Pramāna – J. Phys., Vol. 26, No. 4, April 1986, pp. 337–353. © Printed in India.

Inelastic scattering of pions on ^{7}Li

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MS received 21 June 1985; revised 30 December 1985

Abstract. The recent experimental results of Gibson and coworkers, for inelastic scattering of pions on ^{7}Li, leading to the first five excited states of ^{7}Li are analyzed using a shell model description including configuration admixtures and representing the transition nucleon densities following Helm model, used extensively in the context of inelastic scattering of electrons. With the transition radii for protons and neutrons chosen to be 3·9 fm and 4·1 fm respectively, good fits to the experimental cross sections are obtained. In contrast to the distorted wave Born approximation (DWBA) calculations using optical model codes, we find the present work is able to reproduce the observed maxima and minima precisely. Moreover this analysis reveals the importance of considering the configuration admixtures especially in the transitions to $^{24}P_{5/2}$ (7·48 MeV) and $^{24}D_{7/2}$ (9·67 MeV) levels.

Keywords. Inelastic pion scattering; scattering cross-section; form factor; transition charge density; surface smearing; convolution; nucleon wave functions; configuration admixture of states.

PACS No. 25·80; 21·10; 21·60; 13·75

1. Introduction

The advent of meson factories at LAMPF, TRIUMF, CERN, SIN etc, highlights the great importance being attached at present, to pion scattering studies on nuclei, since the pion, in contrast to the electron, is capable of probing the matter distribution inside the nucleus. Since the pion occurs in three different charge states, it can also probe selectively the proton and neutron distributions. The fact that the spin-dependent part of the pion scattering amplitude is comparable to the spin-independent part, encourages one to expect sizable nuclear polarization and asymmetry effects, which complement the information derivable from the differential scattering cross-section studies. Thus pion scattering could richly complement the electron scattering studies to elucidate nuclear structure.

Rose (1948) was perhaps one of the first to point out that the deviations from the Mott scattering provided the possibility for determining the size and shape of nuclear charge distribution. The most primitive way of considering the nuclear charge and mass distribution is through the so-called 'uniform model' wherein the density $\rho(r)$ is given by a step function

$$\begin{aligned} \rho_R(r) &= 3Z/4\pi R^3 \qquad \text{if } r < R, \\ &= 0 \qquad \text{if } r > R. \end{aligned} \tag{1}$$

Since it appears unrealistic that a nucleus should exhibit a sharp edge at a distance R, a simple prescription of rounding off the edge was suggested by Helm (1956), which consists in using a convolution of the uniform charge density ρ_R with a surface smearing density ρ_S which is taken to be either another uniform density ρ_u or a Gaussian density ρ_g normalized to unity; that is the density $\rho(\mathbf{r})$ is now given by

$$\rho(\mathbf{r}) = \int \rho_R(\mathbf{r}')\rho_S(\mathbf{r}-\mathbf{r}')\,\mathrm{d}^3r'. \tag{2}$$

The main advantage of the above form of charge distribution is that the form factor $F(q)$ of $\rho(r)$ is given by convolution theorem as

$$\begin{aligned} F(q) &= \int \exp(i\mathbf{q}\cdot\mathbf{r})\rho(\mathbf{r})\,\mathrm{d}^3r, \\ &= F^R(q)\cdot F_S(q), \end{aligned} \tag{3}$$

where

$$F^R(q) = \frac{3}{qR}j_1(qR), \tag{4}$$

and $F_S(q)$ is given either by

$$\frac{3}{qu}j_1(qu), \tag{5a}$$

for uniform smearing of smearing width u, or by

$$\exp(-g^2q^2/2) \tag{5b}$$

for the Gaussian smearing of smearing width g. This simple picture which was quite successful in the case of elastic scattering of electrons, has been extended to inelastic scattering of electrons leading to isolated nuclear levels by assuming that the 'transition charge density' is concentrated on a shell of radius R in the form of a delta function which is again smeared out by a convolution with Gaussian smearing density. Following Uberall (1971) transition charge density for inelastic transitions can be written as

$$\rho_{Rl}(\mathbf{r}) = (i)^l(2J_i+1)^{1/2}\beta_l^{J_iJ_f}R^{-2}\delta(\mathbf{r}-\mathbf{R}), \tag{6}$$

where $\beta_l^{J_iJ_f}$ is called the 'strength parameter' for the given transition. Then the convolution of $\rho_{Rl}(\mathbf{r})$ with $\rho_g(\mathbf{r})$ gives the form factor

$$F(q) = (2J_i+1)^{1/2}\beta_l^{J_iJ_f}F_g(q)j_1(qR). \tag{7}$$

Such an analysis has been carried out on inelastic scattering of electrons on an impressively large number of nuclei (Uberall 1971), and the nuclear model parameters fitted to both electron scattering and photoabsorption (Nagl *et al* 1975; Nagl and Uberall 1976). This model has been extended to discuss processes other than electron scattering, like muon capture (Devanathan *et al* 1975), photoproduction of charged pions on initially spin zero nuclei like ^{12}C (Seaborn *et al* 1974), and this has further been generalized by Graves *et al* (1980) to the case of non-zero spin targets. Such a generalized Helm model has been considered to be some kind of extension of the two-parameter Fermi distribution, to the case of inelastic processes leading to the isolated excited nuclear states. Seaborn *et al* (1974) nevertheless pointed out the importance of providing a more fundamental microscopic picture of the nuclear transitions using the shell model including the configuration mixing.

At this juncture, it is worth examining, if one could incorporate the various advantages inherent in the shell model description with configuration admixtures and parametrize the reduced matrix elements by the Helm model prescription. This is more advantageous since the harmonic oscillator shell model is intrinsically incapable of describing the entire q dependance of a given form factor either due to distortion effects or due to the presence of short range correlations, while the Helm model could be capable of such a simple description (which indirectly takes care of correlation and distortion effects). If necessary, one could introduce superposition of several transition radii, which could give the model a greater flexibility.

We therefore have made an attempt here to examine, using the above point of view, the recent experimental results reported by Gibson *et al* (1982) on inelastic scattering of $\pi^{\pm}$ on ^{7}Li at 143 MeV, leading to the excited states $^{22}P_{1/2}$ (0·48 MeV), $^{22}F_{7/2}$ (4·63 MeV), $^{22}F_{5/2}$ (6·68 MeV), $^{24}P_{5/2}$ (7·48 MeV) and $^{24}D_{7/2}$ (9·67 MeV). We have carried out a shell model calculation including the configuration admixtures and then parametrized the reduced matrix elements using the Helm model, of course extending it to both proton and neutron distributions. The authors of the experiment have themselves analyzed the data by carrying out distorted wave calculations using a programme referred to as CHOPIN and using an optical model potential code referred to as DUMIT (Gibson *et al* 1982). These programmes, however, do not lead to correct predictions regarding even the general shape of the angular distribution of scattered pions at large angles (for example, they get a minimum where there is an experimental maximum at about 100°), whereas our results obtained by carrying out a simple calculation as above, show a better agreement to the experimental data. Earlier Sparrow (1977) studied pion reactions on ^{7}Li using an optical potential generated using the local Laplacian prescription wherein a simplified operator is used to describe the deformed spin-orbit interaction. He concluded that the sensitivity to the target spin structure is generally small though the procedure in the author's own words, "is probably not sufficiently precise to allow the analysis of pion scattering data to determine nuclear structure". On the contrary the procedure used in this paper demonstrates that the configuration admixtures perceptibly change the differential cross-sections particularly in the cases of excitations to the $^{24}P_{5/2}$ (7·46 MeV) level and $^{24}D_{7/2}$ (9·61 MeV) level. By suitable choice of proton matter radius R_p, neutron matter radius R_n and convolution width a, fairly good fits are obtained here to the experimental results in the case of all the five excited states studied by Gibson *et al* (1982). The ratio of isovector and isoscalar amplitudes, and the relative total cross section w.r.t. the state at 4·63 MeV have also been calculated, and we find that our results are in better agreement with the experimental values than the corresponding theoretical values given by Gibson *et al* (1982).

In §2, we derive general expressions for both elastic and inelastic differential scattering cross sections, taking fully into account both isoscalar and isovector parts as well as the spin-independent and spin dependent parts of the amplitudes and including the configuration admixtures in the nuclear states. In §3 our numerical results are presented along with the theoretical and experimental results of Gibson *et al* (1982), with our conclusions in §4.

2. Evaluation of the nuclear cross-sections with configuration admixtures of shell model states

2.1 *Resume of the general formalism*

The pion scattering amplitude on a nucleon, depending both on the spin and isospin, may be written in the form

$$t = \sum_{\tau=0,1} [i\sigma\cdot\mathbf{K}_\tau + L_\tau]\exp(i\mathbf{k}\cdot\mathbf{r}),$$

$$= \sum_{\tau=0,1}\sum_{s=0,1} (i)^s(\sigma^s\cdot K_\tau^s)\exp(i\mathbf{k}\cdot\mathbf{r}), \tag{8}$$

where $\mathbf{K}$ and L are spin-dependent and spin-independent amplitudes, wherein $\mathbf{K}_0$ and L_0 are the isoscalar parts; $\mathbf{K}_1$ and L_1 are the isovector parts and $\mathbf{k}$ is the momentum transferred to the nucleus. Following essentially the algebra developed by Ramachandran (1966) for evaluating the nuclear cross-section, using shell model and plane wave impulse approximation (PWIA), the differential cross-section for a pion to scatter from an initial state $|i\rangle$ to a final state $|f\rangle$ of a nucleus, may be written in the form

$$\frac{d\sigma}{d\Omega} = \frac{2\pi\rho}{v}\left|\langle f|T|i\rangle\right|^2$$

$$= \frac{2\pi\rho}{v}[\delta_{fi}|Q_{c.s}|_L^2 + |Q|_L^2 + |Q|_K^2], \tag{9}$$

where ρ denotes the density of states, v the relative velocity of the pion in the initial state $|Q_{c.s}|_L^2$ is the closed shell contribution whereas $|Q|_L^2$ and $|Q|_K^2$ are spin-independent and spin-dependent scattering contributions by open shell nucleons. It may be clearly noted that the closed shell nucleons contribute only to spin-independent scattering and that too in the case of elastic scattering i.e., to the ground state only, whereas open shell nucleons can contribute both spin-dependent and spin-independent amplitudes to the elastic scattering as well as to the scattering leading to excited states. Explicit expressions for these contributions may be written as

$$|Q_{c.s}|_L^2 = n_c^2|L_0|^2\cdot F_{0s} + 2n_c\cdot n\cdot F_{0s}\cdot F_{op}\cdot \mathrm{Re}\sum_\tau(-1)^\tau L_\tau R_{000\tau}, \tag{10}$$

$$|Q|_L^2 = n^2[J_f]^2[J_i]^{-2}\sum_{\tau,\tau'}\sum_\lambda[\lambda]^{-2}A^\lambda(-1)^{\tau+\tau'} \times R_{\lambda\lambda 0\tau}R_{\lambda\lambda 0\tau'}L_\tau L_{\tau'}^* F_{\lambda p}, \tag{11}$$

$$|Q|_K^2 = n^2[J_f]^2[J_i]^{-2}\sum_{\tau\tau'}\sum_\lambda\sum_{ll'}[\lambda]^{-2}B_{ll'}^\lambda R_{\lambda l1\tau}R_{\lambda l'1\tau'}\mathbf{K}_\tau\cdot\mathbf{K}_{\tau'}^* F_{lp}F_{l'p}, \tag{12}$$

where n_c is the number of nucleons in the closed shell and n in the open shell. F_{0s} and F_{lp}, $(l = 0, 2)$ are nuclear structure form factors for the S and P shell nucleons respectively. The geometrical factors A^λ and $B_{ll'}^\lambda$ are explicitly given and are numerically tabulated for P shell transitions in table 1 of Ramachandran (1966). $R_{\lambda ls\tau}$ are reduction factors in L.S. coupling connecting the nuclear and nucleonic amplitudes. When configuration admixtures of wave functions are taken into account both in the initial and final states, the geometrical reduction factors can be written as

$$R_{\lambda l s \tau} = \sum_{\{\alpha_i\}} \sum_{\{\alpha_f\}} a^*_{\{\alpha_f\}} a_{\{\alpha_i\}} R^{\text{L.S.}}_{\lambda l s \tau}, \tag{13}$$

where $a_{\{\alpha\}}$ are admixture coefficients in a state denoted by collective quantum number $\{\alpha\}$ and $R^{\text{L.S.}}_{\lambda l s \tau}$ are given by (10) of Ramachandran (1966). $F_{l,p}$ are radial integrals (or structure functions) given by

$$\begin{aligned} F_{l,j} &= \int_0^\infty u_f^*(r) j_l(kr) u_j(r)\, r^2 \mathrm{d}r, \\ &= \int_0^\infty j_l(kr) \rho(r)\, r^2 \mathrm{d}r. \end{aligned} \tag{14}$$

2.2 *Synthesis with the Helm model*

In the light of the Helm model, each of the F_{lj} is now represented in the modified form as follows

$$F_{lj} = \beta_l^{J_i J_f} F_{lj}(k) \cdot F_H(k), \tag{15}$$

where $F_{lj}(k) = F^R(q)$ of (3) and $F_H(k) = F_S(q)$ of (5). The strength parameter $\beta_l^{J_i J_f}$, which is introduced ad hoc, may be looked upon, in the spirit of the Helm model, as incorporating all distortion effects too. The pion wave with initial momentum q_1, incident on a nucleus is likely to get distorted before getting scattered on a nucleon, and the scattered pion wave could once again be distorted before emerging out of the nucleus with the final momentum $\mathbf{q}_2$. If $\phi_i(\mathbf{r})$ and $\phi_f(\mathbf{r})$ denote the distorted pion wave functions before and after scattering on a nucleon located at $\mathbf{r}$, then the scattering amplitude t in (8) can be written in the form

$$t = \int \mathrm{d}^3 q_1' \mathrm{d}^3 q_2' \phi_f^*(q_2') t' \phi_i(q_1'), \tag{16}$$

where

$$t' = (i\sigma \cdot \mathbf{K}' + L') \exp i(\mathbf{q}_1' - \mathbf{q}_2') \cdot \mathbf{r}$$

denotes the amplitude for the pion to get scattered from momentum $\mathbf{q}_1'$ to $\mathbf{q}_2'$ on a nucleon; which in reality could also be off the energy shell. Moreover the resulting form factor F_{lj} being function of $\mathbf{k}' = \mathbf{q}_1' - \mathbf{q}_2'$, would also occur inside the integral (16). Thus the problem becomes quite involved. In evaluating t between nuclear states we assume

$$\int \phi_f^*(q_2') t' \phi_i(q_1') F_l(k') \mathrm{d}^3 q_1' \mathrm{d}^3 q_2' = \beta_l^{J_i J_f} t F_l(k), \tag{17}$$

where t and F_l are given by (8) and (14) respectively. Then $\beta_l^{J_i J_f}$ is the effective distortion factor for a given transition and it may be identified with the strength parameter of the Helm model.

2.3 *Inclusion of configuration admixtures of ^7Li wave functions*

The general formalism developed in § 2.1 for the pion scattering on a nucleus can now be applied to the particular case of scattering of pions on ^{7}Li. The wave functions including configuration admixtures for the ground state as well as for the five excited states of ^{7}Li studied by Gibson *et al* (1982), are taken, following Barker (1966). Then using (10), (11) and (12), one can write expressions for the scattering amplitudes Q, for the elastic as well as inelastic scattering leading to different excited states of ^{7}Li, taking into consideration, the modified structure factors, in the spirit of Helm model. The results are

$$|Q|_L^2 = \delta_{fi}|Q_{\mathrm{c.s}}|^2 + a|L_p F_{2p}^p + 2L_n F_{2p}^n|^2 \beta^2 \cdot F_H^2(k), \tag{18}$$

$$\begin{aligned} |Q|_K^2 = \beta^2 F_H^2(k)[&\{b_1|F_{0p}^p|^2 + b_2 F_{0p}^{*p} \cdot F_{2p}^p + b_3|F_{2p}^p|^2\}|K_p|^2 \\ &+ \{c_1|F_{0p}^n|^2 + c_2 F_{0p}^{*n} \cdot F_{2p}^n + c_3|F_{2p}^n|^2\}|K_n|^2 \\ &+ \{d_1 F_{0p}^{*p} \cdot F_{0p}^n + d_2(F_{0p}^p \cdot F_{2p}^{*n} + F_{0p}^n \cdot F_{2p}^{*p}) \\ &+ d_3 F_{2p}^p \cdot F_{2p}^n\}, \quad \mathrm{Re}(K_p \cdot K_n^*)], \end{aligned} \tag{19}$$

where $|Q_{\mathrm{c.s}}|^2 \delta_{fi}$ contributes only to the ground state and is given by

$$|Q_{\mathrm{c.s}}|^2 = \beta^2 F_H^2(k)|[2(L_p \cdot F_{0s}^p + L_n F_{0s}^n) + (L_p F_{0p}^p + 2L_n F_{0p}^n)]|^2. \tag{20}$$

The coefficients a, b, c and d are certain geometrical factors whose values depend on the two states between which the transition takes place. The appropriate reduction factors $R_{\lambda l s \tau}$ corresponding to these states can now be calculated (1) by approximating the initial and final states, by retaining only the leading state i.e., pure L.S. coupling state in each case, and (2) by considering all the admixtures of configurations, as given by Barker (1966). The coefficients of fractional parentage are obtained from Jahn and Wieringen (1951). Then the numerical values of the constants a, b, c and d are calculated and are tabulated in table 1 for both (i) pure and (ii) admixtures of configurations, for all the states to which transitions have been studied by Gibson *et al* (1982). F_{lp}^p and F_{lp}^n are structure functions of proton and neutron. F_H and $\beta_l^{J_i J_f}$ are surface smearing factors and the effective distortion factors of the Helm model as discussed in §2.2 above.

3. Numerical results and discussion

Differential cross-sections for all the excited states of ^{7}Li studied by Gibson *et al* (1982) have been calculated using (18) and (19), for both π^+ and π^- scattering. Elementary amplitudes L_p, L_n, K_p and K_n were calculated using the empirical formulae of Berends and Donnachie (1975), for the π-N phase shifts. The structure functions F_{lp}^p and F_{lp}^n, for proton and neutron distributions, were calculated by choosing the transition matter density to be in the form of delta function, in the spirit of Helm model, given by (6). Then by suitable choice of proton and neutron matter distribution radii, the smearing width g and the strength factor $\beta_l^{J_i J_f}$, the differential cross-sections for all the excited states were calculated. Optimum fits to the experimental data of Gibson *et al* (1982) have been obtained, for the choice of $R_p = 3{\cdot}9$ fm; $R_n = 4{\cdot}1$ fm and $g = 0{\cdot}4$ fm. The values of $\beta_l^{J_i J_f}$ for both π^+ and π^- scattering leading to different excited states are listed in table 2. The calculations are also made by taking the value of $\beta_l^{J_i J_f}$ to be equal to 1 for all the cases. The differential scattering cross-sections have been calculated by representing each excited states as (i) a pure shell model and (ii) by including admixtures of configurations. The results of all these calculations are shown in figures 1–10, along

Table 1. Coefficients *a*, *b*, *c* and *d* used in the cross-section expressions.

	Ground State $^{22}P_{3/2}$		$^{22}P_{1/2}$ (0·48 MeV)		$^{22}F_{7/2}$ (4·63 MeV)	
Coefficients	Pure	Admixture	Pure	Admixture	Pure	Admixture
a	9/25	0·3268	9/25	0·3241	16/25	0·5523
b_1	5/9	0·5418	4/9	0·4108	—	—
b_2	−2/15	−0·1477	2/15	0·1182	—	—
b_3	11/25	0·4330	7/25	0·2057	$\frac{1365}{2573}$	0·4727
c_1	—	0·0005	—	0·0005	—	—
c_2	—	0·0025	—	−0·0007	—	—
c_3	—	0·0038	—	0·0017	—	0·0211
d_1	—	0·0330	—	0·0281	—	—
d_2	—	0·0237	—	−0·0084	—	—
d_3	—	0·0388	—	−0·0370	—	−0·0482

	$^{22}F_{5/2}$ (6·68 MeV)		$^{24}P_{5/2}$ (7·48 MeV)		$^{24}D_{7/2}$ (9·67 MeV)	
Coefficients	Pure	Admixture	Pure	Admixture	Pure	Admixture
a	8/75	0·0906	—	0·0105	—	0·0748
b_1	—	0·0004	—	0·0043	—	—
b_2	—	0·0131	—	−0·0111	—	—
b_3	184/525	0·3052	—	0·0115	—	0·15
c_1	—	0·0003	—	0·0083	—	—
c_2	—	0·0009	—	0·0063	—	—
c_3	—	0·0928	27/200	0·1454	62/525	0·0058
d_1	—	0·0006	—	0·0119	—	—
d_2	—	0·0061	—	−0·0087	—	—
d_3	—	−0·1391	—	−0·0385	—	0·0414

Table 2. Optimum values of the strength parameter, used in our work, for various excited states.

	Strength parameters for	
Levels of ^{7}Li	π^+ scattering	π^- scattering
$^{22}P_{1/2}$ (0·48 MeV)	1	1
$^{22}F_{7/2}$ (4·63 MeV)	1	1
$^{22}F_{5/2}$ (6·68 MeV)	1·43	1·25
$^{24}P_{5/2}$ (7·48 MeV)	1·73	1·43
$^{24}D_{7/2}$ (9·67 MeV)	0·64	0·71

with the results of DWBA calculations made by Gibson *et al* (1982) and also their experimental results.

A look at these figures shows that the model adopted by us leads to far better fits, both qualitatively and quantitatively, to the experimental data, than those of DWBA calculations made by Gibson *et al* (1982) using sophisticated codes like CHOPIN and DUMIT. In spite of the fact that they have normalized the DWBA calculations, to get

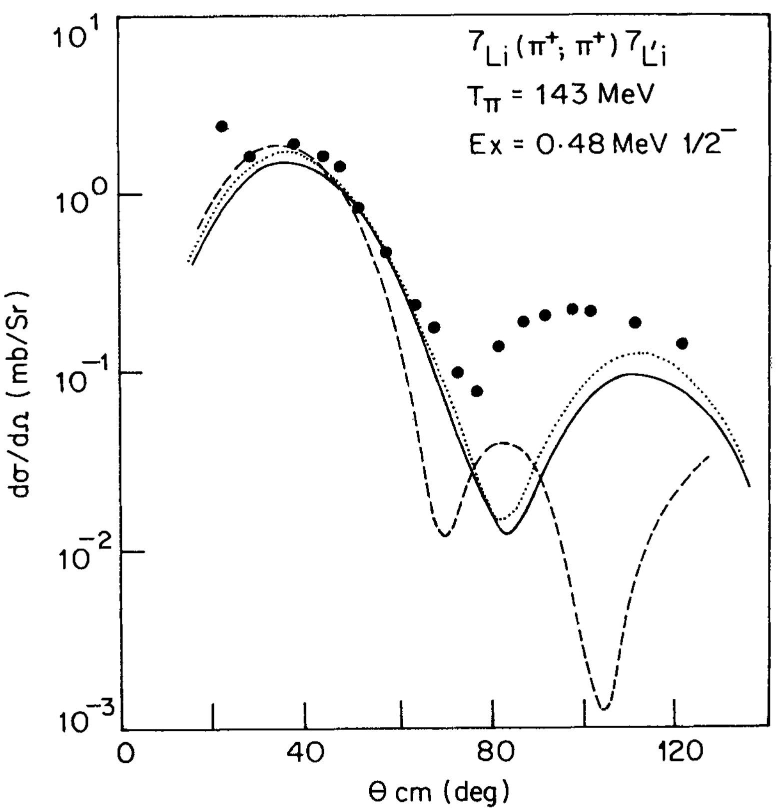

Figure 1. Angular distribution of π^+ for the excitation of $^{22}P_{1/2}$ state at 0·48 MeV. Notations as in figure 3.

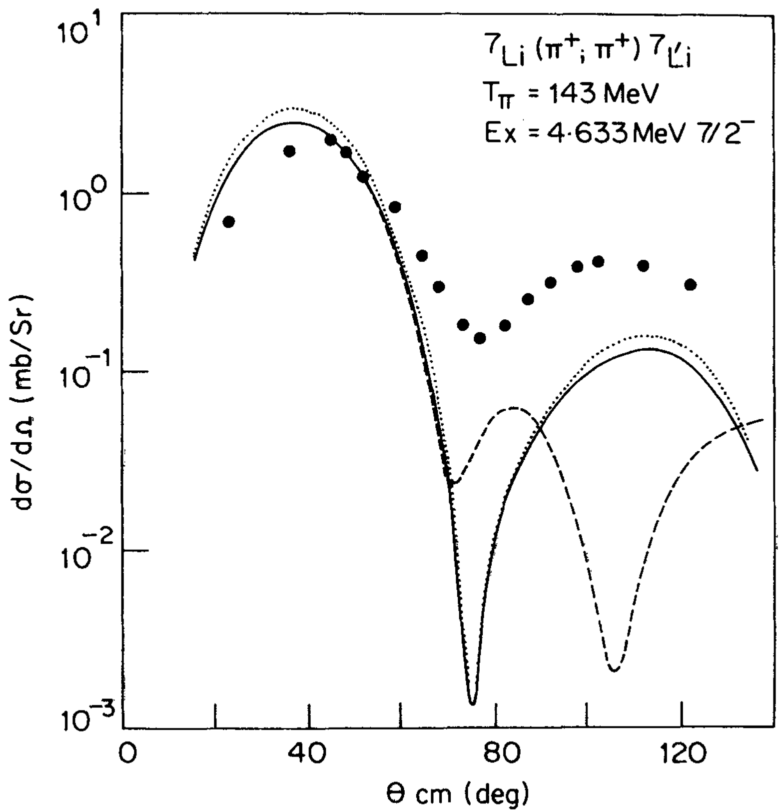

Figure 2. Angular distribution of π^+, for the excitation of $^{22}F_{7/2}$ state at 4·633 MeV. Notations as in figure 3.

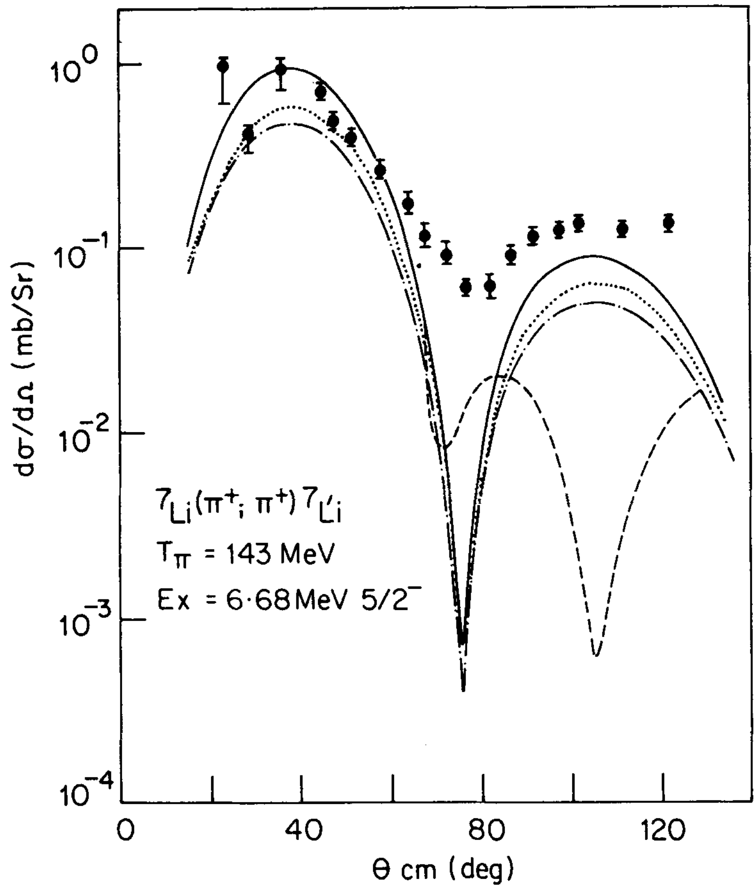

Figure 3. Angular distribution of π^{+}, for the excitation of $^{22}F_{5/2}$ state at 6·68 MeV. (i) The heavy solid line is the result of our calculations with inclusion of strength parameter and admixtures. (ii) The dot-dash line is the result of our calculations with admixtures only. (iii) The dot-dot line is the result of our calculations without admixtures. (iv) The dash-dash line is the result of DWBA calculations from Gibson *et al* (1982). Experimental results are also from Gibson *et al* (1982).

reasonable fit in the region of first maximum, their results fail to give good fits for all scattering angles. They fail to predict correct minima at about 75° and the second maxima at about 100° where they get, instead, a deep minimum. We get correct maxima at about 40°, correct minima at about 75° and correct second maxima at about 100°. Though in some cases this second maximum is shifted up to 110°, we get good qualitative fit in this region. It is worth noting that in the cases of π^{+} scattering to the states $^{22}F_{5/2}$ (6·68 MeV), $^{24}P_{5/2}$ (7·48 MeV), $^{24}D_{7/2}$ (9·67 MeV) and π^{-} scattering to the states $^{24}P_{5/2}$ (7·48 MeV), we get very good agreement, both qualitatively and quantitatively in all the regions.

3.1 *Admixture effects*

The differential cross-sections calculated by considering each state to be (i) pure and (ii) admixtures of different configurations, are shown, for each case, in figures from 1–10. It is found that in the case of inelastic scattering leading to the excited states $^{22}P_{1/2}$

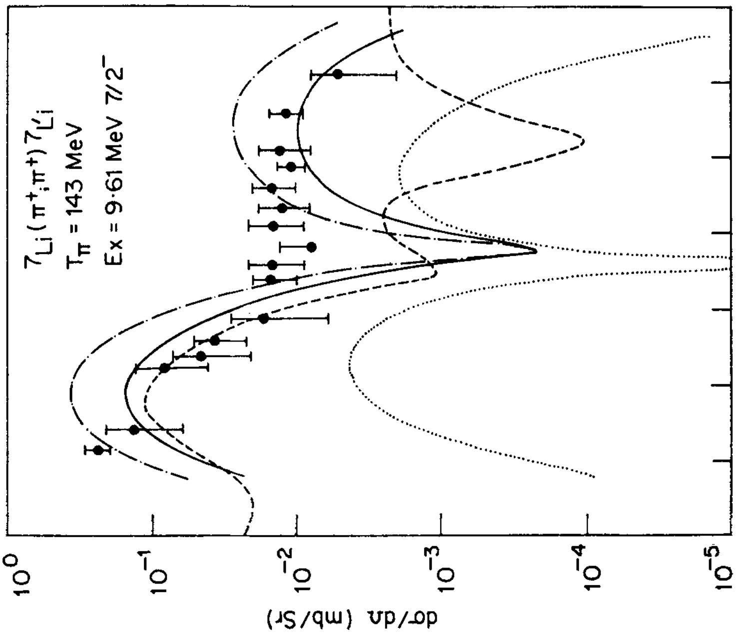

Figure 5. Angular distribution of π^+, for the excitation of $^{24}D_{7/2}$ state at 9·61 MeV. Notations as in figure 3.

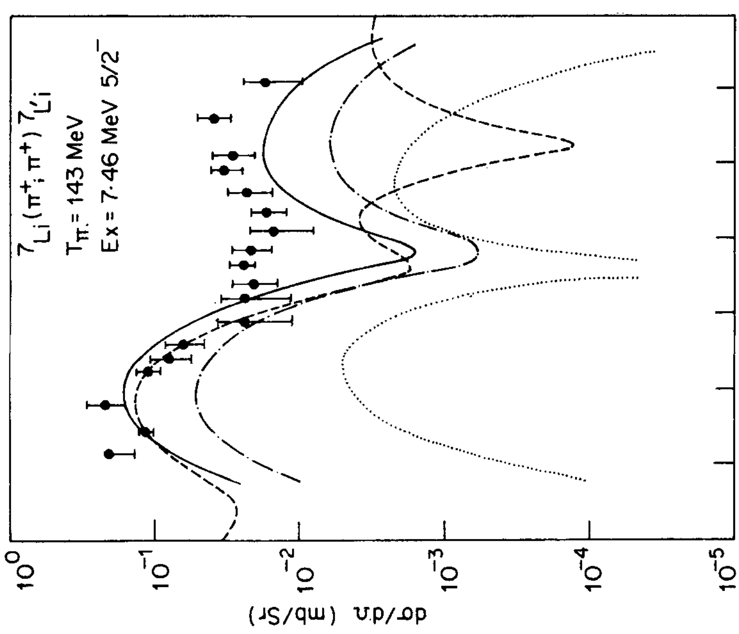

Figure 4. Angular distribution of π^+, for the excitation of $^{24}P_{5/2}$ state at 7·46 MeV. Notations as in figure 3.

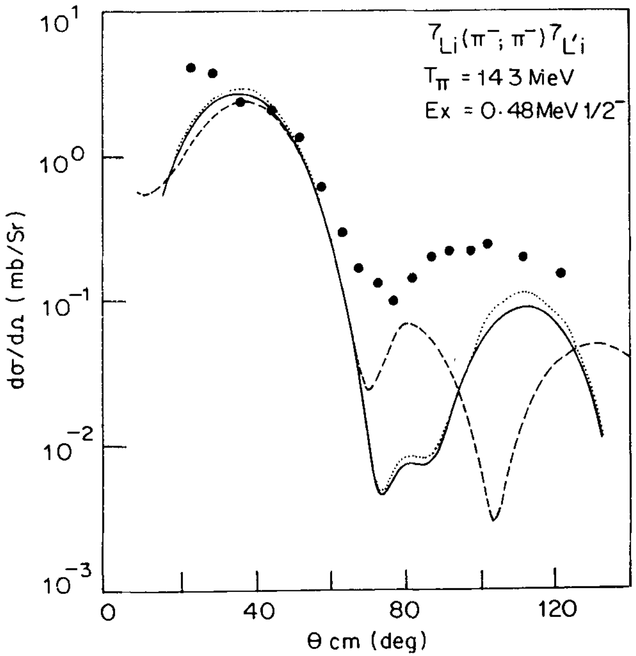

Figure 6. Angular distribution of π^-, for the excitation of $^{22}P_{1/2}$ state at 0·48 MeV. Notations as in figure 3.

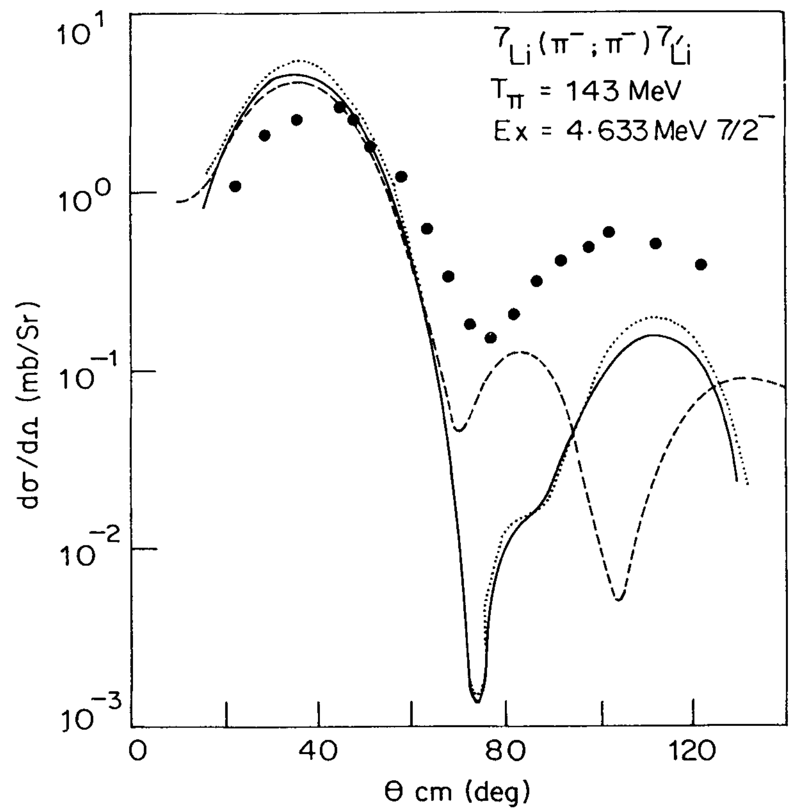

Figure 7. Angular distribution of π^-, for the excitation of $^{22}F_{7/2}$ state at 4·633 MeV. Notations as in figure 3.

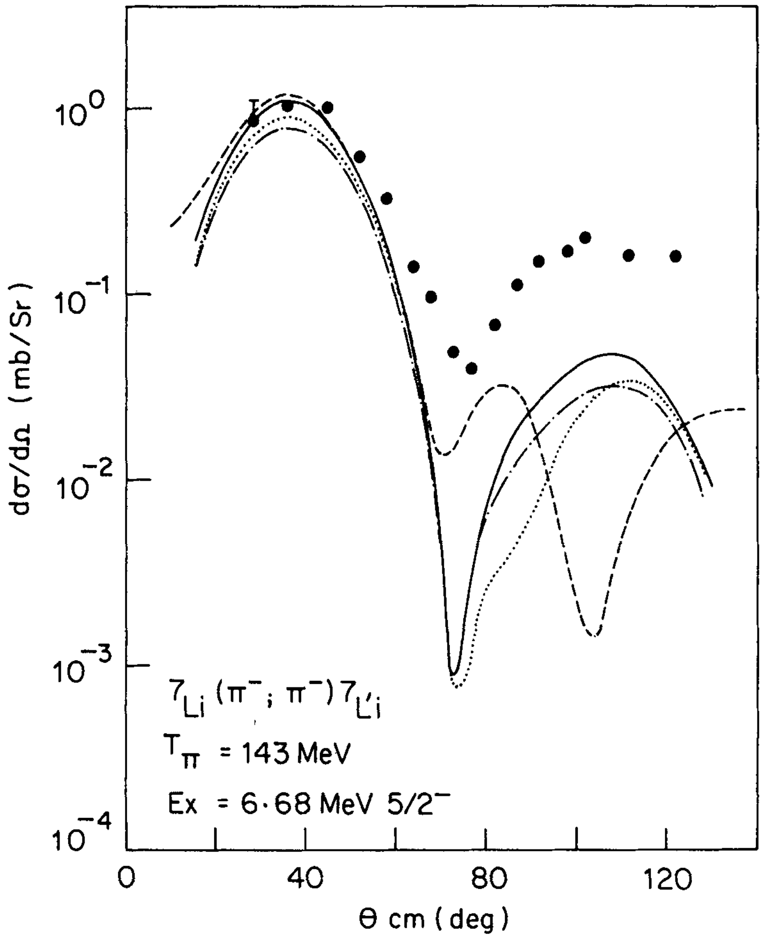

Figure 8. Angular distribution of π^-, for the excitation of $^{22}F_{5/2}$ state at 6·68 MeV. Notations as in figure 3.

(0·48 MeV), $^{22}F_{7/2}$ (4·63 MeV) and $^{22}F_{5/2}$ (6·68 MeV), the admixture of states does not appreciably change the cross-sections. They reduce the cross-sections by a small margin which is in conformity with the findings of Sparrow (1977). But in the cases of the 4th excited state $^{24}P_{5/2}$ (7·48 MeV) and fifth excited state $^{24}D_{7/2}$ (9·67 MeV), we find that the admixture effects are very large, especially at small angles. In the region of first maximum the cross-sections are increased by an order of 10^2. These increased values of cross-sections give very good quantitative fits to the experimental values, both at small and large angles of scattering, than the cross-sections calculated without the configuration admixtures.

This large enhancement of the cross-section is due to the fact that, if the ground state is described purely by the $^{22}P_{3/2}$ state, then there is no contribution of spin-independent amplitude, when the excitation takes place to the states $^{24}P_{5/2}$ and $^{24}D_{1/2}$. On the other hand a small percentage of ^{24}P and ^{24}D admixtures in the ground state will contribute spin-independent amplitudes also, to these scatterings. Knowing that at small angles of scattering it is the spin-independent amplitude which is very dominant, even a small admixture of such amplitudes enhances the cross-section to a large extent.

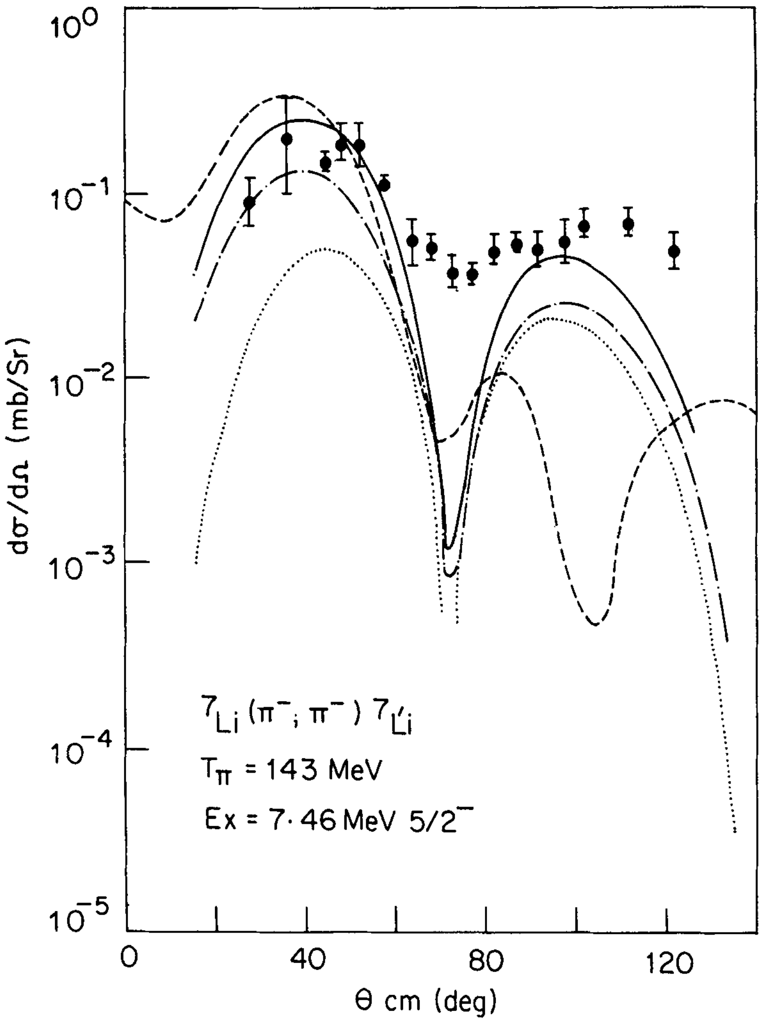

Figure 9. Angular distribution of π^-, for the excitation of $^{24}P_{5/2}$ state at 7·46 MeV. Notations as in figure 3.

Sparrow (1977) who earlier discussed some effects of configuration mixing for the first and second excited states, has stated that "it seems improbable that any information about nuclear wave functions can be obtained from pion scattering". On the other hand we find that the small admixtures of wave functions can play an important role, at least in some cases, which shows that pions can be used to study the spin structure of the nucleus.

3.2 *Transition strength parameter*

The best values for the transition strength parameter $\beta_l^{J_i J_f}$, identified earlier as effectively representing the distortion effects were found to be 1 for the transition to the $^{22}P_{1/2}$ and $^{22}F_{7/2}$ states, while for other transitions, as shown in detail in table 2, range between 0·63 and 1·73. Unfortunately we do not find any estimates of $\beta_l^{J_i J_f}$ for these transitions in ^{7}Li from the analysis of either inelastic electron scattering or from pion

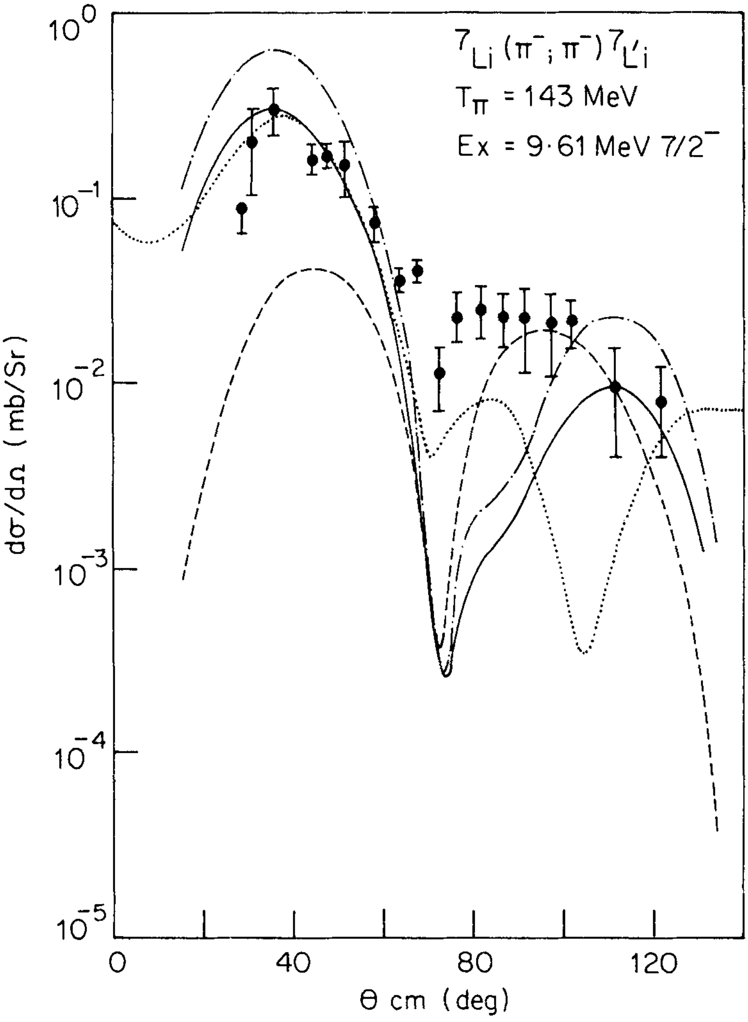

Figure 10. Angular distribution of π^-, for the excitation of $^{24}D_{7/2}$ state at 9·61 MeV. Notations as in figure 3.

photoproduction or muon capture. It may however be mentioned that, considering transitions on various nuclei, ranging from ^{12}C to ^{88}Sr, the value of $\beta_l^{J_i J_f}$ in the case of inelastic scattering of electron ranges from 0·033 to 0·40 (Helm 1956) while it ranges from −0·0604 to 1·21 (Graves *et al* 1980) in the case of pion photoproduction and muon capture. Compared to such wide ranges of values for $\beta_l^{J_i J_f}$, the values presented in table 2 may be considered to be not too far away from 1. These results may be interpreted as indicating that the distortion effects are small if one chooses Helm model for nuclear matter distributions. In this model matter being distributed over a spherical shell, a pion need not enter deep into the nucleus, before scattering on a nucleon. On the other hand, in the case of elastic scattering matter being assumed to be uniformally distributed throughout the volume of a nucleus, the distortion effects may be felt much more. This may be the reason why the DWBA calculations of Gibson *et al* lead to poor fits in the case of inelastic scattering leading to the excited states, while it is quite successful in explaining the elastic scattering.

3.3 *Relative parameters*

The relative strength of isovector to isoscalar amplitudes may be calculated by calculating the ratio R, of π^+ and π^- cross-sections (Peterson 1980)

$$R = \frac{\sigma(\pi^-)}{\sigma(\pi^+)} = \left|\frac{A_0 + \frac{1}{2}A_1}{A_0 - \frac{1}{2}A_1}\right|^2, \tag{21}$$

where A_0 and A_1 are isoscalar and isovector amplitudes, and $\sigma(\pi^-)$ and $\sigma(\pi^+)$ are total cross-sections for π^- and π^+ scatterings respectively. The total cross-section was calculated by integrating the differential cross-section over all angles. The values of R and the ratio A_1/A_0 are tabulated in table 3 along with the experimental results of Gibson *et al* (1982) and those of Bolger *et al* (1979) for the case of scattering of 164 MeV pions on ^{7}Li, as quoted in Gibson *et al* (1982). As has been pointed out already by Gibson *et al* (1982), the value of R can be expected to be around 1·96, if one takes a simplistic view of the nuclear structure and consider 2 neutrons and 1 proton outside the ^{4}He core, and $\pi^+ - p\,(\pi^- - n)$ scattering to be 3 times more than $\pi^- - p\,(\pi^+ - n)$ scattering, in view of the dominance of isospin 3/2 channel. It is interesting to note that, the values of R, based on the present detailed theoretical analysis, are fairly close to this number. The experimental results of Gibson *et al* (1982) for R increase slowly from 1·1 to 2·4, with the increasing excitation energies. The ratio A_1/A_0 obtained in the present work ranges between 0·134 and 0·464, while the experimental values of Gibson (1982) range from 0·05 to 0·43 and of Bolger *et al* (1979) from 0·006 to 0·386. It is interesting to note that our results in some cases favour the measurements of Gibson *et al* and in some other cases favour the measurements of Bolger *et al* (1979). Since there are discrepancies between the two experimental measurements, one cannot draw definite conclusions at this stage.

Another comparison made is the relative total cross-section of a transition w.r.t. total cross-section for the transition at 4·63 MeV state. The results are tabulated in table 4, along with the other results quoted by Gibson *et al* (1982). It is seen that our findings are in good agreement with the experimental results of Gibson *et al* in the case of

Table 3. Ratios of total cross-sections $\sigma(\pi^-)/\sigma(\pi^+)$ and the ratios of isovector to the isoscalar amplitudes.

E (MeV)	(*a*) R	(*b*) R	(*a*) A_1/A_0	(*b*) A_1/A_0	(*c*) A_1/A_0
0·478	1·476	1·11 ± 0·03	0·194	0·05 ± 0·03	0·094 ± 0·030
4·630	1·601	1·39 ± 0·02	0·234	0·16 ± 0·02	0·194 ± 0·025
6·68	1·307	1·26 ± 0·1	0·134	0·12 ± 0·08	0·006 ± 0·049
7·456	2·572	2·00 ± 0·3	0·464	0·33 ± 0·17	0·386 ± 0·102
9·67	1·401	2·40 ± 0·5	0·168	0·43 ± 0·21	0·108 ± 0·055

(a) Our present theoretical work; (b) experimental values of Gibson *et al* (1982); (c) inelastic π^+ scattering at 164 MeV (Bolger *et al* 1979).

Table 4. Ratios r of the total cross-sections relative to the state at 4·63 MeV.

E (MeV)	(a) $r(\pi^+,\pi^{+'})$	(a) $r(\pi^-,\pi^{-'})$	(b) $r(\pi^+,\pi^{+'})$	(b) $r(\pi^-,\pi^{-'})$	(c) $r(\pi^+,\pi^{+'})$	(c) $r(d,d')$	(d) r_{theor}
0·478	0·68	0·59	0·67	0·54	0·92	0·72	0·39
4·630	1	1	1	1	1	1	1
6·68	0·22	0·18 }	0·41	0·41	0·29	—	0·17
7·456	0·025	0·041 }					
9·67	0·16	0·14	—	—	—	—	—

(a) Our present theoretical work; (b) experimental results of Gibson *et al* (1982); (c) inelastic scattering of π^+ at 164 MeV Bolger *et al* (1969); (d) theoretical result of Gibson *et al* (1982). c, d are as quoted in Gibson *et al* (1982).

0·478 MeV state, while in the case of 6·68 MeV state, our results agree with the results of Bolger *et al* and theoretical results quoted in Gibson *et al* (1982).

4. Conclusions and outlook

We find that our calculations for the differential and total cross-sections for π^+ and π^- scattering, at 143 MeV, where the Helm model notions have been introduced instead of oscillatory wave functions in an otherwise basically shell model theory, leads to far better agreement with experiments than the sophisticated DWBA calculations and optical model. These results are encouraging and indicate that the Helm model which is quite successful in electron scattering studies, is also a potentially powerful tool for studying pion scattering on light nuclei. Our analysis also shows that the admixture of ^{24}P and ^{24}D states in the ground states of ^{7}Li, though small, makes major contributions to the inelastic scattering leading to the states $^{24}P_{5/2}$ and $^{24}D_{7/2}$ which are not connected through spin-independent interaction to the pure configuration $^{22}P_{3/2}$ of the ground state. Such situations could be used to study the small admixtures in configuration mixing. A more incisive approach would be to study the final nuclear state polarization parameters which would be highly sensitive to the configuration admixtures. In fact it has been suggested recently (Ramachandran and Ravishankar 1985) that the study of the angular distribution of the intensity and circular polarization asymmetry parameters could lead to the complete determination of the density matrix of the excited state which decays through γ emission.,

Acknowledgements

We thank Sri V Ravishankar for many useful discussions. We acknowledge with thanks the assistance rendered by Dr K S Gundu Rao, Dr S Prakash and other members of the University Computer Centre, Mysore, in processing the numerical results reported here. One of us (SBP) thanks the University Grants Commission and the Management of Shri Sharanabasaveshwara College of Science, Gulbarga, for having provided research opportunity under FIP scheme. The authors also thank the referees for their suggestions, in the light of which the paper has been revised.

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