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Velocity-dependent inertial induction and secular retardation of the earth's rotation

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Abstract. According to the model of inertial induction proposed earlier, the inertia force consists of an acceleration-dependent term which comes out as identically equal to $-m\mathbf{a}$. Besides, there is a velocity-dependent term which is exceedingly small to be easily detected. However, it has been shown that this results in a cosmological red shift of light coming from distant stars and galaxies; the magnitude of the red shift agrees very well with the observed values. Though this model yields correct results when applied to photons it needs modification before applying to other bodies. A modified form of the inertial induction model is now proposed where the proposed velocity-dependent inertia forces, when applied to the solar system, yields correct order of magnitude for the secular retardation of the earth's rotation. Moreover, a combined model using the velocity term and the tidal friction also does not suggest any close proximity of the moon to the earth in the past. When the model is applied to the case of Phobos, a secular acceleration of the order of magnitude of 10^{-3} deg yr^{-2} is obtained.

Keywords. Inertial induction; secular retardation; earth's rotation; tidal friction; velocity drag.

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1. Introduction

In an earlier paper the present author proposed a model of inertial induction including a term dependent on the relative velocity between two interacting objects (Ghosh 1984, hereafter referred to as I). Though that model yields correct results in case of objects moving with the velocity of light it needs modification for treating objects moving with any velocity. In the present paper a more generalized model is presented which results in the same cosmological red shift as shown earlier and the acceleration-dependent term becomes identically equal to the product of the mass and the acceleration.

When the model is applied to the earth-moon-sun system the local effects produce secular accelerations in good agreement with observations. It also explains the riddle of the absence of any evidence of moon's close approach 1300 million years ago as predicted by the tidal friction theory. This model also results in a large secular acceleration of the Phobos.

2. Theoretical model

As before we consider the universe to be homogeneous, infinite and in a steady state. The inertia force due to inertial interaction of a body with gravitational mass δM upon another mass m is proposed to be as follows:

$$\delta\mathbf{F} = -\frac{G\delta Mm}{r^2}\,\hat{u}_r - \frac{G\delta Mm}{c^2r^2}v^2\hat{u}_r\cos\theta|\cos\theta|$$
$$-\frac{G\delta Mm}{c^2r}a\hat{u}_r\cos\phi|\cos\phi|, \tag{1}$$

where G is the constant of gravitation, c is the velocity of light $\mathbf{r}(=\hat{u}_r r)$, $\mathbf{v}(=\hat{u}_v v)$ and $\mathbf{a}(=\hat{u}_a a)$ indicate the position, velocity and acceleration of m with respect to δM, and $\cos\theta = \hat{u}_r\cdot\hat{u}_v$, $\cos\phi = \hat{u}_r\cdot\hat{u}_a$. The total force due to inertial induction acting on a body of gravitational mass m moving with respect to the rest of the universe is given by

$$\mathbf{F} = -\int_0^\infty \frac{Gmv^2\hat{u}_v}{c^2r^2}\; 2\;\, 2\pi r^2\rho \int_0^{\pi/2} \sin\theta\cos^3\theta\, d\theta dr$$
$$-\int_0^\infty \frac{Gma\hat{u}_a}{c^2r}\; 2\;\, 2\pi r^2\rho \int_0^{\pi/2} \sin\phi\cos^3\phi d\phi dr,$$

where ρ is the density of matter in the universe. The position-dependent terms add up to zero because of symmetry. After integration over θ and ϕ we get

$$\mathbf{F} = -\int_0^\infty \frac{Gmv^2\hat{u}_v}{c^2r^2}\,\pi r^2\rho dr - \int_0^\infty \frac{Gma\hat{u}_a}{c^2r}\pi r^2\rho dr. \tag{2}$$

Using the same procedure as in I we assume the first term on the right side of (2) to be $-\frac{k}{c}mv^2\hat{u}_v$ after integration and following the same logic as explained in I we get $G = G_0 \exp[-(k/c)r]$ where G_0 is the gravitational constant near the source. Substituting the expression for G in (2)

$$\mathbf{F} = -\frac{G_0mv^2\hat{u}_v}{c^2}\,\pi\rho\int_0^\infty \exp[-(k/c)r]dr$$
$$-\frac{G_0ma\hat{u}_a}{c^2}\,\pi\rho\int_0^\infty r\exp[-(k/c)r]dr$$
$$= -\frac{\pi G_0\rho}{kc}mv^2\hat{u}_v - \frac{\pi G_0\rho}{k^2}\,ma\hat{u}_a. \tag{3}$$

But the coefficient of $mv^2\hat{u}_v$ has already been assumed to be $-(k/c)$. Hence, we get the following expression for $\mathbf{F}$:

$$\mathbf{F} = -\frac{k}{c}mv^2\hat{u}_v - ma\hat{u}_a, \tag{4}$$

and $k = (\pi G_0\rho)^{1/2}$. Using $G_0 = 6{\cdot}67\times10^{-11}$ m^3/kg sec^2 and $\rho = 7\times10^{-27}$ kg/m^3

$$k = 1{\cdot}21\times10^{-18}\ \text{sec}^{-1}$$

As explained in I it can again be shown that the velocity-dependent term gives rise to a cosmological red shift proportional to the distance of the source and k is nothing but the Hubble constant and this value agrees with the observed value of $1{\cdot}6\times10^{-18}$ sec^{-1}.

3. Tidal friction and secular retardation of earth's rotation

It is well established that the earth is gradually slowing down. Considerable work has been done on the subject and the references are too many to be listed here. However, a few could be mentioned (Munk and Macdonald 1960; Dicke 1966; Rosenberg and Runcorn 1975; Stacey 1977; Melchior 1978; McElhinny 1979). The only major contributing factor is the tidal friction according to the current theory. Available evidences are mainly of two types—astronomical and palaeontological. The currently accepted accelerations deduced from observations (Stacey 1977) are as follows:

$$\dot{\omega}_M \sim -1\cdot3 \times 10^{-23}\,\text{rad sec}^{-2},$$
$$\dot{\Omega} \sim -6 \times 10^{-22}\,\text{rad sec}^{-2},$$
$$\dot{R}_M \sim 1\cdot3 \times 10^{-9}\,\text{m sec}^{-1},$$

where $\dot{\omega}_M$ is the orbital angular velocity of the moon around the earth, Ω is the angular velocity of earth's spin and R_M is the radius of the moon's orbit. Though the frictional torque due to tide has been estimated based on the observational results and it is found that the magnitude is feasible the tidal friction theory suggests a very close approach of the moon to the earth about 1300 million years ago.* This does not agree with geological evidences which clearly indicate the presence of tidal phenomenon but no catastrophic event during the last 3500 million years.

Without going into further details it can be mentioned that the present theory is not completely satisfactory as reflected in a comment by Calame and Mulholland (1978)—"we must abandon the habit of treating the unexplained acceleration as being entirely of tidal origin and search for other causes that might contribute to it".

4. Theory combining velocity-dependent inertial induction and tidal friction

From equation (4) it is clear that for objects moving with velocities much less than that of light the velocity-dependent term due to universal interaction will be negligibly small. However, the presence of local massive objects can give rise to much higher values of velocity-dependent drag. Therefore, we consider the effect of sun's mass on the orbital motion of the moon and the earth's spin. In this paper exact analysis is not attempted and analyses yielding approximate values and orders of magnitudes will be presented.

Figure 1 shows a simplified model of the sun-earth-moon system. The effect due to the tilt of the axis of the earth's spin will be considered. Calculation shows that the effects due to other planets in the solar system are much smaller and can be neglected as a first approximation. The various effects** due to the velocity-dependent inertial induction are as follows:

(i) A resisting torque on the spinning earth due to the sun. This results in retardation of the earth's spin ($\dot{\Omega}$) and a retardation of the earth's orbital speed ($\dot{\omega}_E$). The order of

* Such a close approach would have been devastating both for the earth and the moon. Apart from this, tides would have been much stronger in the past as the tide amplitude is proportional to R_M^{-3}. This is also not supported by any sedimentological or other data.

** Since most of the sun's mass is concentrated towards its centre the effect of sun's spin has not been considered in this paper.

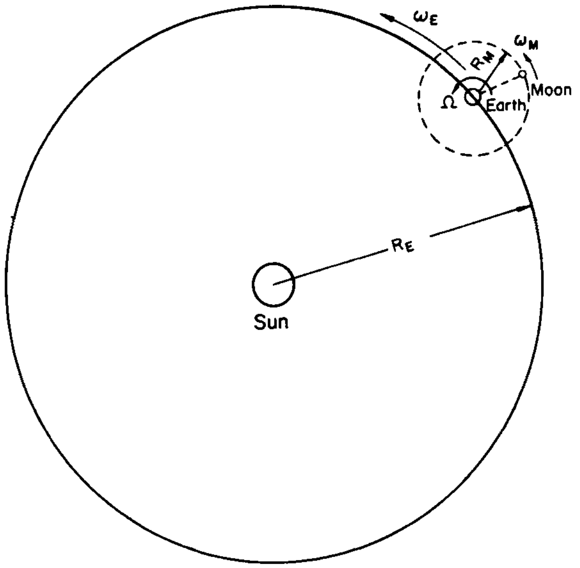

Figure 1. Simplified sun-earth-moon system.

magnitude of $\dot{\omega}_E$ is, however, very small compared to the other secular acceleration and retardation terms and can be neglected.

(ii) A resisting torque on the earth-moon system (due to their rotation) because of the sun. This results in an acceleration of the moon's orbital speed ($\dot{\omega}_M$). This also contributes a very small amount to $\dot{\omega}_E$ and can be neglected.

(iii) Resisting torque on the earth due to its spin because of the moon. This will result in a retardation of the earth's spin and a retardation of the moon's orbital motion ($\dot{\omega}_M$). The magnitude of this effect is also very small and can be ignored for getting approximate results.

The effect of tidal friction will be superimposed on all the above mentioned effects.

While estimating the torque on the spinning earth the radial variation of the earth's density has been taken into consideration. A linear variation* has been assumed. Since the sun is quite far all the lines joining the points on the earth and the sun have been considered to be parallel. The resisting torque due to the earth's spin can be expressed as follows:

$$T_{ES} = 0{\cdot}92 \cdot \tfrac{1}{3}\left(\frac{\alpha}{6} - \frac{\beta}{7}\right)\frac{G_0 M_S M_E \Omega^2 r_E^3}{c^2 R_E^2}, \tag{5}$$

where α and β are terms due to density variation. M_S is the sun's mass, M_E is the earth's mass, Ω is the earth's spin velocity, r_E is the earth's radius and R_E is the distance of the earth from the sun. The factor 0·92 has been introduced to take care of the tilt of the earth's axis. Using $\alpha = 6$ and $\beta = 5$, which is quite reasonable in case of the earth,

$$T_{ES} \sim 4{\cdot}75 \times 10^{16}\ \mathrm{N-m}.$$

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*$\rho(r) = [\alpha - \beta(r/r_E)]\rho_0$ is the form of density variation.

Using the same density variation the moment of inertia of the earth turns out to be** $8{\cdot}6 \times 10^{37}$ kg m^2. Thus

$$\dot{\Omega} \sim -5{\cdot}52 \times 10^{-22} \text{ rad sec}^{-2}.$$

Figure 2 shows the change of the number of days in a year (Ny) as obtained from the palaeontological studies (Scrutton 1978). Now

$$N_y = (\Omega/\omega_E) - 1$$

and since $\dot{\omega}_E$ is very small compared to $\dot{\Omega}$

$$\dot{N}_y \approx \dot{\Omega}/\omega_E. \tag{6}$$

From figure 2 it is seen that the required value of $\dot{N}_y$ is -3×10^{-45} sec^{-1} which yields $\dot{\Omega} = -6 \times 10^{-22}$ rad sec^{-2}. The extra amount of $-0{\cdot}48 \times 10^{-22}$ rad sec^{-2} is supplied by the tidal friction. Hence the magnitude of the torque due to tidal friction becomes $\sim 0{\cdot}41 \times 10^{16}$ N – m.

The torque on the rotating earth-moon system due to sun will be a fluctuating one and the average value of the resisting torque

$$T_{EM_s} \approx 0{\cdot}42 \frac{G_0 M_S M_E}{c^2 R_E^2} \left(\frac{M_M}{M_M + M_E} \right)^2 \omega_M^2 R_M^3, \tag{7}$$

where M_M is the mass of the moon and R_M is the distance of the moon from the earth.

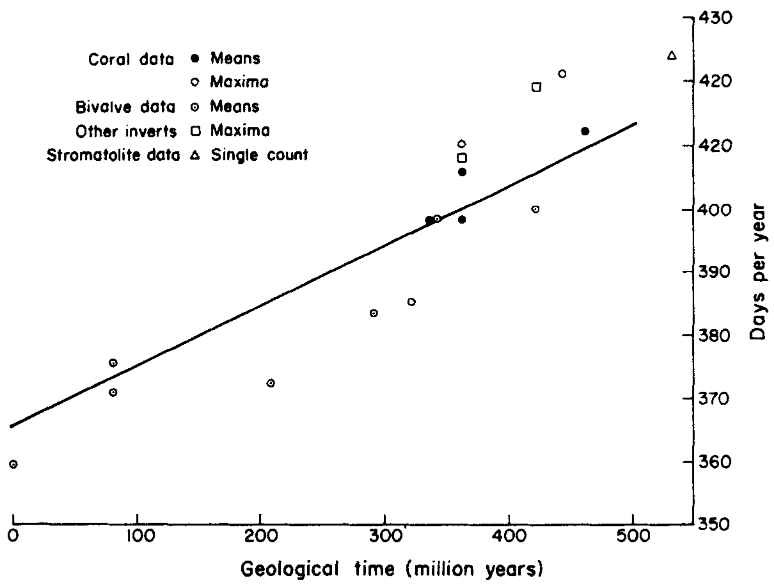

Figure 2. Graphical representation of palaeontological data for days per year.

** The exact value is $8{\cdot}04 \times 10^{37}$ kg m^2. This mismatch is because of the fact that actual density variation in the radial direction is not linear.

The angular momentum of the earth-moon system

$$H_{EM} = \frac{M_E M_M}{M_E + M_M} \cdot R_M^2 \cdot \omega_M. \tag{8}$$

Again the acceleration of the moon's orbital speed due to the sun

$$\dot{\omega}^S_{M_{av}} = -(3\omega_M T_{EM_s})/H_{EM}. \tag{9}$$

Equations (7), (8) and (9) finally results in

$$\dot{\omega}^S_{M'_{av}} \sim 0{\cdot}27 \times 10^{-23} \text{ rad sec}^{-2}.$$

However, the tidal friction will contribute and the amount of $-0{\cdot}11 \times 10^{-23}$ rad sec^{-2} towards $\dot{\omega}_{M_{av}}$ and finally the secular acceleration of the moon's orbital motion

$$\dot{\omega}_M \sim 0{\cdot}15 \times 10^{-23} \text{ rad sec}^{-2},$$

Again

$$\dot{R}_M = -\frac{2R_M}{3\omega_M}\dot{\omega}_M \sim -0{\cdot}15 \times 10^{-9} \text{ m sec}^{-1}.$$

The effect of moon on the earth's spin is about 0·5 % of that of the sun and it is neglected both for $\dot{\Omega}$ and $\dot{\omega}_M$ calculations.

Using the above results the apparent solar and lunar accelerations can be estimated as follows:

$$\dot{\omega}^{ap}_S = \dot{\omega}_E - \dot{\Omega}\frac{\omega_E}{\Omega} \sim 1{\cdot}65 \times 10^{-24} \text{ rad sec}^{-2},$$

$$\dot{\omega}^{ap}_M = \dot{\omega}_M - \dot{\Omega}\frac{\omega_M}{\Omega} \sim 2{\cdot}3 \times 10^{-23} \text{ rad sec}^{-2}.$$

5. Velocity-dependent inertial induction and the secular acceleration of Phobos

There are two possible ways in which the orbital motion of Phobos gets affected. There will be a secular acceleration of its orbital motion due to the velocity-dependent inertial induction with the sun. Combining the equations for the sun-mars-phobos system similar to those given by (7), (8) and (9) the secular acceleration of Phobos due to the sun

$$\dot{\omega}^S_{\text{Ph}_{av}} \approx \frac{4}{\pi} \cdot \frac{G_0 M_S}{c^2 R^2_{\text{Mars}}}\left(\frac{M_{\text{Ph}}}{M_{\text{Mars}}}\right)\omega^2_{\text{Ph}} R_{\text{Ph}},$$

where R_{Mars} is the distance of the Mars from the sun, M_{Ph} is the mass of Phobos, M_{Mars} is the mass of the Mars, ω_{Ph} is the orbital angular speed of Phobos and R_{Ph} is the orbital radius of Phobos. Substituting the respective values we get

$$\dot{\omega}^S_{\text{Ph}_{av}} \sim 5{\cdot}7 \times 10^{-28} \text{ rad sec}^{-2}.$$

Apart from this a retarding torque will act on the orbital rotation of the Phobos due to the relative spin of the Mars with respect to the Phobos. Considering a radial density variation of the Mars similar to that in case of the earth and neglecting the inclination effects (i.e. assuming all the lines joining points on the Mars and the Phobos to be approximately parallel) we can use an equation similar to (5). Thus the orbital acceleration due to the spin of the Mars can be expressed as follows:

$$\dot{\omega}_{\mathrm{Ph}}^{\mathrm{Mars}} \approx \frac{2}{7} \cdot \frac{G M_{\mathrm{Mars}} (\Omega_{\mathrm{Mars}}^{\mathrm{rel}})^2 R_{\mathrm{Mars}}^3}{c^2 R_{\mathrm{Ph}}^4},$$

where $\Omega_{\mathrm{Mars}}^{\mathrm{rel}}$ is the relative spin velocity of the Mars with respect to the Phobos. Substituting the values we get

$$\dot{\omega}_{\mathrm{Ph}}^{\mathrm{Mars}} \sim 26\cdot5 \times 10^{-21} \,\mathrm{rad\,sec}^{-2}$$

Since $\dot{\omega}_{\mathrm{Ph}_{\mathrm{av}}}^{\mathrm{S}} \ll \dot{\omega}_{\mathrm{Ph}}^{\mathrm{Mars}}$ we may take

$$\dot{\omega}_{\mathrm{Ph}} \sim 26\cdot5 \times 10^{-21} \,\mathrm{rad\,sec}^{-2} \equiv 1\cdot5 \times 10^{-3} \,\mathrm{deg\,yr}^{-2}.$$

Actually the value will be somewhat less when all the lines joining the points on the Mars and the Phobos are not assumed to be parallel. A secular acceleration of 1×10^{-3} deg yr^{-2} (of the Phobos) has been observed and reported (Pollack 1977) and it had not been very easy to explain this with the help of only tidal friction theory.

6. Discussion and conclusions

A direct comparison of $\dot{\Omega}$, $\dot{\omega}_M$ and $\dot{R}_M$ obtained by the tidal friction theory and the proposed model is not possible. Direct observation of $\dot{N}_y$, $\dot{\omega}_S^{\mathrm{ap}}$ and $\dot{\omega}_M^{\mathrm{ap}}$ is possible and it is seen that these values according to the proposed model are of correct orders of magnitude.

However, a very important difference with the tidal friction model emerges when the effect of the velocity-dependent drag is taken into consideration. According to this combined model $\dot{R}_M$ is very small and no close approach of the moon in the past is suggested. Thus a major difficulty can be resolved. If this theory is accepted the history of moon's orbital evolution has to be revised.

The velocity-dependent inertial induction also gives rise to a secular acceleration of the Phobos which agrees quite well with the observational results.

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