of weak coupling $G_{ij}^{\lambda\mu}$ has the form

$$G_{ij}^{\lambda\mu} = -(\hbar^2/4B_{\lambda\mu}C_{\lambda\mu})^{1/4} \langle \Psi_i | k(r) Y_\lambda^\mu(\theta, \phi) | \Psi_j \rangle; \tag{3}$$

here Ψ's are single particle nucleonic wavefunctions, $B_{\lambda\mu}$'s are inertia tensors and $C_{\lambda\mu}$'s are suitable spring constants. We shall use new operators and coupling constants defined by

$$b_{\lambda\mu} = (c_{\lambda\mu} + c_{\lambda\mu}^+)/\sqrt{2} = b_{\lambda\mu}^+, \quad V_{ij}^{\lambda\mu} = \sqrt{2}\, G_{ij}^{\lambda\mu} = V_{ji}^{\lambda\mu *},$$

with $\quad [b_{\lambda\mu}, b_{\lambda'\mu'}]_- = 0.$

The hermitean operators $b_{\lambda\mu}$'s are used to save space and no deeper significance is attached to them. The time-dependent interaction hamiltonian will now be of the form

$$H'(t) = \sum_{\lambda\mu ij} V_{ij}^{\lambda\mu}(t)\, a_i^+ a_j b_{\lambda\mu}. \tag{4}$$

The time dependence in V indicates the random fluctuating nature of the interaction discussed above.

3. Density matrix formalism

In this section we give a short resume of the method of density matrix when the system is subjected to a stochastic interaction with an external bath (Abragam 1961).

The statistical system is described by the density matrix $\rho(t)$, which in Heisenberg picture satisfies the Liouville equation

$$i\hbar \partial\rho/\partial t = [H, \rho]_- \text{ with } H = H_0 + H'(t). \tag{5}$$

In the Dirac picture any Heisenberg operator $A(t)$ has the form

$$A_D(t) = (\exp iH_0 t/\hbar) A(t) (\exp -iH_0 t/\hbar),$$

and we have the corresponding Liouville equation

$$i\hbar\partial\rho_D(t)/\partial t = [H'_D(t), \rho_D(t)]_- . \tag{6}$$

Since this has the formal solution

$$\rho_D(t) = \rho_D(0) + (i\hbar)^{-1}\int_0^t \mathrm{d}t'[H'_D(t'), \rho_D(t')]_- , \tag{7}$$

Equation (6) can be written in the form

$$i\hbar\partial\rho_D(t)/\partial t = [H'_D(t), \rho_D(0)]_- + (i\hbar)^{-1}\int_0^t \mathrm{d}t'\,[H'_D(t), [H'_D(t'), \rho_D(t')]_-]_- . \tag{8}$$

We take matrix elements of (8) over the many-particle states $|\alpha\rangle$ and are confronted with terms like

$$I_1 = \int_0^t \mathrm{d}t'\langle\alpha|H'_D(t)\rho_D(t')H'_D(t')|\alpha'\rangle \tag{9}$$

from the second member on the right side of (8). I_1 can be simplified to

$$\begin{aligned} I_1 &= \int_0^t \mathrm{d}t' \sum_{\gamma\gamma'} \langle\alpha|H'(t)|\gamma\rangle\langle\gamma|\rho_D(t')|\gamma'\rangle\langle\gamma'|H'(t')|\alpha'\rangle \\ &\quad\times [\exp i(\varepsilon_\alpha - \varepsilon_\gamma)t/\hbar][\exp i(\varepsilon_{\gamma'} - \varepsilon_{\alpha'})t'/\hbar] \\ &= \int_0^t \mathrm{d}t' \sum_{\gamma\gamma'} \langle\alpha|H'(t)|\gamma\rangle\langle\gamma'|H'(t-t')|\alpha'\rangle\langle\gamma|\rho_D(t-t')|\gamma'\rangle \\ &\quad\times [\exp -i(\varepsilon_{\gamma'} - \varepsilon_{\alpha'})t'/\hbar][\exp i(\varepsilon_\alpha - \varepsilon_\gamma + \varepsilon_{\gamma'} - \varepsilon_{\alpha'})t/\hbar]. \end{aligned}$$

At this stage we take ensemble average (denoted by overhead bar) of the equation and take into account the stochasttic nature of the interaction:

(i) $\overline{\langle\alpha|H'(t)|\beta\rangle} = 0$;

(ii) $\overline{\langle\alpha|H'(t)|\beta\rangle\langle\beta'|H'(t-t')|\alpha'\rangle}$

is independent of t, and tends to zero as $t' \gg t_c$, a correlation time;

(iii) $\rho_D(t-t') \approx \rho_D(t) \approx \rho_D(0)$.

The upper limit of integration in I_1 can now be extended to infinity, and we get in the Dirac picture

$$\partial\overline{\langle\alpha|\rho_D(t)|\alpha'\rangle}/\partial t = \sum_{\beta\beta'} R_{\alpha\alpha',\beta\beta'}\,\overline{\langle\beta|\rho_D(t)|\beta'\rangle}\,[\exp i(\varepsilon_\alpha - \varepsilon_\beta + \varepsilon_{\beta'} - \varepsilon_{\alpha'})t/\hbar], \tag{10a}$$

and in the Heisenberg picture

$$\partial\overline{\langle\alpha|\rho(t)|\alpha'\rangle}/\partial t = (i\hbar)^{-1}\,\overline{\langle\alpha|H_0, \rho(t)]_-|\alpha'\rangle} + \sum_{\beta\beta'} R_{\alpha\alpha',\beta\beta'}\,\overline{\langle\beta|\rho(t)|\beta'\rangle}, \tag{10b}$$

where the Redfield matrix $R_{\alpha\alpha',\beta\beta'}$ is given by

$$
\begin{aligned}
R_{\alpha\alpha',\beta\beta'} = (1/\hbar^2)\int_0^\infty dt' \{ & \overline{\langle\alpha|H'(t)|\beta\rangle\langle\beta'|H'(t-t')|\alpha'\rangle} \\
& \times [\exp - i(\varepsilon_{\beta'} - \varepsilon_{\alpha'})t'/\hbar] \\
& + \overline{\langle\beta'|H'(t)|\alpha'\rangle\langle\alpha|H'(t-t')\ |\beta\rangle}[\exp - i(\varepsilon_\alpha - \varepsilon_\beta)t'/\hbar] \\
& - \delta_{\alpha',\beta'}\sum_\gamma \overline{\langle\alpha|H'(t)|\gamma\rangle\langle\gamma|H'(t-t')|\beta\rangle}\,[\exp - i(\varepsilon_\gamma - \varepsilon_\beta)t'/\hbar] \\
& - \delta_{\alpha,\beta}\sum_\gamma \overline{\langle\gamma|H'(t)|\alpha'\rangle\langle\beta'|H'(t-t')|\gamma\rangle}\,[\exp - i(\varepsilon_{\beta'} - \varepsilon_\gamma)t'/\hbar]\}.
\end{aligned}
\tag{11}
$$

This is the famous Redfield equation (Abragam 1961) used for the calculation of relaxation times in nuclear resonance processes.

We can now write the rate equations for any set of operators $\langle M_i(t)\rangle$ as

$$
\begin{aligned}
& \partial[\langle M_i(t)\rangle - \langle M_i(\infty)\rangle]/\partial t \\
& = (\partial/\partial t)\sum_{\alpha\alpha'} \langle\alpha'|M_i(t) - \langle M_i(\infty)\rangle|\alpha\rangle\,\overline{\langle\alpha|\rho(t)|\alpha'\rangle} \\
& = (i\hbar)^{-1}\langle[M_i, H_0]_-\rangle + \sum_{\alpha\alpha'\beta\beta'} \langle\alpha'|M_i - \langle M_i(\infty)\rangle|\alpha\rangle \\
& \qquad\qquad \times R_{\alpha\alpha',\beta\beta'}\,\overline{\langle\beta|\rho(t)|\beta'\rangle},
\end{aligned}
\tag{12}
$$

where $\langle M_i(\infty)\rangle$ is the equilibrium value of $\langle M_i(t)\rangle$ at $t = \infty$. In the next section we apply this formalism to obtain the rate equations for the particle correlation functions and find that the second term on the right side of (12) can be written as linear combinations of particle correlation functions.

4. Mathematical analysis

In this section we use the model given in § 2 to calculate the Redfield matrix $R_{\alpha\alpha',\beta\beta'}$ of (11). We assume an exponentially correlated randomness of interaction hamiltonian of (4):

$$
\begin{aligned}
& \overline{V_{ij}^{\lambda\mu}(t)\,V_{i'j'}^{\lambda'\mu'}(t-t')b_{\lambda\mu}b_{\lambda'\mu'}} \\
& = \delta_{\lambda\mu,\lambda'\mu'}\,\delta_{ij,i'j'}\,\langle b_{\lambda\mu}^2\rangle\,|V_{ij}^{\lambda\mu}|^2\,[\exp - |t'|/t_0],
\end{aligned}
\tag{13}
$$

where t_0 is the decay time of the correlation of interactions at different times.

This form of the randomness of interaction corresponds to stationary Ornstein-Uhlenbeck stochastic process. In this process the probability $P(y, t)$ that the random variable $y(t)$ has the value y at time t, has the form

$$P(y, t) = (2\pi)^{-1/2}\exp(-y^2/2)$$

and the conditional probability $P(y_2, t_2/y_1, t_1)$ that the variable has value y_2 at t_2 given that the value is y_1 at t_1, has the form

$$P(y_2, t_2/y_1, t_1) = [2\pi(1-\exp(-2t))]^{-1/2} \times \exp - [(y_2 - y_1 e^{-t})^2/(2-2e^{-2t})]$$

where $t = t_2 - t_1 > 0$. Apart from changes in the scale of y and t, this is essentially the only stationary Gaussian Markov process (van Kampen 1978).

We now evaluate the Redfield matrix with the stochastic assumption of (13). If the form (4) of the interaction hamiltonian is inserted in (11) of the Redfield matrix, then with the stochastic assumption of (13) we get

$$\begin{aligned} R_{\alpha\alpha',\beta\beta'} = \sum_{ij} \overline{A_{ij}^2} (t_0/\hbar^2) \Bigg[& \frac{1 - i\omega_{\beta'\alpha'} t_0}{1 + \omega_{\beta'\alpha'}^2 t_0^2} \langle \alpha | a_i^+ a_j | \beta \rangle \langle \beta' | a_i^+ a_j | \alpha' \rangle \\ & + \frac{1 - i\omega_{\alpha\beta} t_0}{1 + \omega_{\alpha\beta}^2 t_0^2} \langle \beta' | a_i^+ a_j | \alpha' \rangle \langle \alpha | a_i^+ a_j | \beta \rangle \\ & - \delta_{\alpha',\beta'} \sum_{\gamma} \frac{1 - i\omega_{\gamma\beta} t_0}{1 + \omega_{\gamma\beta}^2 t_0^2} \langle \alpha | a_i^+ a_j | \gamma \rangle \langle \gamma | a_i^+ a_j | \beta \rangle \\ & - \delta_{\alpha,\beta} \sum_{\gamma} \frac{1 - i\omega_{\beta'\gamma} t_0}{1 + \omega_{\beta'\gamma}^2 t_0^2} \langle \gamma | a_i^+ a_j | \alpha' \rangle \langle \beta' | a_i^+ a_j | \gamma \rangle \Bigg], \end{aligned} \tag{14}$$

where

$$\overline{A_{ij}^2} = \sum_{\lambda\mu} | V_{ij}^{\lambda\mu} |^2 \langle b_{\lambda\mu}^2 \rangle, \quad \text{and} \quad \omega_{\alpha\beta} = (E_\alpha - E_\beta)/\hbar,$$

E_α being the energy of the state $|\alpha\rangle$ of the total system. Taking $\langle a_m^+ a_n \rangle$ as the operators whose rate equations we shall investigate, we get from (12) and (14) after some algebraic calculation:

$$\begin{aligned} \partial \langle a_m^+ a_n \rangle / \partial t &= (i\hbar)^{-1} \langle [a_m^+ a_n, H_0]_- \rangle \\ & \quad + \sum_{\alpha\alpha'\beta\beta'} [\langle \alpha' | a_m^+ a_n | \alpha \rangle - \langle \alpha' | a_m^+ a_n | \alpha \rangle_{t=\infty}] R_{\alpha\alpha',\beta\beta'} \overline{\langle \beta | \rho(t) | \beta' \rangle} \\ &= (i\hbar)^{-1} (\varepsilon_n - \varepsilon_m) \langle a_m^+ a_n \rangle \\ & \quad - (t_0/\hbar^2)(\overline{A_{mn}^2} + \overline{A_{nm}^2}) [\langle a_m^+ a_n \rangle - \langle a_m^+ a_n \rangle_{t=\infty}] \\ & \quad + (t_0/\hbar^2) \overline{A_{nm}^2} \frac{1 - it_0(\varepsilon_n - \varepsilon_m)/\hbar}{1 + t_0^2 (\varepsilon_n - \varepsilon_m)^2/\hbar^2} [\langle a_n^+ a_m \rangle - \langle a_n^+ a_m \rangle_{t=\infty}]. \end{aligned} \tag{15}$$

For $m = n$, we get the rate equation for the occupation number $N_m(t)$ for the m-th single particle state

$$\partial N_m(t)/\partial t = -\overline{A_{mm}^2} (t_0/\hbar^2) [N_m(t) - N_m(\infty)] \tag{16}$$

and $N_m(t)$ decays exponentially to $N_m(\infty)$ with decay time T_m,

where $$1/T_m = \overline{A_{mm}^2} t_0/\hbar^2. \tag{17}$$

The width of the m-th single particle state will be

$$\Gamma_m = \hbar/T_m = \overline{A_{mm}^2} t_0/\hbar. \tag{18}$$

We express $\overline{A^2_{mm}}$ in terms of the number of phonons excited

$$\overline{A^2_{mn}} = \sum_{\lambda\mu} |V^{\lambda\mu}_{mn}|^2 \langle b^2_{\lambda\mu} \rangle = \sum_{\lambda\mu} |V^{\lambda\mu}_{mn}|^2 [\langle c^+_{\lambda\mu} c_{\lambda\mu} \rangle + \tfrac{1}{2}]. \tag{19}$$

We can also write the solution for the single particle correlation function $\langle a^+_m a_n \rangle$, $m \neq n$, assuming that $\langle a^+_m a_n \rangle_{t=\infty} = 0$:

$$\langle a^+_m a_n \rangle = \langle a^+_m a_n \rangle_{t=0} [\exp - t/T_{mn}],$$

where
$$1/T_{mn} = (t_0/\hbar^2)[(\overline{A^2_{mm}} + \overline{A^2_{nn}}) \pm \{(\overline{A^2_{mn}})^2/[1 + t_0^2(\varepsilon_n - \varepsilon_m)^2/\hbar^2] - \hbar^2(\varepsilon_n - \varepsilon_m)^2/t_0^2\}^{1/2}]. \tag{20}$$

Here we have a possibility that

if
$$\hbar^2(\varepsilon_n - \varepsilon_m)^2/t_0^2 > (\overline{A^2_{mn}})^2/[1 + t_0^2(\varepsilon_n - \varepsilon_m)^2/\hbar^2],$$

i.e if
$$2(\varepsilon_n - \varepsilon_m)^2 t_0^2/\hbar^2 > (1 + 4t_0^4(\overline{A^2_{mn}})^2/\hbar^4)^{\frac{1}{2}} - 1, \tag{21}$$

then the single particle correlation function has an oscillatory decay form:

$$\langle a^+_m a_n \rangle = \langle a^+_m a_n \rangle_{t=0} [\exp(i\omega_{mn} t - t/T'_{mn})]$$

with the decay time T'_{mn} given by

$$1/T'_{mn} = (\overline{A^2_{mm}} + \overline{A^2_{nn}}) t_0/\hbar^2$$

and the oscillatory frequency ω_{mn} given by

$$\hbar\omega_{mn} = \{(\varepsilon_n - \varepsilon_m)^2 - (\overline{A^2_{mn}} t_0/\hbar)^2/[1 + t_0^2(\varepsilon_n - \varepsilon_m)^2/\hbar^2]\}^{1/2}. \tag{22}$$

5. Numerical estimation and discussion

From (19) we find that the width of the levels depends on the number of phonons present. It is also to be noted that even when no phonon is present the level has an intrinsic width. This state of affairs is comparable to the case of spontaneous decay probability in atomic levels giving rise to intrinsic width in atomic levels. We restrict ourselves to this case of approximation in calculating widths for low lying particle states.

To get numerical values for the levelwidth we require knowledge of $\overline{A^2_{mm}}$ through $V^{\lambda\mu}_{ij}$ and an estimate of t_0. We take the case of ${}^{207}_{82}Pb_{125}$ which has neutron hole states as single particle states. We assume that the single particle states are the (nlj)-states of Woods-Saxon potential (Blomqvist and Wahlborn 1959) for which experimental values of levelwidth Γ_{nlj} are available (Bortignon and Broglia 1981). The total width Γ_{nlj} is the sum of partial widths of the different azimuthal quantum numbers. So, for the levelwidth, we get the expression

$$\Gamma_{nlj} = (t_0/4\pi\hbar) \sum_{\lambda = \text{even}} (\hbar\omega_\lambda/2C_\lambda)|(2j+1)^{\frac{1}{2}} \langle nlj|k(r)|nlj \rangle|^2 \times |[4\pi/(2\lambda+1)]^{1/2} \langle l\tfrac{1}{2}j \| Y_\lambda \| l\tfrac{1}{2}j \rangle|^2,$$

where
$$[4\pi/(2\lambda+1)]^{1/2}\langle l\tfrac{1}{2}j\|Y_\lambda\|l\tfrac{1}{2}j\rangle = (-1)^{l+\lambda/2}\frac{(l+\lambda/2)!\lambda!}{(l-\lambda/2)!(\lambda/2)!(\lambda/2)!}\left(\frac{(2l+1)(2l-\lambda)!}{(2l+\lambda+1)!}\right)^{1/2} \times\left[\delta_{j,l+1/2}\left(\frac{(2j+1+\lambda)(2j-\lambda)}{2j(2j+1)}\right)^{1/2} + \delta_{j,l-1/2}\left(\frac{(2j+2+\lambda)(2j+1-\lambda)}{(2j+1)(2j+2)}\right)^{1/2}\right]. \tag{23}$$

With an adiabatic assumption for the relative orders of the vibrational and other energies Bertsch *et al* (1983) obtained a similar expression for the variance of damping of single particle states by vibrational excitations. Our analysis shows that this expression is of more general applicability.

The radial integration $\langle nlj|k(r)|nlj\rangle$ is done with the nucleonic wavefunctions of Blomqvist and Wahlborn (1959) and the coupling strength $k(r)$ of Bortignon and Broglia (1981). The summation over even λ in (23) has been restricted only to quadrupole mode. Since $\hbar\omega_2$ and $2C_2$ was not available for ${}^{207}_{82}Pb_{125}$, we have treated $(t_0/4\pi\hbar)(\hbar\omega_2/2C_2)$ as the adjustable parameter and obtained a value $2{\cdot}2502\times10^{-1}$ $(MeV)^{-1}$ for it when the expression of Γ_{nlj} for the $1i^{-1}_{13/2}$ state has been equated to the corresponding experimental value of 3·6 MeV (Bortignon and Broglia 1981). If we take the value of $(\hbar\omega_2/2C_2)$ as that (Broglia *et al* 1970) corresponding to the neighbouring nucleus ${}^{209}_{83}Bi_{126}$ which is $(2{\cdot}5\times10^{-2})^2$ we get an estimate of t_0 as $2{\cdot}98\times10^{-21}$ sec. The results of calculation are given in table 1. For the calculation of $\langle nlj|k(r)|nlj\rangle$ for the level $1h^{-1}_{11/2}$ we have taken the same radial wavefunction as that for $1h^{-1}_{9/2}$, since the radial wavefunctions are not very sensitive to j. The calculated value for $1h^{-1}_{9/2}$ level is considerably different from the experimental value. This, we believe, is because $1h^{-1}_{9/2}$ level is not a pure single particle state, having a considerable admixture of octupole vibration, and any calculation with pure single particle state is certain to be in error.

It is to be noted that if the single particle states have single l-values then the expressions for Γ_{nlj} in (23) have contributions only from vibrations of even polarity, thus excluding any contribution from octupole vibrations. Because of the coupling

Table 1. Calculated and experimental values of Γ_{nlj}, the levelwidth for different Woods-Saxon states.

State[a] (nlj)	Energy[a] (MeV)	$\sqrt{\frac{4\pi}{5}}\langle l\frac{1}{2}j\|Y_2\|l\frac{1}{2}j\rangle$	$-\langle nlj\|k(r)\|nlj\rangle$ (MeV)	Γ_{nlj} (MeV) Calc.	Exp.[a]
$3p^{-1}_{1/2}$	0·0	0	—	0·0	—
$2f^{-1}_{5/2}$	0·57	$(8/35)^{1/2}$	54·776	0·93	—
$3p^{-1}_{3/2}$	0·89	$(1/5)^{1/2}$	53·363	0·51	—
$1i^{-1}_{13/2}$	2·39	$-(16/65)^{1/2}$	68·136	3·6	3.6
$2f^{-1}_{7/2}$	3·35	$(5/21)^{1/2}$	55·600	1·3	1·2
$1h^{-1}_{9/2}$	4·83	$(8/33)^{1/2}$	47·015	1·3	3·0
$1h^{-1}_{11/2}$	10·25	$(35/143)^{1/2}$	47·015	1·5	1·75

[a] Bortignon and Broglia (1981)

itself the states themselves with fixed j become admixtured with different orbital angular momentum states. This is a separate problem from the one treated here. If such admixtures are taken into account then vibrations of odd polarity will contribute to Γ_{nlj}. We believe this accounts for the disagreement between our calculated and experimental values for the $1h_{9/2}^{-1}$ state. The agreement in the other cases and the fact that the estimated value of t_0 is in the proper nuclear time scale, show that the model presented here is of great practical value, provided the coupling strength $k(r)$ and the single particle wavefunctions are known.

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