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# Transport coefficients of quark-gluon plasma

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**Abstract.** Transport coefficients of quark-gluon plasma are discussed in the framework of relativistic kinetic theory with the relaxation time approximation of Boltzmann transport equation. The expressions for the coefficients of shear and volume viscosities and heat conductivity are derived assuming quark-gluon plasma to be a non-reactive mixture of quarks, anti-quarks and gluons. A lowest order in deviations from local thermal equilibrium and in plasma phase, lowest order in coupling constant are assumed. Entropy production due to irreversible processes is discussed.

**Keywords.** Quark gluon plasma; neutron star; early universe; big bang; relaxation time; lattice Monte Carlo.

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## 1. Introduction

The possible formation of quark-gluon plasma (QGP) in ultra-relativistic heavy-ion collisions has recently attracted widespread interest (Anishetty *et al* 1980; Shuryak 1980). The most promising technique to produce extended regions of QGP is the collisions of heavy-ions at very high energies (c.m. energies $\sqrt{S} = 25$ GeV/nucleon). Then one may hope to achieve densities comparable to those in the core of a neutron star or in the early universe ($10^{-6}$ sec after big bang). For $T \gtrsim T_c \sim 200$ MeV or densities $n \gtrsim n_c =$ (few times) $n_0$, where $n_0$ is the density of cold nuclear matter ($= 0{\cdot}15$ nucleon/fm$^3$), a transition from colour-confined phase to colour-deconfined phase takes place.

So far the study of QGP has mainly been directed towards clarifying its equilibrium properties (Bjorken 1983; Baym *et al* 1983; Kajantie *et al* 1983). In this paper we report our study on some of the non-equilibrium properties, shear viscosity, volume viscosity and heat conductivity of QGP using relativistic version of Boltzmann equation, assuming first order in deviation from local thermal equilibrium and in the quark-gluon phase to the lowest order in the coupling constant.

The method of calculating kinetic coefficients are well established and have been applied, for instance, to classical gases (Chapman and Cowling 1970), Fermi liquid (Hojgaard *et al* 1969; de Groot *et al* 1980). In this paper, our calculations are mainly based on the work of van Erkenlens and van Leeuwen (1977, 1978) with some modification. In QGP we take the $u$ and $d$ quarks, their antiparticles and gluons only. The mixture of quarks and gluons is assumed to be non-reactive and the collisions are assumed to be binary. We shall consider central rapidity region and fragmentation regions separately.

## 2. Transport coefficients in the relaxation time approximation

Hydrodynamics is the most powerful technique to describe the macroscopic properties of QGP, when there is a small departure of the system from its local equilibrium configuration.

The interplay between microscopic (collisions) and macroscopic phenomena is supposed to be described by the relativistic form of the Boltzmann equation and is given by (Ehlers 1971)

$$L(f) = C(f), \tag{1}$$

where $L$ is the differential operator, given by

$$L \equiv p^\mu \frac{\partial}{\partial x^\mu} + \Gamma^\mu_{\nu\lambda} p^\nu p^\lambda \frac{\partial}{\partial p^\mu}, \tag{2}$$

and $C(x, p)$ is the collision integral, given by

$$\begin{aligned} C(x, p) = \tfrac{1}{2}\int \mathrm{d}\Gamma_1 \mathrm{d}\Gamma' \mathrm{d}\Gamma'_1 [ & f(x, p'_1) f(x, p')(1 \pm f(x, p))(1 \pm f(x, p_1)) \\ & \times W(p', p'_1 \,|\, p, p_1) - f(x, p) f(x, p_1)(1 \pm f(x, p'))(1 \pm f(x, p'_1)) \\ & \times W(p, p_1 \,|\, p', p'_1). \end{aligned} \tag{3}$$

The plus sign is for fermions, and the minus is for bosons, $\mathrm{d}\Gamma = g\mathrm{d}^3p/p^0(2\pi)^3$ $p^0$ is the zeroth component of four momentum $p^\mu \equiv (p^0, \mathbf{p})$ and $g$ is the effective number of degrees of freedom, $f(x, p)$ is the single particle distribution function, $W(p, p_1 \,|\, p', p'_1)$ is the transition rate from the momentum state $(p, p_1)$ to $(p', p'_1)$ of the colliding particles through binary collisions.

To solve the relativistic Boltzmann equation (1) for non-equilibrium processes approximation method had to be developed, just as in the classical case. Studies by Chernikov (1963), Marle (1969), Stewart (1969) and Anderson (1970) led to the formulation of relativistic Grad moment method which allows one to formally solve (1). This method suffers, however, from the practical problem of one's inability to evaluate the resulting collision integral (even numerically, particularly for QGP case, where we have to take all sorts of interaction between quarks, antiquarks and gluons) unless we restrict ourselves to expansions about the Maxwell-Boltzmann distribution.

The same problem exists with the classical Grad moment method and can overcome by use of the BGK (Bhatnagar *et al* 1954) relaxation-time model. The BGK model allows one to effectively handle distributions other than Maxwell-Boltzmann and is relatively easier to deal with than the Grad moment method, albeit one must pay the price of less accuracy. To handle distributions other than Maxwell-Boltzmann, it is necessary to develop a relativistic BGK model with

$$C(x, p) = -\frac{p^0}{\tau}(f(x, p) - f^{(0)}(p)), \tag{4}$$

where $\tau$ is the relaxation time,

$$f^{(0)}(p) = \frac{1}{\exp[\beta(p^\mu u_\mu - \alpha)] + 1}, \tag{5}$$

is the equilibrium single particle distribution function for fermions (with + sign) or bosons (with − sign), $\alpha$ is the chemical potential, $\beta = 1/T$ is a measure of the equilibrium temperature and $u^\mu \equiv (\gamma, \gamma v)$ is the hydrodynamic four velocity with $\gamma = 1/(1-v^2)^{1/2}$.

Then in a flat Minkowski space-time, (1) reduces to

$$[\Gamma^\mu_{\nu\lambda} = 0],$$

$$p^\mu \partial_\mu f(x, p) = -\frac{p^0}{\tau}(f - f^{(0)}). \tag{6}$$

The conservation laws for energy-momentum and baryon number current are given by

$$\partial_\mu T^{\mu\nu} = 0, \tag{7}$$

and

$$\partial_\mu j^\mu = 0 \tag{8}$$

respectively,
where

$$T^{\mu\nu} = \int d\Gamma_q p^\mu p^\nu (f_q + f_{\bar{q}}) + \int d\Gamma_g p^\mu p^\nu f_g \tag{9}$$

and

$$j^\mu = \int d\Gamma_q p^\mu (f_q - f_{\bar{q}}), \tag{10}$$

are respectively the energy-momentum tensor and baryon current density, the symbols $q, \bar{q}$ and $g$ stand for quark, anti-quark and gluon respectively. From these conservation laws one can derive the following dynamical equations of motion:

$$Dn = -n\nabla_\mu u^\mu, \tag{11}$$

$$Du^\mu = \frac{1}{nh}\nabla^\mu P, \tag{12}$$

$$C_v DT = -T\nabla_\mu u^\mu, \tag{13}$$

where $D \equiv u^\mu \partial_\mu$ is the convective time derivative and $\nabla^\mu$ is the gradient operator

$$\Delta^{\mu\nu}\partial_\nu, \Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu, g^{\mu\nu} = \mathrm{diag}(1, -1, -1, -1),$$

is the metric tensor,

$$n = \int d\Gamma p^\mu u_\mu f^{(0)}(p), \tag{14}$$

is the equilibrium particle density,

$$h = e + Pn^{-1}, \tag{15}$$

is the enthalpy per particle,

$$e = \frac{1}{n}\int d\Gamma (p^\mu u_\mu)^2 f^{(0)}(p), \tag{16}$$

is the energy per particle and

$$P = -\tfrac{1}{3}\int d\Gamma \Delta_{\mu\nu} p^\mu p^\nu f^{(0)}(p), \tag{17}$$

is the hydrodynamic pressure, $C_v = (\partial e/\partial T)_v$ is the specific heat per particle at constant volume.

The space-time gradient $\partial^\mu$ may be decomposed with respect to the hydrodynamic four-velocity $u^\mu$ into a time-like and a space-like part, according to

$$\partial^\mu = u^\mu D + \nabla^\mu. \tag{18}$$

Since our interest lies in the regime of small amplitude disturbances, we linearize the transport equation (6) by the ansatz

$$f(x, p) = f^{(0)}(p)(1 + \chi(x, p)). \tag{19}$$

Substituting (18) into (6) and using (19) and (5) for equilibrium distribution functions and eliminating $Dn$, $DT$ and $Du^\mu$ with the dynamical equations of motion and also using the relativistic Gibbs-Duhem relation in the form

$$\frac{1}{n}\nabla^\mu P = h\frac{\nabla^\mu T}{T} + T\nabla^\mu\left(\frac{\alpha}{T}\right), \tag{20}$$

we get

$$\chi = \frac{\tau(1 \pm f^{(0)})}{p^0 T}(QX + p^\mu p^\nu \mathring{X}_{\mu\nu} - p_\nu(p^\mu u_\mu - h)X_q^\nu), \tag{21}$$

where plus is for bosons (gluons) and minus is for fermions (quarks), $X$, $\mathring{X}_{\mu\nu}$ and $X_q^\nu$ are respectively thermodynamic forces for bulk viscosity, shear viscosity and heat conductivity and are defined as,

$$X = -\nabla^\mu u_\mu, \tag{22}$$

$$\mathring{X}_{\mu\nu} = (\nabla_\mu u_\nu - \tfrac{1}{3}\Delta_{\mu\nu}\nabla_\sigma u^\sigma), \tag{23}$$

$$X_q^\mu = \left(\frac{\nabla^\mu T}{T} - \frac{\nabla^\mu P}{nh}\right), \tag{24}$$

the quantity $Q$ is given by

$$Q = -\tfrac{1}{3}\Delta^{\mu\nu}p_\mu p_\nu + (1-\gamma)(p^\mu u_\mu)^2 + \left[T^2(1-\gamma) + \frac{\partial}{\partial T}\left(\frac{\alpha}{T}\right) - n\left(\frac{\partial\alpha}{\partial n}\right)\right]p^\mu u_\mu, \tag{25}$$

and is different for quarks, antiquarks and gluons, because of the difference in $\alpha$ and $\gamma = C_p/C_v$, $C_p = (\partial h/\partial T)_p$ is the specific heat per particle at constant pressure. The thermodynamic driving forces (22)–(24) are independent of each other, so that after substituting $\chi$ [equation (21)] in some expression, we can equate the coefficients of each of the thermodynamic forces independently from both sides (which we shall see later).

### 2.1 *Central rapidity region*

In the central rapidity region the net baryon content is effectively small; therefore the baryon chemical potential can be put equal to zero. This is also true for pure gluonic matter.

Using relativistic Gibbs-Duhem relation [equation (20)] it can be shown that in the central rapidity region, the heat conductivity term in (21) becomes zero and (25) reduces to

$$Q = (\tfrac{4}{3} - \gamma)(p^\mu u_\mu)^2 - \tfrac{1}{3}m^2. \tag{26}$$

In the central rapidity region, the quantity $Q$ is the same for both quarks and antiquarks, while for mass-less gluons, since $\gamma = \tfrac{4}{3}$, this quantity becomes zero.

Now in the presence of dissipative processes, one can decompose the energy-momentum tensor, the pressure tensor and the heat flow term into a reversible and an irreversible parts. These quantities are defined as

$$T^{\mu\nu} = \int \mathrm{d}\Gamma p^\mu p^\nu f(x, p), \tag{27}$$

is the energy momentum tensor,

$$\Pi^{\mu\nu} = \Delta^\mu_\sigma T^{\sigma\tau} \Delta^\nu_\tau + P \Delta^{\mu\nu}, \tag{28}$$

is the pressure tensor and

$$I^\mu_q = (u_\nu T^{\nu\sigma} - hn^\sigma) \Delta^\mu_\sigma, \tag{29}$$

is the heat flow, $n^\mu = nu^\mu$ is the particle four flow.

One can show that the reversible parts of the last two quantities are zero, while the reversible part of the energy-momentum tensor is given by

$$T^{(0)\mu\nu} = enu^\mu u^\nu - P\Delta^{\mu\nu}. \tag{30}$$

The irreversible parts of the three quantities are given by

$$T^{(1)\mu\nu} = \int \mathrm{d}\Gamma p^\mu p^\nu \chi(x, p) f^{(0)}(r), \tag{31}$$

$$\Pi^{(1)\mu\nu} = \int \mathrm{d}\Gamma \Delta^\mu_\sigma \Delta^\nu_\tau p^\sigma p^\tau \chi(x, p) f^{(0)}(p), \tag{32}$$

and

$$I^\mu_q = \int \mathrm{d}\Gamma p^\mu (p^\lambda u_\lambda - h) \chi(x, p) f^{(0)}(p). \tag{33}$$

To determine the shear viscosity coefficient of quark-gluon plasma, let us consider an one-dimensional steady motion of QGP having cylindrical symmetry. Then using the space-space component of the dissipative part of energy-momentum tensor (Landau and Lifshitz 1959), which is given by

$$T^{(1)rz} = -\eta \frac{\partial uz}{\partial r}, \tag{34}$$

and (31) and (26), we obtain after equating the coefficients of the driving forces for shear viscosity

$$\eta_g = \frac{\tau_g}{T} \int \frac{\mathrm{d}\Gamma}{p^0} (p^r p^z)^2 f^{(0)}_g (1 + f^{(0)}_g), \tag{35}$$

for gluonic matter, and

$$\eta_{q\bar{q}} = \frac{2\tau_q}{T} \int \frac{\mathrm{d}\Gamma}{p^0} (p^r p^z)^2 f^{(0)}_q (1 - f^{(0)}_q) \tag{36}$$

for quark-antiquark mixture, where $\eta$ is the coefficient of shear viscosity, $z$ is the direction of motion, $r$ is the axis of symmetry.

It can be shown that for mass-less gluons, the shear viscosity coefficient reduces to

$$\eta_g = 4{\cdot}329 \cdot \frac{g\tau_g T^4}{\pi^2} - \frac{gT\tau_g}{2\pi^2} \sum_{s=1}^{\infty} \frac{1}{s} \int_0^\infty p_z^2 z^2 \bar{E}(z) \bar{e}^z \mathrm{d}p_z, \tag{37}$$

where $\bar{E}(z) = \int_0^\infty \frac{e^{-x}}{x+z} \mathrm{d}x$ and $z = \beta p_z s$.

Let us calculate the volume viscosity coefficient of quark-gluon plasma, which arises from the departure of the system from thermodynamic equilibrium as it is compressed. Since for mass-less gluons $\gamma = \frac{4}{3}$, the volume viscosity term becomes zero, whereas for quark-antiquark mixture this term is non-zero.

We can obtain the volume viscosity coefficient $\eta_{vq\bar{q}}$ of quark-antiquark mixture from the relation

$$\Pi^{\mu}_{\mu} = -3\eta_v \nabla \cdot \boldsymbol{u} \tag{38}$$

and (32), by equating the coefficients of the corresponding driving forces from both sides. Then we have

$$\eta_{vq\bar{q}} = \frac{2\tau_q}{3T} \int \frac{\mathrm{d}\Gamma}{p^0} \Delta_{\mu\nu} p^{\mu} p^{\nu} Q_q f^{(0)}_q (1 - f^{(0)}_q). \tag{39}$$

Since thermal conduction involves the relative flow of baryon number, in the central rapidity region, for both the gluonic matter and quark-antiquark mixture, the heat conductivity term is zero.

The transport coefficients $\eta_g$, $\eta_{q\bar{q}}$ and $\eta_{vq\bar{q}}$ are evaluated numerically and the variation of the quantities $\eta_g/\tau_g, \eta_{q\bar{q}}/\tau_g$ and $\eta_{vq\bar{q}}/\tau_q$ with temperature has been shown in figures 1 and 2. It is seen that, the volume viscosity coefficient is $\sim 0.1\%$ of shear viscosity, which is in agreement with the results obtained by Gavin (1985).

## 2.2 *Fragmentation region*

In the fragmentation regions, the baryon chemical potential $\alpha \neq 0$, consequently the equilibrium distribution functions for quarks and gluons are different and are given by

$$f^{(0)}_q(p) = \frac{1}{\exp[\beta(p^{\mu}u_{\mu} - \alpha)] + 1}, \tag{40}$$

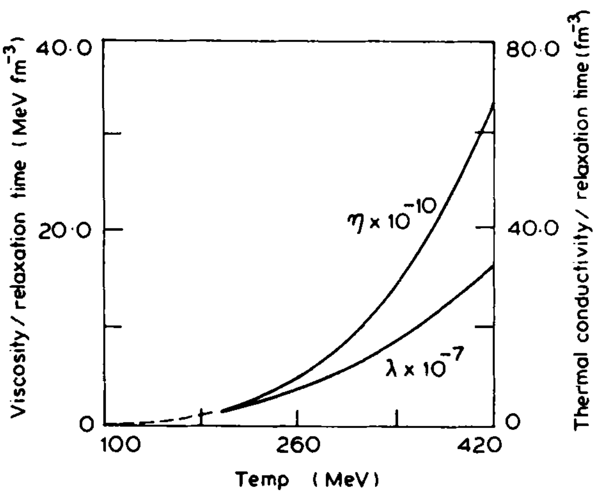


**Figure 1.** Variations of shear viscosity coefficient ($\eta$) and heat conductivity ($\lambda$) of gluonic matter, with temperature ($g = 16, m = 0$). Heat conductivity is non-zero only when the number of gluons are conserved.

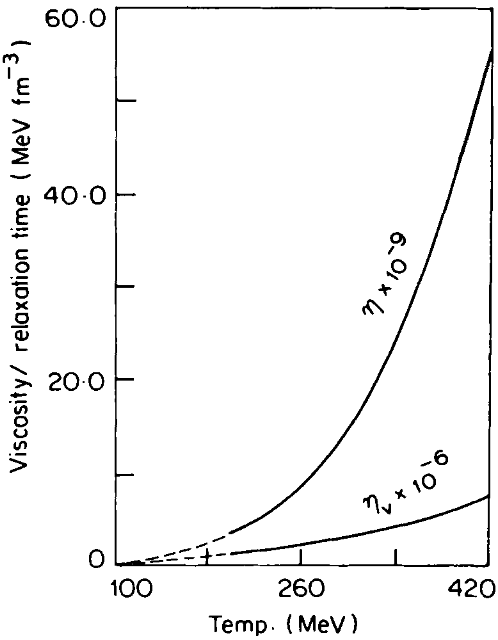


**Figure 2.** Variations of shear and bulk viscosity coefficients ($\eta$ and $\eta_v$) of quark-antiquark mixture with temperature in the central region of rapidity. ($g = 6$, $m = 10$ MeV, $\mu = 0$, contributions from current quarks).

$$f_{\bar{q}}^{(0)}(p) = \frac{1}{\exp[\beta(p^\mu u_\mu + \alpha)] + 1}. \tag{41}$$

In the fragmentation regions, all the driving force terms in the expression of $\chi$ for quark-antiquark mixture are non-zero.

Doing all the calculation in the same manner as have been done in the central rapidity region, leads to the shear viscosity coefficient and volume viscosity coefficient of quark-antiquark mixture

$$\eta_{q\bar{q}} = \frac{\tau_q}{T} \int \frac{\mathrm{d}\Gamma}{p^0} (p^r p^z)^2 [f_q^{(0)}(1 - f_q^{(0)}) + f_{\bar{q}}^{(0)}(1 - f_{\bar{q}}^{(0)})] \tag{42}$$

and

$$\eta_{vq\bar{q}} = \frac{\tau_q}{3T} \int \frac{\mathrm{d}\Gamma}{p^0} \Delta_{\mu\nu} p^\mu p^\nu [Q_q f_q^{(0)}(1 - f_q^{(0)}) + Q_{\bar{q}} f_{\bar{q}}^{(0)}(1 - f_{\bar{q}}^{(0)})]. \tag{43}$$

In the fragmentation region, the heat conduction term for quark-antiquark mixture is also non-zero, and the heat conductivity is given by

$$I_q^\mu = \lambda^{\mu\nu}\left(\nabla_\nu T - \frac{T}{nh}\nabla_\nu P\right), \tag{44}$$

where $\lambda^{\mu\nu}$ is some coefficient, related with the heat conductivity coefficient by the relation

$$\lambda = -\tfrac{1}{3}\Delta_{\mu\nu}\lambda^{\mu\nu}. \tag{45}$$

Then using (29), (44) and (45), we get

$$\lambda_{q\bar{q}} = -\frac{\tau_q}{3T^2}\int \frac{d\Gamma}{p^0}[(p^\lambda u_\lambda - h_q)^2 f_q^{(0)}(1-f_q^{(0)}) + (p^\lambda u_\lambda - h_{\bar{q}})^2 f_{\bar{q}}^{(0)} \times (1-f_{\bar{q}}^{(0)})]\Delta_{\mu\nu}p^\mu p^\nu. \tag{46}$$

The transport quantities $\eta_{q\bar{q}}$, $\eta_{vq\bar{q}}$ and $\lambda_{q\bar{q}}$ are evaluated numerically and the variations of $\eta_{q\bar{q}}/\tau_q$, $\eta_{vq\bar{q}}/\tau_q$ and $\lambda_{q\bar{q}}/\tau_q$ with temperature have been shown in figures 3 and 4 with chemical potential $\alpha = 100$ and 200 respectively. In this region also the volume viscosity coefficient is $\sim 0{\cdot}1\,\%$ of shear viscosity coefficient, which indicates that the volume viscosity is negligible with respect to the shear viscosity. If the total number of gluons in the QGP is conserved, then heat can be conducted through gluons also, despite the fact, that the gluons themselves carry zero baryon number. The thermal conductivity of gluonic matter can easily be derived following the methods of the previous treatment. Assuming gluons to be mass-less we have

$$\lambda_g = [4{\cdot}329T^3 + 0{\cdot}548\,h^2T - 2{\cdot}404\,hT^2]\frac{2\tau_g}{\pi^2}. \tag{47}$$

The variation of $\lambda_g/\tau_g$ with temperature has been shown in figure 1. It is seen from the numerical results that the major contribution of heat conduction comes from quark-antiquark mixture. Gluonic heat conductivity is $\sim 10^{-7}$ times that of heat conductivity of quark-antiquark mixture.

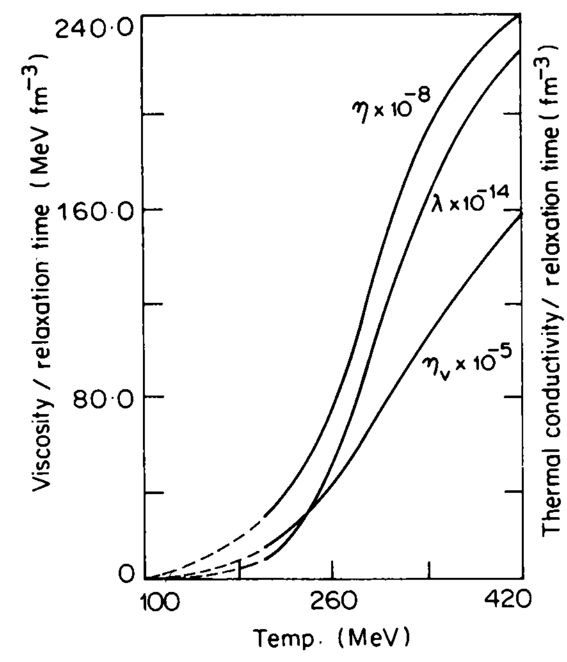


**Figure 3.** Variations of shear and bulk viscosity coefficients ($\eta$ and $\eta_v$) and heat conductivity ($\lambda$) with temperature, in the fragmentation region ($g = 6$, $m = 250$ MeV, $\mu = 100$ MeV, contributions from constituent quarks).

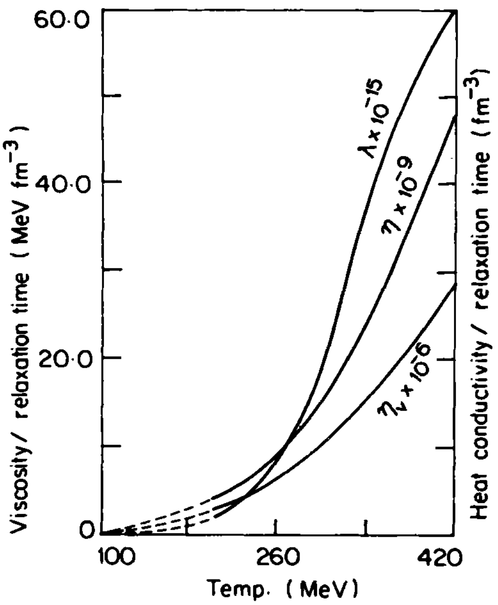


**Figure 4.** Variation of shear and bulk viscosity coefficients ($\eta$ and $\eta_v$) and heat conductivity ($\lambda$) with temperature in the fragmentation region ($g = 6$, $m = 250$, $\mu = 200$ MeV, contributions from constituent quarks).

## 3. Entropy production

Because of the presence of dissipative processes in QGP, entropy will be produced through irreversible processes. The entropy production $\sigma$, defined as

$$\sigma = \partial_\mu s^\mu \tag{48}$$

where $s^\mu$ is the entropy current, defined as

$$s^\mu = \int \mathrm{d}\Gamma p^\mu[-\{(1-f_q)\ln(1-f_q)+f_q\ln f_q\}-\{(1-f_{\bar{q}})\ln(1-f_{\bar{q}}) + f_{\bar{q}}\ln f_{\bar{q}}\}] + \int \mathrm{d}\Gamma p^\mu\{(1+f_g)\ln(1+f_g)-f_g\ln f_g\}, \tag{49}$$

$\sigma > 0$ for irreversible processes and $\sigma = 0$ when the system is in equilibrium. It can be shown that, in the first Chapman-Enskog approximation, the entropy production can be written as the sum of the products of irreversible flows and conjugate thermodynamic driving forces, then

$$\sigma = \frac{1}{T}(\Pi X + \mathring{\Pi}^{\mu\nu}\mathring{X}_{\mu\nu} - I^\mu_q X_{q\mu}), \tag{50}$$

where $\Pi = -\frac{1}{3}\Pi^\mu_\mu$ is the viscous pressure, the first term is the contribution from volume viscosity, the second term is from shear viscosity and the third term is from heat conduction.

## 4. Conclusion

We have studied the simplest transport coefficients, shear viscosity, volume viscosity and heat conductivity of quark-antiquark mixture and gluonic matter. The transport coefficients of QGP will be obtained simply by adding the contribution from quarks, antiquarks and gluons. This is true if QGP is assumed to be a non-reactive mixture of three constituents (which was one of our basic assumptions). But in a relativistic system the conditions are such that besides elastic collisions all kinds of inelastic collisions may occur. Particles may be created or destroyed or transformed into other particles. Thus a realistic description of non-equilibrium phenomena in realistic system must take reactive processes into account (Anderson 1976).

The calculations reported in this paper are also valid for $T \ll T_c$ (i.e. after hadronization) but cannot be applied directly to the region $T = T_c$ (phase transition regions) since the behaviours of transport coefficients are unknown in the critical region $T = T_c$. Transport coefficients of some of their derivatives may exhibit discontinuities at $T = T_c$. To understand the behaviour of transport coefficients near critical region, more powerful numerical methods based on lattice Monte Carlo techniques, should be developed, using Kubo formulas (Hosoya *et al* 1984).

All the calculations have been done with first order in deviation from the local thermal equilibrium and in the quark-gluon plasma phase to lowest order in the coupling constant. To extend to an arbitrary orders in the coupling constant, one has to develop a quantum theoretical formalism.

Another important transport property, the colour-conductivity, should be studied using Vlasov-Boltzmann equation (Ulrich 1983). The variation of colour-conductivity with temperature and its critical behaviour may give some idea of colour deconfinement phase transition (Baym 1979).

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