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Application of current algebra and partial conservation of axial-vector current to kaon-proton interactions

K KRISHNA MURTHY, M L P RAO and PRATHIBHA NUTHAKKI
Department of Physics, Andhra University, Waltair 530 003, India

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Abstract. A soft-pion study using current algebra and the partially conserved axial vector current (PCAC) hypothesis is made of the kaon-proton interaction process $Kp \to Kp2\pi^0$. Considering both pions to be soft, the differential rate for the process is normalized to the differential rate of the corresponding process without the pions. Theoretical predictions for the ratio of cross-sections at various kaon momenta are compared with experimental results.

Keywords. Current algebra; partially conserved axial-vector current hypothesis; kaon-proton interactions; soft pions.

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1. Introduction

In current algebra, the vector and axial vector currents of hadrons are identified as physically observable quantities manifesting the broken chiral SU(3) × SU(3) symmetry. The symmetry is expressed in terms of the equal time commutators of associated charges and currents. That the symmetry is only approximate is given by the partially conserved axial-vector current (PCAC) hypothesis. PCAC hypothesis also enables us to derive scattering amplitudes containing pions from matrix elements of currents.

The current algebra-PCAC formalism has been successful in explaining the experimental results for various processes involving soft-pions. During the past decade, the formalism has been applied to study strong interaction processes, such as $\bar{p}p$ annihilations (Uritam 1972; Nuthakki and Uritam 1973; Greenhut and Intemann 1976; Purushottamudu and Nuthakki 1982; Nuthakki and Purushottamudu 1983). $\bar{p}p$ annihilation occurs in S state and current algebra has been particularly successful in S state. The other strong interaction processes, which occur in S state and can be studied within the formalism, are scattering of kaons by nucleons at low energies. A general feature of the applications of current algebra is that a reaction is related to the ones with one or more additional soft pions. As in the case of $\bar{p}p$ annihilations, the experimental data for kaon-proton interactions are copious and particularly suitable for the application of the formalism. Therefore we consider the process $K^{\pm}p \to K^{\pm}p\,2\pi^0$ and treat both the pions as soft. Taking the pions in the neutral mode has the advantage that one needs to consider gradient coupling only as pointed out by Weinberg (1966). Like Uritam (1972), we also subscribe to van Hove's point of view that the theoretical expressions should be in the most differential form possible rather than have some or all the variables integrated over. So we present the differential rate for the process in all the relevant kinematic variables and normalize it to the differential rate for the process without the pions. In order to make a comparison with experimental results, we finally integrate the resulting expressions numerically over the independent variables to obtain the ratio of cross-sections for various kaon momenta.

2. Amplitude and differential rate

The matrix element for the emission of two pions in the reaction

$$i \to f + \pi^{\alpha}(k_1) + \pi^{\beta}(k_2), \tag{1}$$

is given by the Lehmann, Symanzik and Zimmerman reduction formula to be proportional to

$$\int \mathrm{d}^4x\, \mathrm{d}^4y \exp(-ik_1x)\exp(-ik_2y) \langle f | T[\phi_\pi^\alpha(x), \phi_\pi^\beta(y)] | i \rangle. \tag{2}$$

i and f are the initial and final states containing particles such as K^+, K^- and p. k_1 and k_2 are the momenta of the pions with isospin indices α and β respectively. According to PCAC

$$\partial_\mu A_\mu^\alpha = (C_\pi/\sqrt{2})\,\phi_\pi^\alpha, \tag{3}$$

where $C_\pi = \sqrt{2}\, G_A M_N \mu^2/g_r(0)$; G_A ($\simeq 1{\cdot}18$) is the axial-vector coupling constant, g_r is the rationalized, renormalized pion-nucleon coupling constant ($g_r^2/4\pi \simeq 14{\cdot}6$). ϕ_π^α is the renormalized pion field operator and μ and M_N are the pion and nucleon masses respectively. PCAC given in (3) enables us to rewrite (2) in terms of the divergence of the axial-vector currents A_μ. Bringing the derivatives out of the time-ordered products gives rise to commutators like

$$\delta(x_0 - y_0)[A_0^\alpha(x), \partial_\mu A_\mu^\beta(y)] = \delta_{\alpha\beta}\sigma(x)\,\delta(x-y). \tag{4}$$

These are dropped since they are of the same order as the PCAC correction terms. Integrating the resulting expression by parts, introducing Klein-Gordon operators and again using PCAC we have

$$k_1^\mu k_2^\nu R_{\mu\nu}^{\alpha\beta} = \frac{i}{2}\frac{C_\pi^2}{(\mu^2 + k_1^2)(\mu^2 + k_2^2)} R_{2\pi}^{\alpha\beta} + \frac{i}{2}\varepsilon_{\alpha\beta\gamma}(k_2 - k_1)^\lambda R_\lambda^\gamma, \tag{5}$$

where we have used the relation

$$\delta(x_0 - y_0)[A_0^\alpha(x), A_\mu^\beta(y)] = i\delta(x-y)\varepsilon_{\alpha\beta\gamma} V_\mu^\gamma(x), \tag{6}$$

and

$$R_{\mu\nu}^{\alpha\beta} = i \int \mathrm{d}^4x\, \mathrm{d}^4y \exp(-ik_1x)\exp(-ik_2y) \langle f | T[A_\mu^\alpha(x), A_\nu^\beta(y)] | i \rangle. \tag{7}$$

The expression in (7) is the matrix element for the emission of two axial-vector currents in the process $i \to f$. Pulling out the derivatives from the time-ordered product symmetrically results in the factor 1/2 on the right hand side.

$$R_{2\pi}^{\alpha\beta} = -\int d^4x\, d^4y \exp(-ik_1x)\exp(-ik_2y)(\mu^2-\Box_x)(\mu^2-\Box_y) \times \langle f|T[\phi_\pi^\alpha(x), \phi_\pi^\beta(y)]|i\rangle, \tag{8}$$

is the matrix element for the emission of two pions of momenta k_1 and k_2 and isospins α and β in the process $i \to f$.

$$R_\lambda^\gamma = \int d^4x \exp(-i(k_1+k_2)x)\langle f|\, V_\lambda^\gamma(x)|i\rangle, \tag{9}$$

is the matrix element for the emission of an isovector photon in the process $i \to f$. When both pions are in the same isospin state, this term can be dropped. We have

$$S_{ab} = \delta_{ab} + iR_{ab} = \delta_{ab} + i(2\pi)^4\delta(p_a - p_b)M_{ab}/\sqrt{N_{ab}}. \tag{10}$$

Expression (5) now becomes

$$k_1^\mu k_2^\nu M_{\mu\nu}^{\alpha\beta} = \frac{i}{2}\cdot\frac{C_\pi^2}{(\mu^2+k_1^2)(\mu^2+k_2^2)} M_{2\pi}^{\alpha\beta} + \tfrac{1}{2}\varepsilon_{\alpha\beta\gamma}(k_2-k_1)^\lambda M_\lambda^\gamma. \tag{11}$$

We now set $k_1 = \xi K_1$ and $k_2 = \xi K_2$, so that $\xi \to 0$ corresponds to the soft pion limit. For a process like $\alpha \to \beta + m\pi$ where α and β are different hadronic states, the matrix element is of zeroth order in ξ (Dashen and Weinstein 1969). Therefore to evaluate (11) we look for pole terms that go as k^{-2} in $M_{\mu\nu}^{\alpha\beta}$. M_λ^γ must have pole terms that go as k^{-1}. In evaluating the isovector term one can use the low energy theorem for photon emission due to Low (1958).

Considering neutral pion emission and setting $\alpha = \beta = 3$, the expression in (11) reduces to

$$k_1^\mu k_2^\nu M_{\mu\nu}^{33} = \frac{i\,C_\pi^2}{2\,\mu^4} M_{2\pi}^{33}. \tag{12}$$

The diagrams that give rise to pole terms of order k^{-2} are given in figure 1 (Adler 1965). The central interaction in the diagrams can be written as

$$m = A + B\gamma\cdot Q, \tag{13}$$

where $Q = q_1 + q_2$, the sum of the kaon momenta and A, B are scalar functions of Q^2, K^2 and $Q\cdot K$ with $K = p_1 + p_2$. p_1 and p_2 are the respective initial and final proton momenta. We retain only the zeroth order terms in pion momenta in $k_1^\mu k_2^\nu M_{\mu\nu}^{33}$. Also to facilitate computation we omit the terms proportional to A (Nuthakki and Uritam 1978). This leads to

$$M_{2\pi}^{33} = \frac{-2iBG_A^2\mu^4}{C_\pi^2}\bar{u}(p_2)\left\{\frac{1}{(a+b)a}[-M_N^2 \not{Q}\not{k}_2\not{k}_1 + iM_N b\not{Q}\not{k}_1 - iM_N a\not{Q}\not{k}_2 - ab\not{Q}] + \frac{1}{(a+b)b}[-M_N^2\not{Q}\not{k}_1\not{k}_2 + iM_N a\not{Q}\not{k}_2 - iM_N b\not{Q}\not{k}_1 - ab\not{Q}]\right.$$

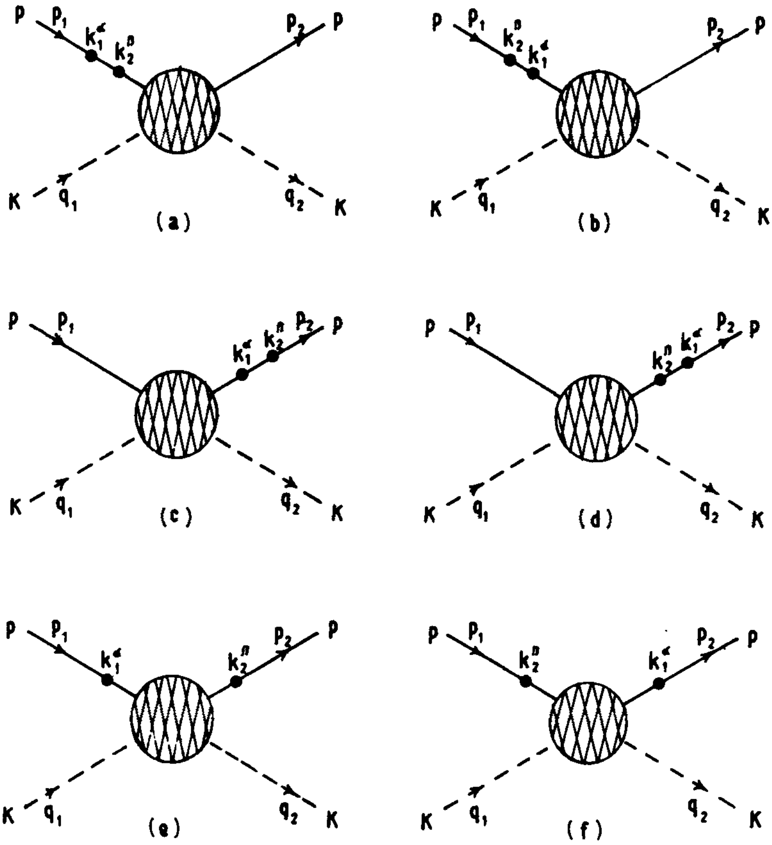

Figure 1. Diagrams of order k^{-2} contributing to $M^{33}_{\mu\nu}$.

$$+\frac{1}{(c+d)d}[-M_N^2\not{k}_2\not{k}_1\not{Q}+iM_Nc\not{k}_2\not{Q}-iM_Nd\not{k}_1\not{Q}-cd\not{Q}]$$

$$+\frac{1}{(c+d)c}[-M_N^2\not{k}_1\not{k}_2\not{Q}+iM_Nd\not{k}_1\not{Q}-iM_Nc\not{k}_2\not{Q}-cd\not{Q}]$$

$$+\frac{1}{da}[-M_N^2\not{k}_2\not{Q}\not{k}_1-iM_Nd\not{Q}\not{k}_1-iM_Na\not{k}_2\not{Q}+ad\not{Q}]$$

$$+\frac{1}{bc}[-M_N^2\not{k}_1\not{Q}\not{k}_2-iM_Nc\not{Q}\not{k}_2-iM_Nb\not{k}_1\not{Q}+bc\not{Q}]\Bigg\}u(p_1), \quad (14)$$

where $a = p_1\cdot k_1$, $b = p_1\cdot k_2$, $c = p_2\cdot k_1$ and $d = p_2\cdot k_2$. The differential cross-section for the reaction $Kp \to Kp + 2\pi^0$ is given by the expression

$$(\mathrm{d}\sigma)^{2\pi^0} = \frac{(2\pi)^4 M_N^2}{2[(p_1\cdot q_1)^2 - m_K^2 M_N^2]^{1/2}}\langle|M^{33}_{2\pi}|^2\rangle\frac{1}{(2\pi)^{12}}\frac{d^3q_2d^3p_2d^3k_1d^3k_2}{8p_2^0q_2^0k_1^0k_2^0}$$
$$\times\,\delta(p_1+q_1-p_2-q_2-k_1-k_2). \quad (15)$$

Finding exact $\langle |M_{2\pi}^{33}|^2 \rangle$ from direct trace calculations is a formidable task. Some approximation has to be made to simplify the calculations. A tedious calculation leads to $\langle |M_{2\pi}^{33}|^2 \rangle$ in the limit $k_1 = k_2 = (\vec{0}, i\mu)$. The resulting expression is given in Krishna Murthy and Nuthakki (1985).

We consider $Kp(p_1 + q_1 = R)$ system decaying into $Kp(p_2 + q_2 = R_1)$ and $2\pi(k_1 + k_2 = R_2)$. R_1 and R_2 themselves decay into a kaon and a proton and two pions respectively. Out of the twelve variables in (15) four can be integrated over trivially and the remaining eight variables are chosen to be the following: $m_{Kp}^2 = -R^2, m_{2\pi}^2 = -R_2^2$, θ_p (the angle between R_2 and p_1), θ_K (the angle between p_2 and R_1), θ_π (the angle between k_2 and R_2), ϕ_1 (the azimuthal angle between the R and R_2 frames in the R_1 rest frame), ϕ_2 (the azimuthal angle between the R and R_1 frames in the R_2 rest frame), ϕ_3 (the azimuthal angle between R_2 and R_1 frames in the R rest frame). In terms of these variables the differential cross-section for the process $Kp \to Kp + 2\pi^0$ is given by

$$
\begin{aligned}
(\mathrm{d}\sigma)^{2\pi^0} = {} & \frac{M_N^2}{2[(p_1 \cdot q_1)^2 - m_K^2 M_N^2]^{1/2}} \frac{\langle |M_{2\pi}^{33}|^2 \rangle}{(2\pi)^8} \times d^2 m_{Kp} \times d^2 m_{\pi\pi} \\
& \times d(\cos\theta_p) d(\cos\theta_K) d(\cos\theta_\pi)\, d\phi_1\, d\phi_2\, d\phi_3 \\
& \times \frac{1}{8M_{Kp}^2} [(M_{Kp}^2 + m_{\pi\pi}^2 - m_{Kp}^2)^2 - 4m_{\pi\pi}^2 M_{Kp}^2]^{1/2} \\
& \times \frac{1}{4m_{Kp}^2} [(m_{Kp}^2 + M_N^2 - m_K^2)^2 - 4M_N^2 m_{Kp}^2]^{1/2} \\
& \times \frac{1}{8m_{\pi\pi}} (m_{\pi\pi}^2 - 4\mu^2)^{1/2}.
\end{aligned} \tag{16}
$$

The variables in $\langle |M_{2\pi}^{33}|^2 \rangle$ have to be expressed in terms of m_{Kp}^2, $m_{\pi\pi}^2$ and the various angles given in (16). The relevant equations are given in Krishna Murthy and Nuthakki (1985).

The unknown parameter B in (14) can be eliminated if we normalize the $(\mathrm{d}\sigma)^{2\pi^0}$ to the differential cross-section for the process $Kp \to Kp$ where soft pions are absent. The relevant expression is

$$
\begin{aligned}
\mathrm{d}\sigma = {} & \frac{M_N^2 \pi}{[(p_1 \cdot q_1)^2 - m_K^2 M_N^2]^{1/2}} \frac{\langle |M|^2 \rangle}{4(2\pi)^2} d(\cos\theta_{p_2}) \\
& \times \frac{1}{M_{Kp}^2} [(M_{Kp}^2 + M_N^2 - m_K^2)^2 - 4M_N^2 M_{Kp}^2]^{1/2},
\end{aligned} \tag{17}
$$

where

$$
\langle |M|^2 \rangle = \tfrac{1}{2} \mathrm{Tr} \left[m \frac{M_N - i\not{p}_1}{2M_N} \gamma_4 m^\dagger \gamma_4 \frac{M_N - i\not{p}_2}{2M_N} \right]. \tag{18}
$$

Again neglecting the terms proportional to A,

$$
\langle |M|^2 \rangle = \frac{-B^2}{2M_N^2} (M_N^2 Q \cdot Q + Q \cdot Q p_1 \cdot p_2 - 2Q \cdot p_1 Q \cdot p_2). \tag{19}
$$

3. Comparison of results

The expressions given in (16) and (17) are valid only for low energies near the threshold, since the pions produced have to be soft. However, for comparison with experimental data, we generalize the expressions to hold at higher centre of mass energies as well. The theoretical predictions obtained after numerical integration for various kaon lab momenta are given in table 1. The results are compared with the experimental data (Rader *et al* 1973; De Bellefon *et al* 1972, 1977; London *et al* 1966; Dauber *et al* 1967a, b; CERN Report 1979) in figure 2. The variation of differential cross-section with respect

Table 1. Calculated ratios of cross-sections for various centre of mass energies.

$K^{\pm}$ lab momenta GeV/c	Centre of mass energies (GeV)	$\frac{\sigma(Kp \to Kp + 2\pi^0)}{\sigma(Kp \to Kp)}$
1·7	2·110	0·0368
1·8	2·152	0·0708
2·0	2·235	0·1946
2·24	2·331	0·4877
2·53	2·443	1·1599
2·72	2·513	1·8209

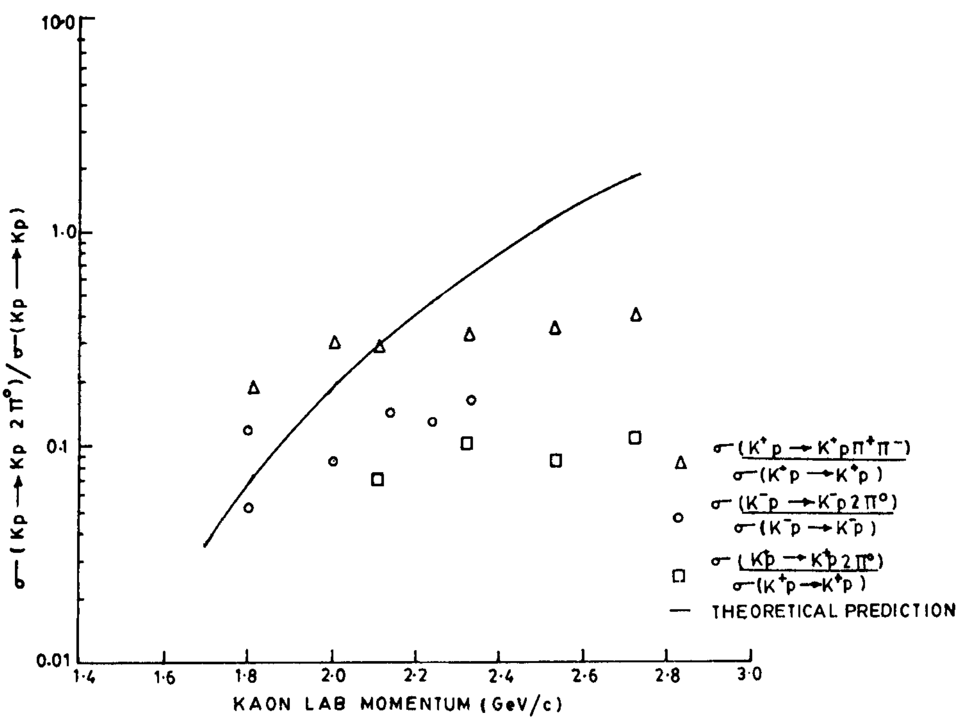

Figure 2. Comparison of theoretical predictions of $\sigma(Kp \to Kp + 2\pi^0)/\sigma(Kp \to Kp)$, with experiment at various kaon laboratory momenta.

to θ_p and m^2_{Kp} is also calculated for different kaon lab momenta and shown in figures 3 and 4. Unfortunately, here, no experimental data is available for comparison.

At low energies where our calculations are meaningful, the theoretical values agree reasonably well with the experimental results. In the kaon-nucleon interaction, especially in the K^-p interaction there are various resonances in the low energy region. One has to subtract the 'on-shell' and 'off-shell' resonances from the data, to get a meaningful comparison of the experimental and theoretical results. The experimental data vary considerably from group to group and therefore no subtraction of 'on-shell' resonances has been possible. The consideration of 'off-shell' resonances forms a separate study. In the present calculations, soft-pion formalism is applied in a straightforward way and no 'on-shell' and 'off-shell' resonances were therefore dealt

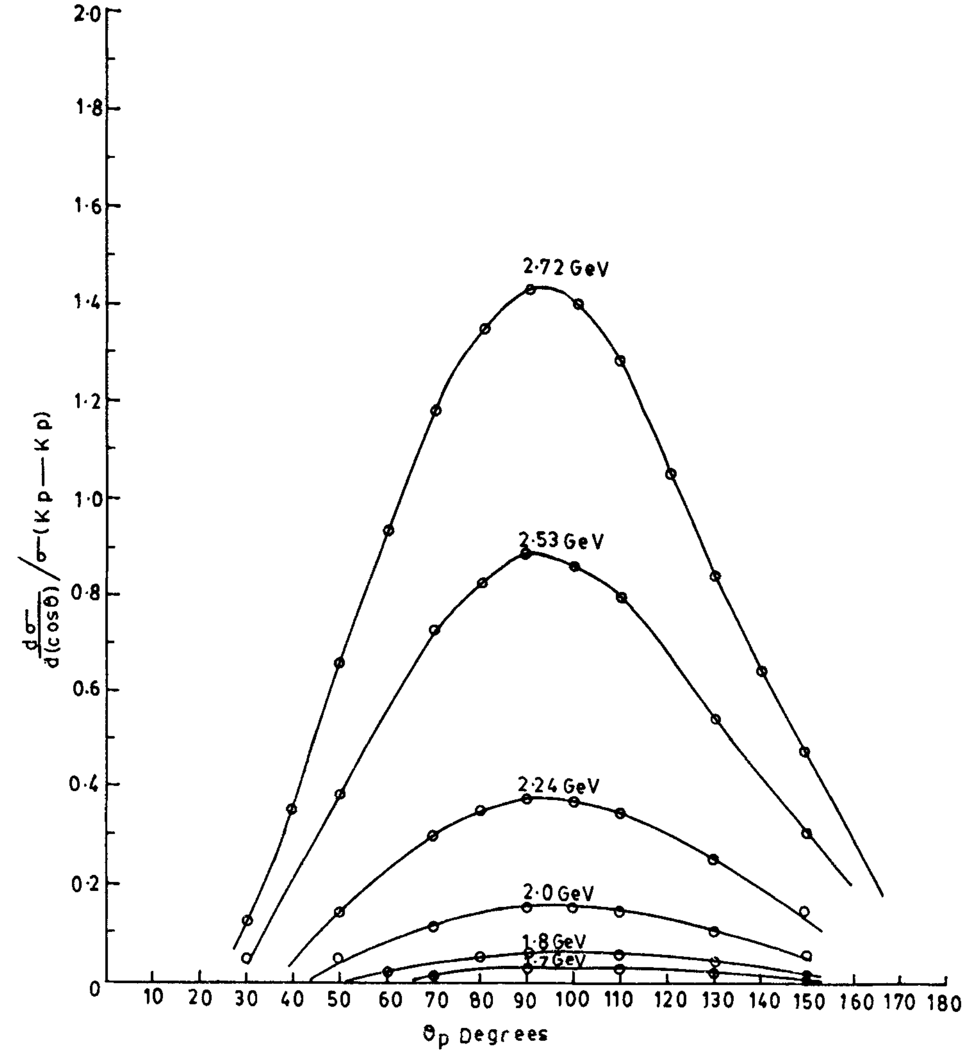

Figure 3. Variation of differential cross-section with θ_p, the angle between the incoming proton and the dipion system.

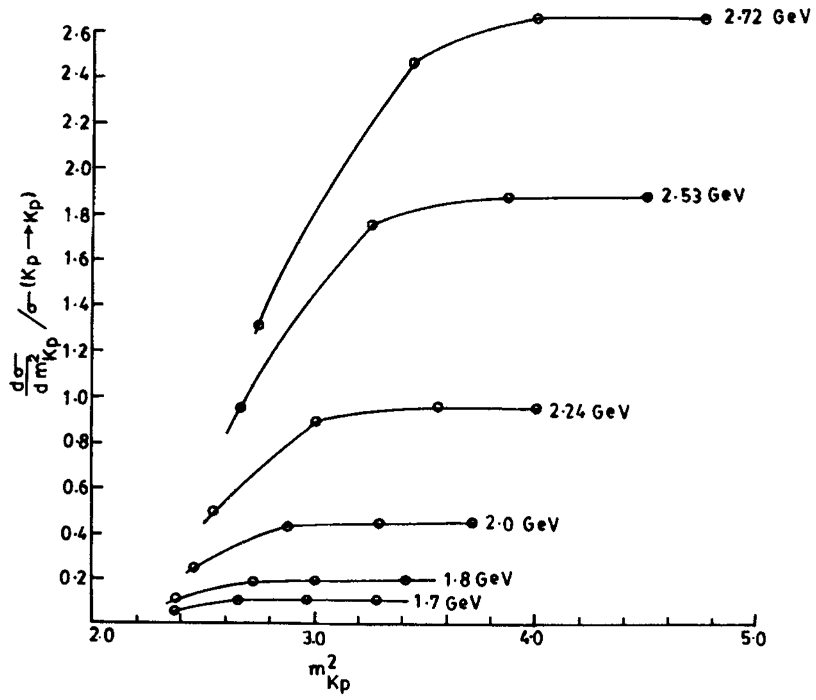

Figure 4. Variation of differential cross-section with m^2_{Kp}, the invariant rest mass of the final kaon-proton system.

with. However, it is worth noting that inclusion of Δ-post emission and nucleon-pre-emission diagrams in the study of $NN \to NN\pi$ by Dubach *et al* (1980) removed the discrepancy only partially. Therefore, it is not clear whether the inclusion of resonances in the Kp system would have the desired effect.

The conclusion is that broken chiral symmetry enables us to make reasonable theoretical predictions for strong processes such as $\bar{p}p$ annihilations and Kp interactions involving pions.

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References

Adler S L 1965 *Phys. Rev.* **B139** 1638
CERN Report 1979 Compilation of cross-sections II-K^- and K^+ induced reactions, HERA 79-02
Dashen R F and Weinstein M 1969 *Phys. Rev.* **183** 1261
Dauber P M, Malamud E I, Schlein P E, Slater W E and Stork D H 1967a *Phys. Lett.* **B24** 525
Dauber P M, Schlein P E, Slater W E and Ticho H K 1967b *Phys. Rev.* **153** 1403

De Bellefon A *et al* 1972 *Nuovo Cimento* **A7** 567

De Bellefon A, Berton A, Billoir P, Brunet J M, Tristram G, Varna J, Baccari B, Poulard G, Revel D and Tallini B 1977 *Nuovo Cimento* **A41** 451

Dubach J, Silbar R R and Kloet W M 1980 *Phys. Rev.* **D22** 2761

Greenhut G K and Intemann G W 1976 *Phys. Rev.* **D14** 764

Krishna Murthy K and Nuthakki P 1985 Emission of two neutral soft-pions in kaon-proton interactions, Andhra University Report No. PHEP-01/85

London G W, Rau R R, Samios N P, Yamamoto S S, Goldberg M, Lichtman S and Prime M 1966 *Phys. Rev.* **143** 1034

Low P E 1958 *Phys. Rev.* **110** 974

Nuthakki P and Purushottamudu R 1983 *Phys. Rev.* **D27** 650

Nuthakki P and Uritam R A 1973 *Phys. Rev.* **D8** 3196

Nuthakki P and Uritam R A 1978 *Phys. Rev.* **D18** 2387

Purushottamudu R and Nuthakki P 1982 *Z. Phys.* **C14** 257

Rader R *et al* 1973 *Nuovo Cimento* **A16** 178

Uritam R A 1972 *Phys. Rev.* **D6** 3233

Weinberg S 1966 *Phys. Rev. Lett.* **16** 879