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Supersymmetric preon models with three-fermion generations

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Abstract. A class of supersymmetric preon models is considered in which the hypercolour group G_{HC} and the unbroken flavour group G_f anomalies are zero without needing spectators. It is shown that for $G_{\text{HC}} = \text{SU}(2)$ and SU(3) quarks and leptons as composites can be obtained satisfying 't Hooft's anomaly matching conditions. For the case of $G_{\text{HC}} = \text{SU}(3)$, G_f can accommodate a horizontal symmetry group to describe just three generations.

Keywords. Supersymmetry; preons; three generations.

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The possibility that quarks and leptons are composites made out of preons has been studied following 't Hooft (1980), by many authors (Barbieri *et al* 1983; Buchmuller *et al* 1983; Gerard *et al* 1982; Greenberg *et al* 1983; Love *et al* 1982 and Gupta *et al* 1984). Non-abelian hypercolour (group G_{HC}) gauge interactions bind the preons to form hypercolour singlet composites. At the large scale, $\Lambda_{\text{HC}} >$ few TeV, where the hypercolour forces become strong, the usual low energy colour and flavour forces can be neglected. The fundamental lagrangian then describes a renormalizable and asymptotically free gauge theory based on G_{HC}. This lagrangian also has a maximal global flavour symmetry, group $\tilde{G}_f$, which depends on the choice of the preons. Depending on the dynamics, $\tilde{G}_f$ may be spontaneously broken to the smaller group G_f which remains unbroken at low energies. For a realistic model, G_f should be large enough to contain the symmetry group needed to describe the interactions of the (composite) quarks and leptons. Further, 't Hooft (1980) argued that the composite fermions (which are massless compared to Λ_{HC}) should satisfy non-trivial anomaly matching conditions (AMC) with respect to G_f.

In this note, we investigate a class of supersymmetric preon models in the 't Hooft framework. Apart from having both scalar and fermion preons, the unbroken supersymmetry provides an interesting constraint on the composite spectrum. The choice of the preons is constrained by the requirement that the hypercolour forces be asymptotically free and that their G_f anomaly be zero (that is, no spectator preons). We show below that for simple choices of $G_{\text{HC}} = \text{SU}(2)$ and SU(3), the AMC can be satisfied by composites corresponding to quarks and leptons. A plus point for the latter case is that G_f can accommodate a horizontal symmetry group in a natural way to describe three generations.

Preons: We assign the preons to be components of two different left-chiral superfields $S_i = (\phi, \chi)_i$ and $T^i = (\eta, \psi)$ which transform as the representations R_0 and $\bar{R}_0$ of G_{HC}. Here $i = 1, 2, \ldots, N$ is the flavour index. In addition, we assume $G_f = \mathrm{SU}(N)$ and choose S and T to transform as the N and $\bar{N}$ representations of G_f respectively. This choice* is consistent with the continuous global symmetry of the hypercolour gauge interactions. These properties of S and T make them 'mirrors' of each other. An immediate consequence is that both the G_{HC} and G_f anomalies, due to the preons, automatically vanish (for any R_0) without requiring any spectator preons. Moreover, the 't Hooft AMC will now require that the G_f anomaly due to massless composite fermions be zero.

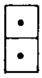
To proceed further, with this general preon model, we specify $G_{\mathrm{HC}} = \mathrm{SU}(n)$, $n \geqslant 2$. Then, for the supersymmetric hypercolour interactions to be asymptotically free,

$$\beta_{\mathrm{HC}} = 6n - 2NC_0 > 0. \tag{1}$$

Here C_0 is the group theoretical index (Slansky 1981) for the representation R_0. Equation (1) provides a constraint both on N and the possible representation R_0. Now, $C_0 = 1$ for the fundamental representation n of SU(n), while its value is larger for the higher dimensional representations. Moreover, for $G_f = \mathrm{SU}(N)$, to encompass the low energy symmetry group one must have $N \geqslant 5$. This is always possible (with $\beta_{\mathrm{HC}} > 0$) for each n if $R_0 = n$. This completes the identification of the properties of the preons for $G_{\mathrm{HC}} = \mathrm{SU}(n)$.

Below, we consider, in detail, the simplest cases $n = 2$ and 3 and show that one can obtain a composite fermion spectrum satisfying the AMC which can be identified with known quarks and leptons.

$G_{\mathrm{HC}} = SU(2)_{\mathrm{HC}}$: The $2N$ preons are described by the two $\mathrm{SU}(2)_{\mathrm{HC}}$ doublets S_{ia} and T^{ia} ($a = 1, 2$ is the $\mathrm{SU}(2)_{\mathrm{HC}}$ index). The largest value allowed by (1) is $N = 5$ and only this is of interest as $G_f = \mathrm{SU}(5)$ would be just the usual grand unification group. The $\mathrm{SU}(2)_{\mathrm{HC}}$ singlet two-preon composites, SS, TT, ST, SS^+ and TT^+ have to be antisymmetric in the hypercolour indices. Consequently, due to the bosonic nature of superfields, the first two are forced to be antisymmetric in the G_f indices and as such they transform like


and


The dot indicates the conjugate representation. Though these simplest possible composites give zero G_f anomaly (as required by the AMC), they do not give all the required quarks and leptons. However, one may try to remedy this situation by including three-preon and four-preon composites. Actually, the two- and four-preon composites which are totally antisymmetric in G_f indices are sufficient for our purpose. These particular representations are listed in table 1. For these the AMC, for $G_f = \mathrm{SU}(N)$, requires

$$(N-4)(l_1 - l_2) + \frac{(N-3)(N-4)(N-8)(l_3 - l_4)}{3!} = 0. \tag{2}$$

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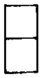
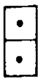
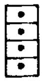
* We have not considered the maximal possible symmetry group for the flavour group. However, it is not necessary to do so. For example, this is quite similar to what is done for quarks. Namely, massless QCD with six-quark flavours has a flavour symmetry U(6) × U(6). However, for the standard flavour group $\mathrm{SU}(2)_L$ × U(1) one *chooses* to put them in doublets and singlets.

Table 1. G_f representations of the two- and four-preon composites, for $G_{\mathrm{HC}} = \mathrm{SU}(2)_{\mathrm{HC}}$ which give the known leptons and quarks. The Young tableaus listed in the second column are only for those representations which are totally antisymmetric in the flavour group indices.

Composites	$G_f = \mathrm{SU}(N)$	$G_f = \mathrm{SU}(5)$	Index
SS		10	l_1
TT		$\overline{10}$	l_2
$SSSS$		$\bar{5}$	l_3
$TTTT$		5	l_4

For the particular case of $N = 5$, this reduces to

$$l_1 - l_2 - l_3 + l_4 = 0. \tag{2a}$$

This equation is easily satisfied in a number of ways with integer l_i. For example, $l_1 = l_3 = k$ and $l_2 = l_4 = m$ would give $(k+m)$ generations of quarks and leptons, where k and m are integers, with the condition $km < 0$. However, the hypercolour forces for S and T should be the same and one would expect $k = m$ i.e. a even number of generations rather than the unsymmetrical solution e.g. $k = 3$, $m = 0$ which would give three generations.

Actually, since SU(2) is a safe group one could have started with only N preons described by one superfield S_{ia}. In this case, $\beta_{\mathrm{HC}} > 0$ permits $N \leqslant 11$. However, the G_f anomaly for the preons is no longer zero and this changes the right hand side of (2) from zero to two. The only solution of academic interest is for $N = 7$ which gives two generations. For further discussion of such a model see Gupta *et al* (1984).

In summary, we get physically interesting solutions for $N = 5$; however, they do not give three generations in a natural way and nor is G_f large enough to accommodate a horizontal symmetry group.

$G_{\mathrm{HC}} = SU(3)_{\mathrm{HC}}$: In this case, the two preons superfields S_{ia} and T^{ia} $(a = 1, 2, 3)$ transform as the 3 and $\bar{3}$ of $\mathrm{SU}(3)_{\mathrm{HC}}$. The asymptotic freedom constraint, equation (1), now restricts $N \leqslant 8$. The physically interesting solutions arise for $N = 8$ and we only discuss these. Moreover, for $N = 8$, G_f is large enough to include the grand unification group SU(5) as well as a horizontal symmetry group G_{H} which can distinguish between the three generations*.

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* For an alternative model see Zhou and Lucio (1983).

Of the two- and three-preon composites the most interesting ones are SSS and TTT as they contain the desired SU(5) representations. Since these are $\mathrm{SU}(3)_{\mathrm{HC}}$ singlets they have to be totally antisymmetric in the hypercolour group indices and consequently, due to the bosonic nature of superfields, they give representations of G_f which are totally antisymmetric in the flavour group indices as shown in table 2. We now consider the AMC for various unbroken flavour groups which are possible starting with $N = 8$. Recall that the G_f anomaly of the preons is zero. Also, except for the composites in table 2, we take the indices for the other composites to be zero.

Case (i) $G_f = SU(8)$: Since the two composites belong to the 56 and $\overline{56}$ their total G_f anomaly vanishes as required. However, in this case unwanted particles are present.

Case (ii) $G_f = SU(5) \times SU(3)$: The flavour representation of the composites together with the corresponding indices p_i and q_i are given in table 2. The two AMC's are:

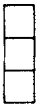
$$[\mathrm{SU}\ (5)]^3:\ \ 3p_1+3p_2-p_3-3q_1-3q_2+q_3=0, \tag{3}$$

$$[\mathrm{SU}\ (3)]^3:\ \ -5p_1+10p_2+5q_1-10q_2=0. \tag{4}$$

The solution $p_3 = q_3 = 0$ and $p_1 = q_1$ and $p_2 = q_2$ satisfies these equations. The representations corresponding to $q_1 = 1$ and $p_2 = 1$ give three generations of quark and leptons in $(\bar{5} + 10)$ of SU(5), if we interpret the SU(3) to be the horizontal symmetry group G_H. However, as can be seen from (3) and (4), we must necessarily have 'mirror' quarks and leptons corresponding to $q_2 = p_1 = 1$*.

Case (iii) $G_f = SU(5) \times SO(3)$: The problem of mirror quarks and leptons can be cured by choosing $G_H = \mathrm{SO}(3)$. This SO(3) will arise from the breaking of the SU(3) in case (ii) and it is fixed by requiring that the original SU(3) triplet goes into a triplet of the surviving SO(3). With this choice of the SO(3) subgroup, the representation content of

Table 2. G_f representations of the three-preon composites for $G_{\mathrm{HC}} = \mathrm{SU}(3)_{\mathrm{HC}}$ which give three generations of quarks and leptons. The index for a SU(5) × SU(3) or SU(5) × SO(3) representation, used in (3) and (4), is given below it.

Composites	$G_f = \mathrm{SU}(N)$	$G_f = \mathrm{SU}(8)$	$G_f = \mathrm{SU}(5) \times \mathrm{SU}(3)$ or $\mathrm{SU}(5) \times \mathrm{SO}(3)$
SSS		56	$(1, 1)+\underset{p_1}{(5, \overline{3})}+\underset{p_2}{(10, 3)}$ $+\underset{p_3}{(\overline{10}, 1)}$
TTT		$\overline{56}$	$(1, 1)+\underset{q_1}{(\overline{5}, 3)}+\underset{q_2}{(\overline{10}+\overline{3})}$ $+\underset{q_3}{(10, 1)}$

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* There are extra massless particles in these models. We do not consider this in detail as we do not want to commit overselves to any particular breaking mechanism.

the *SSS* and *TTT* composites with respect to SU(5) × $SO(3)_H$ is as in the last column of table 2. Now, since SO(3) is a safe group, the only anomaly matching condition which needs to be satisfied is simply (3). It is clear that the solution $q_1 = p_2 = 1$ with $p_1 = p_3 = q_2 = q_3 = 0$ gives exactly the set of known quarks and leptons*.

In conclusion, we have shown that supersymmetric preon models, described by two superfields which are 'mirror' with respect to $G_{\mathrm{HC}} \times G_f$, give simple models which satisfy the AMC and do not require spectator preons. In particular, for the model with $G_{\mathrm{HC}} = \mathrm{SU}(3)$, it is possible to accommodate a horizontal symmetry group to describe three generations (of quarks and leptons) starting from a SU(8) flavour group.

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