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Enhancement of spectral intensities of mercury triplet lines in longitudinal magnetic field

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Abstract. The variation of intensity of the spectral lines of the triplet series of mercury namely ($7^3S_1 \rightarrow 6^3P_{012}$) ($\lambda$5461 Å, λ4358 Å, λ4047 Å) in the presence of a longitudinal magnetic field between zero and 2000 gauss has been investigated. The effect of magnetic field is found to be different as regards the variation of intensity and the occurrence of maxima in the three lines. These variations can be explained by considering the reabsorption of the spectral lines. A mathematical theory has been presented and an expression for $(I_\omega)_B/I_\omega$ has been deduced where $(I_\omega)_B$ and (I_ω) are the intensities of the lines with and without the magnetic field. The experimental results agree fairly well with the theoretical model.

Keywords. Triplet series; self-absorption; electron temperature; spectral intensity; mercury triplet line; longitudinal magnetic field.

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1. Introduction

Enhancement of the intensity of spectral lines in a magnetic field has been considered earlier in inhomogeneous magnetic field (Rokhlin 1939), in longitudinal magnetic field (Forrest and Franklin 1966; Hedge and Ghosh 1979) and in transverse magnetic field (Kulkarni 1944; Sen *et al* 1972). In transverse magnetic field it is generally observed that as the magnetic field increases the spectral intensities of the lines increase attaining a maximum value at a certain magnetic field and then gradually decrease. Sen *et al* (1972) showed that the enhancement process can be quantitatively explained as due to increase of electron temperature and decrease of azimuthal electron density caused by the presence of transverse magnetic field. In the case of longitudinal magnetic field, Hegde and Ghosh (1979) applied a collisional radiative model to the positive column of a helium plasma and interpreted the enhancement of intensity with the magnetic field. Sen *et al* (1972) showed that in a transverse magnetic field, the field at which the line intensity becomes a maximum is a function of the energy of the upper level.

In order to investigate the phenomena in greater detail the enhancement of the intensity of spectral lines of the sharp series triplet radiations of mercury has been studied in longitudinal magnetic field. The sharp series triplet lines have a common upper level. For transitions originating from the same upper level, the relative intensities do not depend on its excitation cross-section but only on the line strength.

The signal strength of the detector should be proportional to the emission rate integrated both along the detector line of sight and over the spectral profile and might be modified by self-absorption; variation of signal strength with the variation of longitudinal magnetic field has been obtained in terms of these parameters and a quantitative explanation of the observed results is presented.

2. Experimental set-up

The intensities of spectral lines were measured for the sharp series triplet lines of the mercury atom in a low pressure mercury arc placed in a longitudinal magnetic field. The vertical mercury arc was placed between the pole pieces of an electromagnet. The arc was constructed of pyrex tube (0·75 cm internal radius, 8 cm in length) and was force-cooled externally. The buffer gas was the dry air whose concentration was regulated through a needle valve. Radiations from the axial region of the positive column of the arc discharge were focussed by lens arrangements on the slit (width 0·5 mm) of an accurately calibrated constant deviation spectrograph. The triplet radiations λ 5461 Å ($7^3S_1 \rightarrow 6^3P_2$) λ 4358 Å ($7^3S_1 \rightarrow 6^3P_1$) and λ 4047 Å ($7^3S_1 \rightarrow 6^3P_0$) were focussed separately on the cathode of the photomultiplier (M 10 FS 29 V_λ) and the intensities of the lines were recorded to study the variation with magnetic field from zero to 2000 G. Details of experimental arrangement and the technique for measuring the intensities of spectral lines employing electronic circuits have been reported earlier (Sen *et al* 1972). In the present investigation the arc current was varied between 2 and 3 amps and the pressure of the air from 0·05 torr to 1 torr.

3. Results and discussion

The triplet radiations ($7^3S_1 \rightarrow 6^3P_{012}$) emitted from the axial region of the positive column of a low pressure mercury arc discharge are found to be enhanced in the presence of longitudinal magnetic field (0–2000 G). The ratio I_B/I, where I_B and I are the intensities of radiation when the magnetic field is present and absent respectively, increases with the magnetic field, thereafter passing through a broad maxima and then decreases slowly. Figure 1 shows the variation of I_B/I with B when the discharge current is 2 amp and the pressure of the air inside the discharge tube is 0·05 torr. It is observed that $(I_B/I)_{max}$ values are different for the three lines. The ratio (I_B/I) for 4047 Å increases rather rapidly and reaches a broad maxima in comparatively low magnetic field. For other two lines, (I_B/I) reaches the maximum value nearly in the same field. The general nature of the curve remains the same when the discharge current is varied from 2 to 3 amps and the pressure of the air inside the discharge tube was changed from 0·05 torr to 1 torr. Table 1 shows the value of B_{max}, the magnetic field at which the intensity becomes a maximum along with the values of $(I_B)_{max}/I$ for the three lines considered for different discharge conditions. It is well known that when a discharge column is subjected to a magnetic field there is a coupled variation of $n_e(0)$, the radial electron density and the electron temperature T_e (T_e is assumed to be uniform along the cross-section of the discharge tube). In the case of longitudinal magnetic field $n_e(0)$ increases and electron temperature decreases whereas when the magnetic field is transverse $n_e(0)$ decreases and electron temperature increases (Beckman 1948). Based upon this theory, the experimental results regarding the enhancement of intensity of spectral lines in a transverse magnetic field have been quantitatively explained by Sen *et al* (1972). Spectral enhancement of intensity is thus dependent upon $n_e(0)$ and T_e. Since in the present

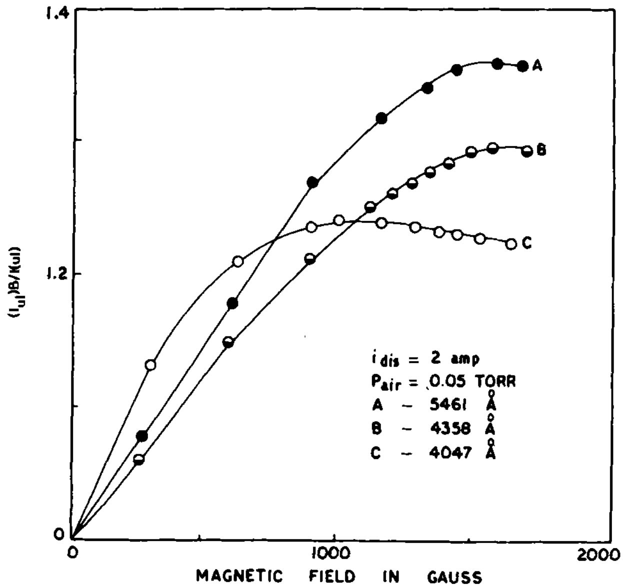

Figure 1. Variation of $(I_{ul})_B/I_{ul}$.

Table 1. Value of magnetic field for maximum intensity (in gauss) with the value of $(I_B/I)_{max}$ for three lines under different discharge conditions.

Wave-length Å	Value of $(B)_{max}$ and $(I_B/I)_{max}$	P_{air} = 0·5 torr		P_{air} = 0·9 torr	
		$i = 3$ A	$i = 2$ A	$i = 2{\cdot}25$ A	$i = 2{\cdot}75$ A
5461	B_{max}	1405	1405	1455	1455
	$(I_B)_{max}/I$	1·312	1·38	1·43	1·46
4358	B_{max}	1455	1425	1495	1455
	$(I_B)_{max}/I$	1·275	1·316	1·332	1·33
4047	B_{max}	1015	930	1055	1015
	$(I_B)_{max}/I$	1·25	1·25	1·22	1·23

investigation all the three lines originate from a single upper level (7^3S_1) their dependence on $n_e(0)$ and T_e should be the same resulting in the same rate of radiation enhancement for the three lines. The present investigation shows that enhancement of spectral intensity is different for different lines and therefore a third factor which is different for the three lines is important in the process of radiation enhancement. Les and Niewodniczauski (1961) observed that the intensity ratios of the visible triplets of mercury atoms differ widely depending upon the condition of the source which has been explained by the phenomena of reabsorption of the lines. The lower levels of the lines are two metastables (6^3P_{02}) and a resonance level (6^3P_1). In the steady state they are supposed to build up appreciable populations, thereby causing self-absorption of the lines. As self-absorption affects the intensity of emission lines and is strongly related to the population of lower levels, the enhancement factor in the presence of magnetic field will also depend on population densities of lower levels of the lines. In the next section a quantitative estimate of this effect has been presented.

4. Self-absorption and enhancement factors in magnetic field

When there is an appreciable self-absorption spectral intensity I_{ul} of a line with upper level U and lower level l is given as

$$I_{ul} = \text{const. } A_{ul} \int_{-R}^{R} n_u(r)\left[\int \alpha(\nu)\exp\left\{-\beta(\nu)\sigma \int_r^R n_l(r)\,\mathrm{d}r\right\}\right]\mathrm{d}r. \tag{1}$$

where $n_{ul}(r)$ are the local number densities of the upper radiating level and lower level as a function of position r along the line of sight, A_{ul} is the transition probability of the line and $\alpha(\nu)$ is the normalized spectral emission profile $\int \alpha(\nu)\,\mathrm{d}\nu = 1$. The fraction of the emitted line which reaches the detector after traversing the medium from position r is

$$\exp\left\{-\sigma\beta(\nu)\int_r^R n_l(r)\,\mathrm{d}r\right\}.$$

σ is the absorption cross-section per atom at the line centre, independent of r and $\beta(\nu)$ is the line profile of absorption normalized to unity at the line centre $\beta(\nu_0) = 1$ and $r = 0$ at the centre of the discharge. When there is no self-absorption

$$I_{ul} = \text{const. } A_{ul} \int_{-R}^{R} n_u(r)\left[\int_{-\infty}^{\infty} \alpha_\nu\,\mathrm{d}\nu\right]\mathrm{d}r.$$

as $\int_{-\infty}^{\infty} \alpha(\nu)\,\mathrm{d}\nu$ the normalized emission profile is unity

$$I_{ul}^0 = \text{const. } A_{ul} \int_{-R}^{R} n_u(r)\,\mathrm{d}r.$$

Vriens *et al* (1978) and Uvarov and Fabrikant (1965) have shown that the excited mercury atom distribution function across the cross-section of a discharge is nearly parabolic. Considering a parabolic distribution of $n_u(r)$ we get

$$I_{ul}^0 = \text{const. } A_{ul}\tfrac{4}{3}n_u(0)R, \tag{2}$$

where $n_u(0)$ is the number density of radiating atoms at the axis of the discharge tube.

The self-absorption A_s of a spectral line is defined as

$$\begin{aligned} I_{ul} &= (1 - A_s)I_{ul}^0 \\ &= \text{const.}\,(1 - A_s)n_u(0)A_{ul}. \end{aligned} \tag{3}$$

When a longitudinal magnetic field B is present

$$(I_{ul})_B = \text{const. } (1 - A_s)_B[n_u(0)]_B A_{ul}. \tag{4}$$

From (3) and (4)

$$\frac{(I_{ul})_B}{I_{ul}} = \frac{(1 - A_s)_B}{(1 - A_s)} \frac{[n_u(0)]_B}{[n_u(0)]}. \tag{5}$$

If both the upper and lower level population densities are parabolic

$$n_x(r) = n_x(0)\left[1 - \frac{r^2}{R^2}\right],$$

we are assuming the source in which the radiating and absorbing atoms are distributed in the same manner. Now

$$1 - A_s = \frac{I_{ul}}{(I_{ul})^0}$$

$$= \frac{\left\{\int_{-R}^{R} n_u(r)\left[\int_{-\infty}^{\infty} \alpha(v)\exp\left(-\beta(v)\sigma\int_r^R n_l(r)\,\mathrm{d}r\right)\right]\right\}\mathrm{d}r}{\frac{4}{3}Rn_u(0)}. \tag{6}$$

Putting $y = r/R$

$$\sigma\int_r^R n_l(r)\,\mathrm{d}r = \sigma R n_l(0)\left[\frac{2}{3} - y\left(1 - \frac{y^2}{3}\right)\right].$$

Putting this value in (6) and replacing the exponential by the power series we get

$$\int_{-\infty}^{\infty} \alpha(v)\exp(-\beta(v)\sigma n_l(0)\left[\frac{2}{3} - y\left(1 - \frac{y^2}{3}\right)\right]\mathrm{d}v$$

$$= -\int_{-\infty}^{\infty} \alpha(v)\sum_{n=0}^{n=\alpha}\frac{(-1)^n R^n\sigma^n\beta^n(v)}{n!} n_l(0)^n\left[\frac{2}{3} - \left(y - \frac{y^3}{3}\right)\right]^n\mathrm{d}v$$

$$= \sum_{n=0}^{n=\infty}\frac{(-1)^n R^n\sigma^n n_l(0)^n}{n!}\left[\frac{2}{3} - \left(y - \frac{y^3}{3}\right)\right]^n\int_{-\infty}^{\infty}\alpha(v)\beta^n(v)\,\mathrm{d}v.$$

For the discharge type which is under consideration we assume that emission and absorption profiles are identical and gaussian in nature which is the outcome of Doppler broadening of spectral lines, all other broadening being neglected. For a Gaussian profile of absorption and emission Mosberg and Wilkie (1978) have shown that

$$\int_{-\infty}^{\infty}\alpha(v)\beta^n(v)\,\mathrm{d}v = \frac{1}{n+1}.$$

Then from (6) we get

$$1 - A_s = 1 - \frac{3}{4}\cdot\sum_{n=1}^{n=\infty}\frac{(-1)^{n+1}\sigma^n n_l(0)^n R^n}{n!\,(n+1)!}\left[\int_{-1}^{1}\left[\frac{2}{3} - \left(y - \frac{y^3}{3}\right)\right]^n(1 - y^2)\,\mathrm{d}y\right.$$

$$= 1 - \frac{3}{4}\sum_{n=1}^{n=\infty}\frac{(-1)^{n+1}\sigma^n n_l(0)^n R^n}{(n+1)!}\left[\int_{-2/3}^{2/3}\left[\frac{2}{3} - \left(y - \frac{y^3}{3}\right)\right]^n\mathrm{d}\left(y - \frac{y^3}{3}\right)\right]. \tag{7}$$

Values of $n_i(0)$'S are calculated utilizing Forrest and Franklin's (1969) equations and the procedure has been given in detail earlier (Sadhya and Sen 1980). To determine the values of $n_0(0)$, $n_1(0)$, and $n_2(0)$ the population densities of 6^3P_{012} levels at the axis of the discharge we have utilized the values of the collision integral $\langle Q_{ij}v \rangle$ given by Johnson *et al* (1978). The density of electrons at the axis $n_i(0)$ without a magnetic field is determined from the expression of current considering a parabolic distribution

$$i = \mu e E 2\pi \int_0^R n_i(0) r \, dr,$$

where μE is the drift velocity of electrons in mercury vapour at the corresponding (E/P) value determined by Nakamura and Lucas (1978). The procedure has been described earlier (Sadhya and Sen 1980). For a discharge with Maxwellian electron energy distribution i and current $= 2{\cdot}0$ amp, $P_{air} = 0{\cdot}5$ torr and $P_{Hg} = 0{\cdot}2729$ torr with the inner wall temperature $T_g = 96°C$ as measured and $T_e = 0{\cdot}412$ eV, the result of calculation is as follows

$$n_g = 9{\cdot}6 \times 10^{15}\ \text{cm}^{-3},$$

$$n_e(0) = 5{\cdot}95 \times 10^{13}\ \text{cm}^{-3},$$

$$n_0(0) = 3{\cdot}92 \times 10^{10}\ \text{cm}^{-3},$$

$$n_1(0) = 2{\cdot}38 \times 10^{10}\ \text{cm}^{-3},$$

$$n_2(0) = 1{\cdot}13 \times 10^{10}\ \text{cm}^{-3}.$$

Again σ the cross-section of absorption at the line centre, when Doppler broadening is considered the sole mechanism for the broadening of the spectral line, is given as

$$\sigma = \pi r_0 C f_{lu} \lambda_{ul} \left(\frac{M}{2\pi K T_g} \right)^{1/2}, \tag{8}$$

where r_0 is the classical electron radius ($2{\cdot}818 \times 10^{-13}$ cm), C is the velocity of light f_{lu} and λ_{ul} are the absorption oscillator strength and the wavelength of transition respectively, M is the mass of the mercury atom and K is the Boltzman constant. Taking f values of the transitions from Gruzdev (1967) the calculated values of σ (equation (8)) and the values of $k_0 = \sigma n_i(0)$ where k_0 is the absorption coefficient of radiation at the line centre have been entered in table 2. It is evident from table 2 that $k_0 R$ ($R = 0{\cdot}75$ cm) which may be called the optical depth is much smaller than unity. Since the series in (7) is a converging one we discard all the terms except the first term $n = 1$.

Table 2. Value of f, σ and k_0 for the triplets.

λ (A)	5461	4358	4047
f Gruzdev (1967)	0·14	0·11	0·10
σ (cm^2)	$6{\cdot}5 \times 10^{-12}$	$4{\cdot}077 \times 10^{-12}$	$3{\cdot}44 \times 10^{-12}$
k_0 (cm^{-1})	0·0735	0·0938	0·1348

Thus

$$1 - A_s = 1 - \tfrac{1}{3}\sigma n_l(0)R.$$
$$= 1 - f_{lu}\,\lambda_{ul}\rho n_l(0). \tag{9}$$

where

$$\rho = \tfrac{1}{3}\pi r_0 C\left[\frac{M}{2\pi K T_g}\right]^{1/2} R. \tag{9'}$$

Putting the value of $(1 - A_s)$ from (9) in (5) we obtain

$$\frac{(I_{ul})_B}{I_{ul}} = \frac{1 - f_{lu}\,\lambda_{ul}\rho[n_l(0)]_B[n_u(0)]_B}{1 - f_{lu}\,\lambda_{ul}\rho\,[n_l(0)]\,[n_u(0)]}$$
$$= \left[1 - f_{lu}\,\lambda_{ul}\rho\left\{[n_l(0)_B - n_l(0)]\frac{[n_u(0)]_B}{n_u(0)}\right\}\right]. \tag{10}$$

As shown earlier (Sadhya and Sen 1980) the electron temperature decreases and electron density along the axis increases when a mercury arc is placed in a longitudinal magnetic field. Equation (10) therefore indicates that due to these changes intensities of spectral lines will change in the presence of longitudinal magnetic field but will be lessened by self-absorption as $[n_l(0)]_B$ increases with B. The effects will be different for the three lines as f, λ and $[n_l(0)]_B$ will be different for them. As the lower levels of lines λ 5461 Å and λ 4047 Å are two metastable states and that of λ 4358 Å is a resonance level we can expect that the variation of intensity of the two lines λ 5461 Å and λ 4047 Å with magnetic field should be similar. In reality, however, the variation will depend upon the effect of magnetic field on the population density of the lower levels. It may be noted here that σ is maximum for λ 5461 Å line while $n_2(0)$ is relatively small. The case is reversed for λ 4047 Å and for λ 4358 Å it lies in between. Hence the variation as expected from the theory is consistent with experimental observation. However, as mentioned earlier, due to the coupled change of electron density and electron temperature with magnetic field both $n_l(0)$ and $n_u(0)$ will attain a saturated upper value. Moreover it is established that for a source of uniform excitation there will be no self-reversal (Cowen and Dieke 1948). On the other hand, the effect due to self-absorption will also reach a saturated maximum value. Thus we can expect that when B is sufficiently large there will be no change in enhancement factor with increase of the magnetic field. Figure 1 however shows a slow decline of the factor at that stage.

We now consider a discharge in sufficiently high magnetic field (B = 1500 gauss) so that all changes are saturated. In that case $[n_l(0)]_B \gg n_l(0)$. Since relative populations of the excited levels always obey a Boltzman distribution with T_e as temperature (Ritcher 1968), (10) may be rewritten as

$$\frac{(I_{ul})_{\max}}{I_{ul}} = \left[1 - f_{lu}\lambda_{ul}\rho\,\{n^0(0)\}_B \exp\frac{-(E_l - E_0)}{KT_B}\right]\frac{[n_u(0)]_B}{n_u(0)}, \tag{11}$$

where E's are energies of corresponding levels. A plot of

$$\frac{(I_{ul})_{\max}}{I_{ul}} \quad \text{against} \quad f_{ul}\lambda_{ul}\exp -\frac{(E_l - E_0)}{KT_B}$$

has been shown in figure 2 for two discharge conditions. The plots are straight lines as predicted by (11). It is observed that

(i) for $i = 2$ amps, $P_{\text{air}} = 0{\cdot}05$ torr,

slope of the line $= 3{\cdot}33 \times 10^4\ \text{cm}^{-1}$

(ii) for $i = 2{\cdot}25$ amp, $P_{\text{air}} = 0{\cdot}9$ torr,

slope is $5{\cdot}19 \times 10^4\ \text{cm}^{-1}$.

The slope is from (11) and (9)

$$\tfrac{1}{3}\pi r_0 CR \left(\frac{M}{2\pi K T_g}\right)^{1/2} \{n_0(0)\}_B \frac{\{n_u(0)\}_B}{n_u(0)}.$$

From the intercept of the curve for current = 2 amp,

$$\frac{\{n_u(0)\}_B}{n_u(0)} = 1{\cdot}4$$

and if $\{n_0(0)\}_B = 10$ times the magnitude of $n_0(0)$ i.e. $\{n_0(0)\}_B = 10^{11}\ \text{cm}^{-3}$ which is quite a plausible assumption, the calculated value of the slope will be given by

$$(\text{slope})_{\text{calc}} = \tfrac{1}{3} \times 3{\cdot}14 \times 2{\cdot}82 \times 10^{-13} \times 3 \times 10^{10} \times 0{\cdot}75$$
$$\times 10^{11} \times 1{\cdot}4 \times \left(\frac{200 \times 1{\cdot}6 \times 10^{-24}}{2 \times 3{\cdot}14 \times 1{\cdot}38 \times 10^{-16} \times 369}\right)^{1/2}$$
$$= 3 \times 10^4\ \text{cm}^{-1}.$$

which is in agreement with the value determined from the graph.

Thus from experimental results we observe that though the triplet radiation lines of mercury namely ($7^3S_1 \rightarrow 6^3P_{012}$) ($\lambda$ 5461 A, λ 4358 A, λ 4047 A) have the same upper

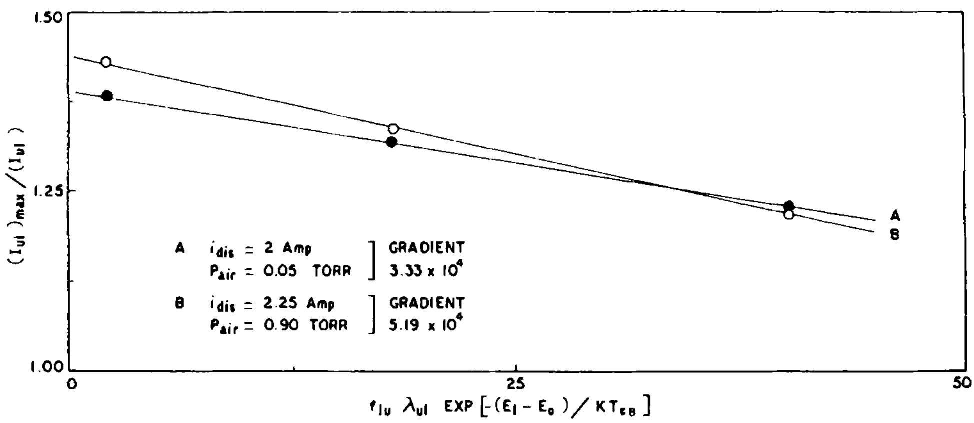

Figure 2. Variation of $(I_{ul})_{\max}/I_{ul}$ with $f_{lu}\lambda_{ul}\exp(-(E_l - E_0)/KT_{eB})$ (A) for arc current 2 amp. (B) for arc current 2·25 amp.

level the effect of magnetic field is different as regards the variation of intensity and the occurrence of maxima. The results have been explained on the assumption that the intênsities of these lines are governed by reabsorption which is different for the three spectral lines in a magnetic field; considering this effect an expression for $(I_{ul})_B/I_{ul}$ has been deduced which can explain qualitatively the behaviour in the variation of spectral intensities of the lines in a magnetic field. The slope of the line when $(I_{ul})_{\max}/I_{ul}$ is plotted against

$$f_{lu}\,\lambda_{ul}\exp\left[-\frac{(E_l-E_0)}{KT_B}\right].$$

gives a value which is in close agreement with the theoretically calculated value

$$\tfrac{1}{3}\pi r_0 CR\left(\frac{M}{2\pi KT_g}\right)^{1/2}\{n_0(0)\}_B\frac{\{n_u(0)_B\}}{n_u(0)}$$

for two different discharge currents. This can be accepted as a justification for the assumption that self-absorption plays a dominant role in the intensity profile of these lines.

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