Pramāṇa, Vol. 26, No. 5, May 1986, pp. 395–398. © Printed in India.

Low energy scattering data and phase shifts

M Z RAHMAN KHAN and M SHOEB

Department of Physics, Aligarh Muslim University, Aligarh 202001, India

MS received 24 January 1986

Abstract. We point out that at any given low energy σ_{tot} and a ratio of integrated scattering data like F/B can, in principle, be used unambiguously to find s- and p-wave phase shifts. Thus efforts to obtain other low energy data like elastic $d\sigma/d\Omega$ and P/E are unnecessary. It is also indicated that the mere knowledge whether F/B is greater than or less than unity enables us to draw important conclusions about the nature of the interaction in the p-state without performing detailed calculations. Thus a strong case is made out for obtaining much more precise F/B data than are presently available. The discussion refers mainly to low energy Λp scattering data.

Keywords. Λp scattering; phase shift; F/B data; p-state interaction.

PACS No. 13·75; 25·80

1. Introduction

Low energy Λp scattering data (Alexander *et al* 1968; Sechi Zorn *et al* 1968) have been analyzed in terms of the effective range parameters (e.g. Ali *et al* 1967; Alexander *et al* 1968; Sechi Zorn *et al* 1968) which are found to have a wide range of variation. To narrow down the variation other analyses (e.g. Bamberger *et al* 1973; Dalitz *et al* 1972; Nagel *et al* 1977, 1979) have included, besides scattering data, other data like ground and excited states of s-shell hypernuclei, etc. Still the parameters have a fairly wide range.

The available low energy data are the total elastic scattering cross-section σ_{tot}, the ratio F/B of the integrated differential elastic scattering in the forward and the backward hemispheres and P/E, the polar to equitorial ratio.

Here we briefly discuss how σ_{tot} and F/B may be used to obtain, at a given low energy, s- and p-wave phase shifts. An almost exact analytical formula has been derived for the p-wave phase shift. It then follows that the full elastic differential cross-section, which is likely to be poorer in accuracy compared to the ratio of the integrated quantities like F/B or P/E, or the polar-to-equitorial ratio, P/E, are not likely to add anything to the information. One could as well have used any other ratio of integrated differential elastic scattering in two suitable regions like P/E then F/B and $d\sigma/d\Omega$ would be redundant. Here we have considered F/B.

If we look at the formula for F/B in terms of the phase shifts, it follows that when F/B is greater then unity p-phase shift is positive and when the ratio is less than unity it is negative. Thus, if reliable F/B data are available it could be safely concluded without performing detailed calculations whether the p-wave interaction is effectively attractive or repulsive. However, the present $\Lambda\, p$ scattering data are not reliable enough to arrive at a definite conclusion in this regard. Therefore, it is suggested that attention may be focussed on obtaining more reliable σ_{tot} and F/B data, the great experimental difficulties notwithstanding.

In the next section we develop our arguments and apply these mainly to $\Lambda\, p$ system at low energies.

2. Formulae, discussion and results

The ratio F/B of the integrated elastic scattering in the forward and the backward hemi-spheres for a spin-independent interaction, upto $l = 2$ partial wave, is given by

$$F/B = \frac{\sin^2\delta_0 + 3\sin\delta_0\sin\delta_1\cos(\delta_0-\delta_1) + 3\sin^2\delta_1 + 15/4\sin\delta_1\sin\delta_2\cos(\delta_1-\delta_2) + 5\sin^2\delta_2}{\sin^2\delta_0 - 3\sin\delta_0\sin\delta_1\cos(\delta_0-\delta_1) + 3\sin^2\delta_1 + 15/4\sin\delta_1\sin\delta_2\cos(\delta_1-\delta_2) + 5\sin^2\delta_2}, \tag{1}$$

where δ_l is the phase shift of the lth partial wave.

For purposes of estimation we take CM energy (E_{cm}) to be 17·7 MeV, the highest scattering energy of the F/B data (Alexander *et al* 1968). At this and lower energies it is easily seen that δ_1 and higher phase shifts make negligible contribution to σ_{tot}. Thus we have

$$\sin^2\delta_0 = \frac{k^2\sigma_{tot}}{4\pi}. \tag{2}$$

For the incident energy considered, we get $\delta_0 = 0{\cdot}453$ rad., taking the positive sign for δ_0 since $\Lambda\, p$ interaction is certainly attractive in the s-state, as otherwise hypernuclei would not be bound entities at all. Phase shifts δ_1 and δ_2 are estimated, for a typical range ($= 1{\cdot}0$ fm) of $\Lambda\, N$ force, from formulae given in the standard literature (e.g. Schiff). Using these, we get, at the incident energy considered, the following numerical estimates for δ_1 and δ_2

$$\delta_1 \approx 0{\cdot}023 \text{ rad}; \quad \delta_2 \approx 0{\cdot}00044 \text{ rad}.$$

The F/B calculated with these values of the phase shifts is in fairly good agreement with the data.

Using these in the formula for F/B, we find that complete neglect of δ_2 terms and neglect of $\sin^2\delta_1$ terms result in an error in F/B of less than 0·25 %. Keeping in mind the possible experimental accuracies, even in the future when these are much improved, we can safely neglect these terms in the expression of F/B. It may, therefore, be written as

$$F/B = \frac{\sin^2\delta_0 + 3\sin\delta_0\sin\delta_1\cos(\delta_0-\delta_1)}{\sin^2\delta_0 - 3\sin\delta_0\sin\delta_1\cos(\delta_0-\delta_1)}.$$

By simple mathematical manipulation we get

$$\cos(\delta_0-\delta_1)\sin\delta_1/\sin\delta_0 = x, \tag{3}$$

where $x = (F/B - 1)/3(F/B + 1)$. Opening out the term, $\cos(\delta_0 - \delta_1)$ and making the usual approximation for small δ_1 we get a simple relation for δ_1 as follows

$$\delta_1 = x/(1/\alpha^2(k) - 1)^{1/2}, \tag{4}$$

where $\alpha^2(k) = k^2 \sigma_{tot}/4\pi$. Thus with the help of (2) and (4), the σ_{tot} and F/B data at any given low energy enable one to obtain δ_0 and δ_1 at that energy so that $d\sigma/d\Omega$ and P/E data are redundant. Of course, a variety of potentials like Gaussian, Yukawa, etc with or without cores would reproduce these phase shifts and hence give a good fit to the data. Phase shifts (in radians) calculated using the above formula (4) at energies (E_{cm}) at which both σ_{tot} and F/B are available for Λp system are given in table 1 (using mean experimental σ_{tot} and F/B from Alexander *et al* 1968; Sechi Zorn *et al* 1968).

From (4) we see that δ_1 is positive when x is positive, i.e. F/B is greater than unity and δ_1 negative when F/B is less than unity (remembering that δ_0 is positive). Thus reliable F/B data could tell us about the nature of the interaction in the p-state. Expression for F/B can also be easily written for a spin-dependent interaction. Then it can be seen that barring a situation of resonance scattering or strong spin-dependence, if F/B is greater than unity either both singlet and triplet are positive or at least the triplet phase shift is positive. If F/B is less than unity either both or atleast the triplet phase shift is negative.

There are strong indications that Λ-nucleon interaction is weakly spin-dependent (Shoeb and Rahman Khan 1984; Bouyssy 1979). The existing low energy scattering data have been fitted with both spin-independent and spin-dependent potentials with no significant difference. The matter of the spin dependence of the interaction can be safely decided only when accurate polarization experiments are performed. Therefore, it is better to apply the above considerations to low energy Λp scattering.

The present F/B data are very imprecise but on the whole there is a slight tendency towards values of F/B greater than unity. If this is taken seriously, potentials which are repulsive (apart from a hard core) in the p-state both in triplet and singlet states or even in the triplet state can be ruled out. Bandō and Nagata (1983) used a Λ-nucleon potential which is repulsive in the triplet p-state. Recently Ansari *et al* (1985) also reported a phenomenological potential of the Bandō type and one which is repulsive in both the p-states. If the faintly discernible trend of F/B being somewhat greater than unity is to be taken seriously all the potentials referred to above would be untenable. However, at the present stage such a conclusion would be unwarranted. The need is to obtain much more accurate F/B data.

The arguments developed here may also be cautiously applied to np scattering provided it is at not too low energies so that resonant singlet scattering does not unduly complicate matters and also not at too high energies where higher partial waves become

Table 1. Phase shifts (in radians) calculated using equation (4).

E_{cm} (MeV)	σ_{tot} (mb)	F/B	δ_0	δ_1
7·7	146	1·0	0·505	~ 0
10·3	101	1·28	0·484	0·008
17·1	52	1·80	0·445	0·030

significant. Remembering that δ_0 is positive the well-known pronounced backward peaking of the experimental elastic differential *np* cross-section at once tells us that at least the triplet *p*-phase shift is negative, a result which has been known for a long time.

Acknowledgement

We are grateful to our colleagues H H Ansari and Dr Q N Usmani for helpful discussions.

References

Alexander G, Karshon U, Shapira A, Yekutieli G, Engelmann R, Filthuth H and Lughofer W 1968 *Phys. Rev.* **173** 1452

Ali S, Grypeos M E and Kok L P 1967 *Phys. Lett.* **B24** 543

Ansari H H, Shoeb M and Rahman Khan M Z 1985 *J. Phys. G* (submitted)

Bamberger A, Faessler M A, Lynen U, Piekarz H, Piekarz J, Pniewski J, Povh B, Ritter H G and Soerghel V 1973 *Nucl. Phys.* **B60** 1

Bandō H and Nagata S 1983 *Prog. Theor. Phys.* **69** 557

Bouyssy A 1979 *Phys. Lett.* **B84** 41

Daltiz R H, Herndon R C and Tang Y C 1972 *Nucl. Phys.* **B47** 109

Nagels M M, Rijken T A and De Swart J J 1977 *Phys. Rev.* **D15** 2547

Nagels M M, Rijken T A and De Swart J J 1979 *Phys. Rev.* **D20** 1633

Schiff L I *Quantum Mechanics* 3rd edn. p. 22

Sechi Zorn B, Kehoe B, Twitty J and Burnstein R 1968 *Phys. Rev.* **175** 1735

Shoeb M and Rahman Khan M Z 1984 *J. Phys.* **G10** 1047