Pramāṇa – J. Phys., Vol. 26, No. 4, April 1986, pp. 327–335. © Printed in India.

Differential incoherent scattering of 279·2 keV photons by Zr, Sn, Ta, Pb and U

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MS received 28 March 1985; revised 24 December 1985

Abstract. Differential incoherent scattering cross-section ratios of 279·2 keV photons by zirconium, tin, tantalum, lead and uranium elements are experimentally determined by comparing the peak areas under the degraded photon energy with that of an equivalent aluminium foil employing a high resolution 35 c.c. coaxial Ge(Li) detector. Studies have been made in single configuration in an angular range of 20° to 115°. The results are compared with theoretical values obtained from the non-relativistic HFS model of Hubbell and co-workers. The cross-section ratios decrease as the atomic number increases for a given scattering angle.

Keywords. Incoherent scattering function; electron binding effect; momentum transfer.

PACS No. 32·80

1. Introduction

Early experimental work on the Compton effect (Du Mond 1933; Du Mond and Kirkpatrick 1937; Kirkpatrick and Du Mond 1938) primarily involved loosely bound electrons ($Z < 10$). The results of these experiments were fairly well established by Klein and Nishina (1929). The cross-section computed from the Klein-Nishina formula gives accurate results for free electrons only. As more and more tightly bound electrons are considered, the scattering cross-section is expected to be less in comparison to that of free electrons particularly in the forward direction (low momentum transfers). The diminution of the scattering cross-section due to the binding effects of electrons is represented by the incoherent scattering function which expresses the ratio of bound electron to free electron scattering.

The theoretical calculations of Compton scattering by bound electrons furnish estimates of a correction factor for the Klein-Nishina formula evaluated for all electrons of a given element. This correction factor $S(x, z)$ is expressed as $d\sigma B/d\Omega = d\sigma KN/d\Omega\, S(x, z)$, where $d\sigma B/d\Omega$ is the scattering cross-section due to bound electrons, and $d\sigma KN/d\Omega$ is the usual Klein-Nishina differential collision cross-section per electron. Different non-relativistic theories based on Thomas-Fermi model (Bewilogua 1931; Wheeler and Lamb 1939, 1956), Hartree-Fock-Slater model (Cromer

and Mann 1967; Cromer 1969), and relativistic theories (Jauch and Rohrlich 1955; Whittingham 1971) are employed to compute the value of incoherent scattering function $S(x, z)$. According to the non-relativistic models the incoherent scattering function is zero in the limit of vanishing energy and unity in the limit of infinite momentum transfer. But according to the relativistic theory of incoherent scattering (Jauch and Rohrlich 1955) it exceeds one at high momentum transfers.

Experimental studies on the effects of binding due to the incoherent scattering process were systematically observed (Brini *et al* 1960; Motz and Missoni 1961; Sujkowski and Nagel 1961; Varma and Eswaran 1962; Pingot 1968, 1971; East and Lewis 1969; Murty *et al* 1971; Krishnareddy *et al* 1974; Kane and Babaprasad 1977) for targets of higher atomic number. Most of this work was carried out on the Compton scattering by K-shell electrons at 662 keV, but very little information is available in the low energy (below 500 keV photons) region (Ramalingareddy *et al* 1967; Pingot 1969, 1972; Murty *et al* 1973; Spitale and Bloom 1977). In all these experiments Compton scattering from K-shell electrons was separately investigated by measuring coincidences between K x-rays and Compton scattered photons for the determination of deviations of the differential cross-section from Klein-Nishina formula. There are considerable divergences in the values of the cross-section ratios obtained by different investigators. It also appears that these experiments had excluded the measurement of photons scattered by other shell electrons and coherently scattered photons since the latter do not leave an excited atom. Whittingham (1971) attempted to compare his theoretical results obtained for Pb at 662 keV but he could not arrive at a definite conclusion since no consistent experimental data were available. Schumacher (1971), and Rullhusen and Schumacher (1976) studied the Compton scattering profiles (electron momentum distribution) for all shell electrons employing a Ge(Li) detector in single mode. The experimental data were compared with theoretical calculations based on relativistic form-factor approximation from second-order perturbation theory. But they found discrepancy at high energy ends of the Compton profiles. Experimental investigations for whole atom were reported (Standing and Jovanovich 1962; Singh *et al* 1963; Anand *et al* 1964; Quivy 1966; Ghumman and Sood 1967) by measuring the scattered gamma-ray spectra using NaI(Tl) detector. The combined effect of all shell electrons was also reported (Ramanarao *et al* 1965; Parthasaradhi 1967, 1969; Gopal and Sanjeevaiah 1973) by subtracting the theoretically computed contribution due to all other processes except the incoherent scattering from the total atomic cross-section. The results of these experiments show deviations from the Klein-Nishina values as the atomic number increases.

A knowledge of the scattering function is desirable to analyze other scattering experiments at low energies since Compton-scattered quanta are, in general, the largest portion of the background. Hence, precise experimental data are needed in the low energy region (below 500 keV), atleast for a few elements for finding out the possible discrepancies between theory and experiment. In the present investigations incoherent scattering functions for the average effect of all shell electrons in elements such as zirconium, tin, tantalum, lead and uranium have been measured using photons with a primary energy of 279·2 keV at scattering angles 20°, 30°, 45°, 60°, 80°, 100° and 115° employing a Ge(Li) detector. The experimental method chosen is qualitatively the same as that used in previous measurements of similar type, namely the observation of scattered spectra. The results are discussed in the light of the values of Hubbell *et al* (1975) calculated on the basis of non-relativistic Hartree-Fock model.

2. Experimental

The experimental arrangement used for the measurement of scattered photons, shown in figure 1, is similar to that used by Schumacher (1971), and Rullhusen and Schumacher (1976) with suitable modifications. The system consists of a source holder with a provision to adjust the source position, a detector, and a target holder which can vary the orientation of the scatterer.

The source holder is a mild steel cylinder filled with lead capable of fixing the source at any desired position along the axial line. The radioactive source ^{203}Hg (279·2 keV photons) with an intensity of about 235 mCi (supplied in a sealed radiographic capsule by the Bhabha Atomic Research Centre, Bombay) is held in a phosphorbronze clip at the end of a lead-filled brass tube that smoothly moves in the axial hole of the source holder. The source holder is mounted on a vertical stand with casters to enable it to rotate about the centre of the target keeping the detector at a fixed position. Lead surfaces facing towards the scatterer and the detector are covered with a thin aluminium sheet to suppress the characteristic lead x-rays. A 35 c.c. coaxial Ge(Li) detector mounted in lead housing coupled with ND 1100 system of multichannel analyser has been used to detect the scattered photons. The detector with a resolution of 3·7 keV at 662 keV photons enables us to separate the elastic and inelastic components of the scattered spectra. The shielding at the face of the detector has an opening of about 2 cm diameter with thin aluminium lining to limit the range of scattering. The lead shielding between the source and the detector is sufficient to reduce the intensity of the photons coming directly from the source at the location of the detector to a negligible amount. Compact geometry is obtained with a distance of 23 cm between detector and target and 18 cm between source and target. The angular resolution of the detector is found to be 2·6 degrees with the present geometrical arrangement. The collimated photon beam, the centre of the target foil and the centre of the face of the detector are aligned in the same plane. The scatterers are cut to 12·5 mm diameter and fixed in the target holder. The target holder is a thin rounded perspex to which the

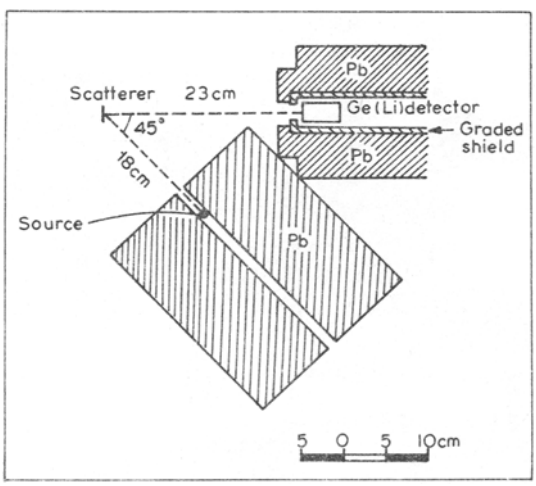

Figure 1. Experimental arrangement for measurement of scattered photons.

targets are fixed and can be rotated on a circular disc of perspex marked with graduations in degrees from 0 to 90 for adjustment of the scatterer. The target holder is turned at an angle in the plane determined by the source, target and the detector such that the absorption of the scattered photons is minimized. The target foils are made from 99·9 % pure metals.

The scattered gamma ray spectra are recorded with the foils of scatterers and aluminium at various angles using a Ge(Li) detector and a 512-channel analyser system. The experiments are repeated with various foils of each scatterer with minimum foil thickness of 46·36 mg/cm^2 (uranium), 13·11 mg/cm^2 (lead), 2·48 mg/cm^2 (tantalum), 5·62 mg/cm^2 (tin) and 8·68 mg/cm^2 (zirconium). The scattered spectra are obtained at angles 20°, 30°, 45°, 60°, 80°, 100° and 115°. A typical scattered photon spectrum at an angle of 45° in tantalum with 18 mg/cm^2 is shown in figure 2. In recording the spectra the time of collection is so adjusted to maintain the statistical error always to be < 1 %. After completing one run with a set of scatterers, usually 4 or 5 in number, the same run with the same set of scatterers is repeated a few times to calculate the standard deviation. The background is separately measured after removing the scatterer from target assembly and is subtracted from each of the scattered spectra obtained for the experimental and aluminium foils. The background corrected spectrum is again corrected for Compton contribution due to coherently scattered photons. For this purpose, an auxiliary source of ^{203}Hg of the scatterer size is placed at the target position after removing the primary source and scatterer. A spectrum recorded under these conditions is normalized to the elastic peak countrate and subtracted from the above background corrected spectrum. The resultant spectrum is finally corrected for finite energy resolution by properly choosing the mono-energetic sources in the scattered energy region. The data for each element are also corrected for self-absorption effects.

The differential cross-section ratios are determined by comparing the intensities of the degraded photon energies of the scatterer materials under investigation and the equivalent aluminium foils. The incoherent scattering cross-section at any angle is

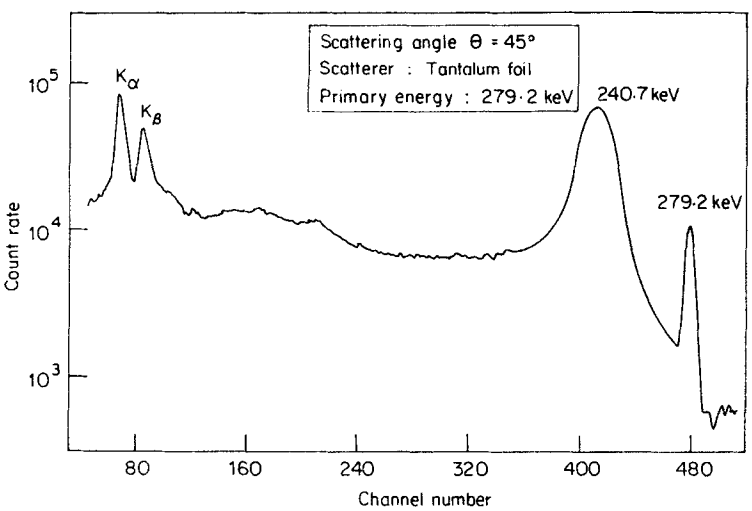

Figure 2. Scattered photon spectrum of primary energy 279·2 keV photons corresponding to 18 mg/cm^2 Ta scatterer at an angle 45°.

determined by measuring the fractional number of photons incoherently scattered at that angle. If C_x is the number of events detected per unit time with a target thickness of N_x expressed as the number of electrons in the scatterer in a defined angle θ, then,

$$C_x = \frac{S}{4\pi r^2} \cdot N_x \cdot \frac{d\sigma_x}{d\Omega}(\theta) \cdot \Omega \cdot \varepsilon_\gamma' \exp(-\mu X_T), \tag{1}$$

where $S/4\pi r^2$ is the number of primary photons with source strength S passing through the scatterer per unit time on an unit area which is kept at a distance r, $d\sigma x/d\Omega$ is the differential incoherent scattering cross-section per electron of the scatterer, ε_γ' is the intrinsic efficiency of the detector for photons scattered in the solid angle Ω, and $\exp(-\mu X_T)$ is the self-absorption correction factor for incoherently scattered gamma rays in the target.

The primary source intensity determination is eliminated by conducting auxiliary experiments with aluminium foils in which the binding effects may be neglected and the Klein-Nishina law (Klein and Nishina 1929) could be employed. The number of events C_{Al} of the incoherently scattered photons by free and stationary electrons of the aluminium scatterer at the same defined angle as that of the experimental scatterers are given by

$$C_{Al} = \frac{S}{4\pi r^2} \cdot N_{Al} \cdot \frac{d\sigma_{Al}}{d\Omega}(\theta) \cdot \Omega \cdot \varepsilon_\gamma' \cdot \exp(-\mu X_{T_{Al}}). \tag{2}$$

Here $\mu X_{T_{Al}}$ corresponds to the absorption coefficient of aluminium at the Compton maximum. The values of C_{Al} are measured before and after each cycle of observation at all angles of interest with various foil thicknesses. From the above two equations the incoherent scattering cross-section ratio is

$$\frac{d\sigma_x}{d\sigma_{Al}} = \frac{C_x}{C_{Al}} \cdot \frac{N_{Al}}{N_x} \cdot \frac{\exp(-\mu X_{T_{Al}})}{\exp(-\mu X_T)}. \tag{3}$$

The cross-section ratios for all scatterer materials at each angle are evaluated using C_x and C_{Al} values after making all possible corrections.

The incoherent scattering spectrum as shown in figure 2 corresponds to the energy range from zero to $(h\nu_0 - B)$, where $h\nu_0$ is the incident photon energy and B is the binding energy of the electron. As the entire scattered energy spectrum is recorded as a single peak, it has to be corrected for certain unwanted events, like (i) elastic events, (ii) Compton continuum of higher energy photons to individual incoherently scattered photon events inside the Ge(Li) crystal, (iii) background due to bremstrahlung radiation resulting from the interaction of photoelectrons as well as Compton electrons produced in the target, (iv) double Compton effect, (v) scattering at wall and air and (vi) background due to cosmic rays.

The contribution due to elastic scattering is eliminated by subtracting the normalized auxiliary spectra taken with a ^{203}Hg source having the dimensions of the scatterer at the position of the target. To obtain the contribution due to Compton continuum of the high energy photons in the scattered spectrum, the auxiliary spectra of mono-energetic sources of ^{109}Cd (88 keV), ^{141}Ce (145 keV), ^{114m}In (191·6 keV) and ^{203}Hg (279·2 keV) are recorded at the Compton energy region. These spectra are normalized to the inelastic counts of the scattered spectra and subtracted to obtain pure incoherent

spectrum without Compton continuum. The contribution due to bremstrahlung radiation, double Compton effect, and the photons scattered from air etc is proportional to the square of the thickness of the target for thicknesses smaller than the range of the photo and Compton electrons. Therefore, corrections are carried out by extrapolating the data to zero foil thickness. A method of least squares fit is adopted to obtain true incoherent events for a unit thickness of mass for each scatterer element (Prasad *et al* 1978).

All the above mentioned corrections are carried out for the determination of C_x and C_{Al}. These correction factors are calculated separately for each of the scatterer and aluminium foils as the peak shapes are different from each other due to the difference of their electronic wave functions. The energy dependence of the detector efficiency has also been considered while calculating these systematic errors. The contribution due to the natural radioactivity in the uranium target is negligible since its daughter energy lines do not contribute significantly to the scattered spectrum and since the incident photon energy is very low. The overall error in the determination of the cross-section ratio $d\sigma_x/d\sigma_{Al}$ is the root mean square value of the individual errors in C_x/C_{Al} and other errors and is found to be 5% in the present investigation.

3. Results and discussion

The experimental data and the theoretical values compiled by Hubbell *et al* (1975) are presented for comparison in tables 1 to 5. The theoretical values of $S(x, z)$ due to Hubbell *et al* (1975) are the recent calculations of Cromer and Mann (1967), and Cromer (1969) for the average-bound electron cross-sections based on non-relativistic Hartree-Fock-Slater model for all elements $Z = 2$ to 100 and x, the momentum transfer from 0·005 to 80 $Å^{-1}$. The experimental values vary from 0·797 in uranium at 20° to 1·069 in lead at 115°. The former value corresponds to a reduction over a free electron whereas the latter corresponds to an enhancement. It is observed that the experimental values are much less than unity at forward scattering angles and the deviations increase progressively with increasing atomic number. The possible reason for the cross-section ratio to be less than unity in most cases at this low incident photon energy is the small size of the momentum transfer on an absolute scale. Although a little discrepancy is noticed at small scattering angles, the present experimental results agree to within the

Table 1. Incoherent scattering functions of 279·2 keV photons by uranium.

Angle (in deg.)	Experimental value $d\sigma_x/d\sigma_{Al}$	Value of Hubbell *et al* (1975) $S(x, z)$
20	0·797 (40)	0·843
30	0·869 (44)	0·912
45	0·927 (46)	0·957
60	0·986 (49)	0·976
80	1·012 (51)	0·987
100	1·069 (54)	0·992
115	1·076 (54)	0·993

Table 2. Incoherent scattering functions of 279·2 keV photons by lead.

Angle (in deg.)	Experimental value $d\sigma_x/d\sigma_{Al}$	Value of Hubbell *et al* 1975 $S(x, z)$
20	0·824 (41)	0·860
30	0·894 (45)	0·925
45	0·968 (48)	0·964
60	0·983 (49)	0·979
80	0·989 (50)	0·987
100	1·059 (53)	0·990
115	1·069 (54)	0·992

Table 3. Incoherent scattering functions of 279·2 keV photons by tantalum.

Angle (in deg.)	Experimental value $d\sigma_x/d\sigma_{Al}$	Value of Hubbell *et al* (1975) $S(x, z)$
20	0·836 (42)	0·876
30	0·904 (45)	0·935
45	0·949 (48)	0·968
60	0·988 (50)	0·981
80	0·973 (50)	0·987
100	1·043 (52)	0·991
115	1·048 (52)	0·992

Table 4. Incoherent scattering functions of 279·2 keV photons by tin.

Angle (in deg.)	Experimental value $d\sigma_x/d\sigma_{Al}$	Value of Hubbell *et al* (1975) $S(x, z)$
20	0·861 (43)	0·913
30	0·922 (46)	0·957
45	0·968 (48)	0·979
60	0·985 (49)	0·989
80	0·992 (50)	0·996
100	0·996 (50)	0·998
115	0·992 (50)	0·999

experimental errors when compared with the theoretical non-relativistic HFS model calculations of Cromer and Mann (1967), and Cromer (1969).

The variation of the incoherent scattering functions with momentum transfers (fixed scattering angles) for each element is shown in figure 3. The momentum transfer has

Table 5. Incoherent scattering functions of 279·2 keV photons by zirconium.

Angle (in deg.)	Experimental value $d\sigma_x/d\sigma_{Al}$	Value of Hubbell *et al* (1975) $S(x, z)$
20	0·913 (46)	0·933
30	0·965 (48)	0·969
45	1·009 (51)	0·987
60	0·998 (50)	0·992
80	0·976 (49)	0·994
100	0·979 (49)	0·995
115	1·002 (50)	0·996

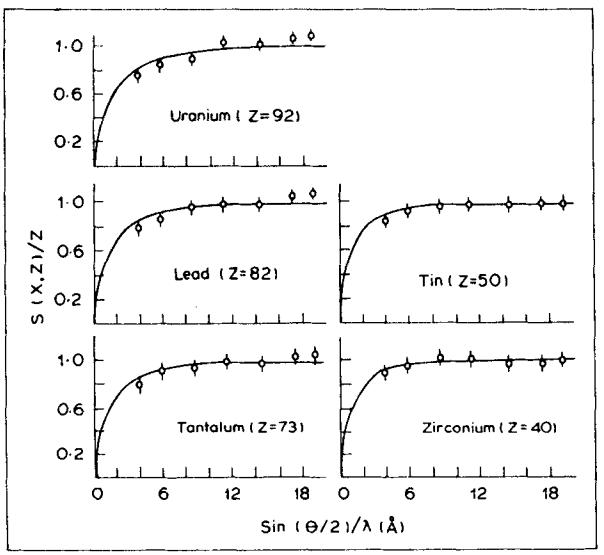

Figure 3. Variation of incoherent scattering function with momentum transfer of 279·2 keV photons by U, Pb, Ta, Sn and Zr. The lines indicate the theoretical values based on the non-relativistic HFS calculation values of Hubbell *et al* (1975). Circles represent experimental points.

been calculated using $x = \sin(\theta/2)/\lambda$ (Å), where λ is the wavelength of the incident radiation expressed in Å with the value of $\lambda = 12398{\cdot}520/E$. Here E is the energy of the incident photon in eV. The trend of the cross-section ratios as seen from figure 3 is found to be less than unity, approaching zero as the momentum transfer decreases to zero for all elements.

Acknowledgements

The authors are grateful to Prof. V Lakshminarayana for useful suggestions. They are also thankful to the referees for raising critical points.

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