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Pionic corrections to the masses of heavier hadrons in the bag model

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Abstract. The influence of pionic corrections to the masses of heavier hadrons is investigated in the cloudy bag model. After determining the bag parameters from ordinary baryons and mesons the heavier hadron mass spectrum is calculated. A sizable amount of pionic correction is observed which has improved the masses favourably.

Keywords. Bag model; heavier hadrons; pionic correction.

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1. Introduction

Quantum chromodynamics (QCD) (Frazer and Henyey 1979) is regarded almost universally as the theory of strong interaction. The main features of QCD are (i) confinement, (ii) asymptotic freedom, and (iii) chiral symmetry (which arises naturally in massless QCD). Unfortunately, because of its complexity it is very difficult to solve QCD except in some limits, therefore a simplified picture is necessary. Guided by QCD, it is possible to derive a consistent phenomenological model which provides a framework to study the properties of hadrons quantitatively. The earliest and probably the most widely successful QCD motivated phenomenological model is the non-relativistic quark model (Rujula *et al* 1975; Isgur and Karl 1978, 1979) where confinement is incorporated through a phenomenological potential but asymptotic freedom and chiral symmetry does not appear naturally. A more plausible model is the MIT bag model (Chodos *et al* 1974a, b; DeGrand *et al* 1975) in which both confinement and asymptotic freedom were incorporated by letting the quarks to move freely inside the cavity only. Since MIT bag model badly violates chiral symmetry the third ingredient of QCD, many hybrid bag models (Chodos and Thorn 1975; Barnhill *et al* 1979; Brown and Rho 1979; Vento 1980; Jaffe 1979; Thomas 1982) have been constructed to include this feature also. The cloudy bag model (CBM) (Miller *et al* 1980; Theberge *et al* 1980; Thomas *et al* 1981) is one such hybrid bag model in which the nucleon consists of a spherical static cavity of radius R filled with three massless free quarks. A vacuum pressure B balances the pressure of the quark field on the surface and then enables the system to reach an equilibrium position. Around and inside the nucleon circulates a cloud of pions moving freely everywhere except at the bag surface where they can be emitted or absorbed. In CBM the size of the bag is determined by nonlinear effect in QCD unrelated to the pion, whose effects can then be treated as quantum fluctuations about the stable MIT bag model.

It has been shown (Thomas 1982; Miller 1984) that CBM has produced a number of striking results for the properties of single hadrons *e.g.* the neutron electric form factor, the magnetic moments of octet baryons, the proton life time and Δ-resonance. It is noticed that where pionic corrections have been computed the agreement with experiment is as good as, and usually better than, the original MIT bag model. The pionic corrections to the masses of hadrons in the bag model have been also calculated (Theberge, *et al* 1980; Thomas *et al* 1981; Cottinghem *et al* 1981; Myhrer *et al* 1981; Mulders and Thomas 1982) and it is found that these corrections are about as important as term form gluon exchange.

The masses of heavier hadrons have already been studied in symmetry considerations (Singh *et al* 1980; Singh 1981a), non-relativistic quark model (Singh and Khanna 1981; Singh 1981b; Singh *et al* 1981) and bag model (Jaffe and Kiskis 1976; Ponce 1979; Izatt *et al* 1982; Chatley 1983). In this paper following Mulders and Thomas (1982), we estimate the pion cloud contributions to the masses of heavier hadrons in the bag model. In §2 the bag energy including pion is discussed whereas §§3 and 4 contain estimation of hadron masses and conclusions, respectively.

2. Bag energy including pions

In the limit of static spherical cavity, the usual expression for the energy of the MIT bag is (DeGrand *et al* 1975)

$$E_{(R)} = E_V + E_Q + E_M, \tag{1}$$

where
$$E_V = \frac{4\pi}{3} BR^3 - \frac{Z_0}{R} \tag{2}$$

is the energy required to make a hole in the vacuum (BV) minus a phenomenological term (Z_0/R) originally attributed to zero point energy.

The quark kinetic energy is

$$E_Q = \sum_i N_i \omega_i, \tag{3}$$

where N_i is the number of quarks and antiquarks of the ith flavor. A quark of mass m moving in a spherical bag of radius R has a kinetic energy ω given by

$$\omega_{(mR)} = \frac{1}{R}[X^2 + (mR)^2]^{1/2}. \tag{4}$$

X is a function of mR and obeys the eigenvalue equation

$$\tan X = \frac{X}{1 - mR - [X^2 + (mR)^2]^{1/2}} \tag{5}$$

for a massless quark $X = 2{\cdot}043$.

The last term in (1) is the colour magnetic interaction associated with the gluon exchange between two quarks inside the bag. It is given as

$$E_M = \sum_{i>j} M_{ij} (F^c \bar{\sigma})_i \cdot (F^c \bar{\sigma})_j, \tag{6}$$

where,

$$M_{ij} = 8\alpha_s \frac{\mu(m_i R)\mu(m_j R)}{R^3} I(m_i R, m_j R), \tag{7}$$

$$\mu(mR) = \frac{R}{6}\left[\frac{4\omega R + 2mR - 3}{2\omega R(\omega R - 1) + mR}\right], \tag{8}$$

α_s is the effective quark gluon coupling constant, F^c and $\bar{\sigma}$ are, respectively, the colour and spin of the quarks. $I(m_i R, m_j R)$ is a slowly varying function of $m_i R$ and $m_j R$ and can be taken to be nearly equal to one.

In the CBM where pionic corrections are included as a perturbative contribution, the masses of hadrons are obtained from

$$M = [E_{(R)}]_{R_{\min}} + E_p, \tag{9}$$

where E_p is a phenomenological representative of the pion self energy. It has the form

$$E_p = -\frac{1}{pR^3_{\min}} \sum_{i,j} (\bar{\sigma}, \tau)_i \cdot (\bar{\sigma}\tau)_j, \tag{10}$$

where p is a phenomenological constant. This isospin-spin structure corresponds to keeping only the intermediate states with quarks in the lowest radial state, and treating all such states as degenerate. The eigenvalues of the operator

$$\Sigma_{\text{op}} = -\sum_{(i,j)\,\omega\,(u,d)} (\bar{\sigma}\tau)_i \cdot (\bar{\sigma}\tau)_j \tag{11}$$

are found to be (Mulders and Thomas 1982)

$$= \tfrac{7}{3}N^2 - 28N + 8f_c^2 + 4S(S+1) + 4I(I+1), \tag{12}$$

where N is the number of u/d quarks and S and I, respectively, are total spin and isospin of the ordinary quarks only. f_c^2 is the eigenvalue of colour, quadratic casimir operator.

3. Estimation of hadron masses

Using the expressions of § 2, the masses of different ground state hadrons can be written in terms of various parameters. Now, in order to estimate hadron masses numerically we have to first determine the values of the model parameters *viz* B, Z_0, α_s, m_s, m_c, m_b and p. For this we choose the known masses of p, Σ, Λ, Δ, ρ and Λ_c as input and get the following values of parameters.

$B^{1/4} = 0{\cdot}159$ GeV, $\quad Z_0 = 1{\cdot}804$

$\alpha_s = 1{\cdot}371$, $\quad m_s = 0{\cdot}215$ GeV

$m_c = 1{\cdot}63$ GeV, $\quad p^{1/2} = 1{\cdot}63$ GeV

For the b-quark mass we take $m_b = 4{\cdot}9$ GeV as found in some phenomenological calculations. Using these values of parameters the different contributions to the masses of mesons and baryons are calculated and are given in tables 1 and 2 respectively.

Table 1. Meson masses including pionic corrections.

Particles	Radius (GeV^{-1})	E_Q (GeV)	E_V (GeV)	E_M (GeV)	E_p (GeV)	Total mass (GeV)	MIT values (De Grand *et al* 1975; Jaffe and Kiskis 1976)	Chiral bag model (De Tar 1981)	Expt. values (GeV)
π	3·34	1·223	−0·440	−0·268	−0·243	0·271	0·280	0·355	0·139
K	3·26	1·386	−0·460	−0·235	−0·098	0·592	0·497	—	0·496
η_s	4·015	1·287	−0·276	−0·051	—	0·960	1·068	—	0·958
D_c	2·8	2·645	−0·585	−0·142	−0·155	1·763	1·726	—	1·865
F_c	2·84	2·764	−0·574	−0·119	—	2·070	1·885	—	2·03
η_c	2·31	4·039	−0·748	−0·099	—	3·193	2·931	—	2·96
D_b	4·18	5·446	−0·236	−0·025	−0·047	5·14	—	—	5·271
F_b	4·096	5·595	−0·256	−0·025	—	5·314	—	—	—
G_b	3·546	6·799	−0·389	−0·013	—	6·397	—	—	—
η_b	2·965	10·026	−0·539	−0·008	—	9·479	—	—	—
ρ	4·71	0·867	−0·103	−0·064	−0·058	0·769	0·783	0·824	0·776
ω	4·71	0·867	−0·103	0·064	−0·049	0·779	—	0·770	0·782
K^*	4·65	1·011	−0·119	0·052	−0·034	0·911	0·928	—	0·892
ϕ	4·61	1·154	−0·129	0·042	—	1·067	1·02	—	1·02
D_c^*	4·18	2·260	−0·236	0·023	−0·046	2·001	1·969	—	2·007
F_c^*	4·12	2·407	−0·251	0·019	—	2·176	2·099	—	2·14
ψ	3·53	3·641	−0·393	0·012	—	3·259	3·095	—	3·095
D_b^*	4·44	5·402	−0·172	0·007	−0·039	5·20	—	—	5·321
F_b^*	4·38	5·543	−0·186	0·006	—	5·363	—	—	—
G_b^*	3·837	6·751	−0·319	0·004	—	6·436	—	—	—
γ	3·21	9·967	−0·466	0·022	—	9·50	—	—	9·46

Table 2. Baryon masses including pionic corrections.

Particles	R (GeV^{-1})	E_Q (GeV)	E_V (GeV)	E_M (GeV)	E_p (GeV)	Total mass (GeV)	MIT values (De Grand *et al* 1975; Jaffe and Kiskis 1976)	Chiral bag model (De Tar 1981)	Expt. values (GeV)
N	5·00	1·226	−0·026	−0·090	−0·172	0·938	0·938	0·939	0·938
Λ	4·95	1·371	−0·040	−0·091	−0·112	1·129	1·105	—	1·116
Σ	4·95	1·371	−0·040	−0·066	−0·062	1·203	1·114	—	1·193
Ξ	4·91	1·515	−0·051	−0·078	−0·029	1·35	1·289	—	1·318
Σ_c	4·79	2·594	−0·082	−0·005	−0·069	2·437	2·357	—	2·43
Ξ_c	4·75	2·735	−0·093	−0·004	−0·032	2·607	2·507	—	—
Ω_c	4·71	2·872	−0·103	−0·010	—	2·759	2·653	—	—
Λ_c'	4·63	2·631	−0·124	−0·097	−0·137	2·272	2·214	—	2·273
Ξ_c'	4·75	2·735	−0·093	−0·080	−0·053	2·52	2·396	—	—
Ξ_{cc}	4·27	4·010	−0·022	−0·041	−0·044	3·708	3·538	—	—
Ω_{cc}	4·25	4·145	−0·219	−0·034	—	3·893	3·690	—	—
Δ	5·48	1·118	0·111	0·082	−0·076	1·236	1·233	1·230	1·236
Σ^*	5·43	1·263	0·096	0·071	−0·047	1·383	1·382	—	1·385
Ξ^*	5·39	1·403	0·085	0·061	−0·022	1·526	1·529	—	1·533
Ω	5·35	1·543	0·073	0·052	—	1·667	1·672	—	1·672
Σ_c^*	5·12	2·526	0·007	0·045	−0·056	2·525	2·461	—	2·48
Ξ_c^*	5·07	2·603	−0·007	0·039	−0·026	2·609	2·603	—	—
Ω_c^*	5·02	2·808	−0·021	0·032	—	2·819	2·742	—	—
Ξ_{cc}^*	4·69	3·924	−0·108	0·022	−0·033	3·805	3·661	—	—
Ω_{cc}^*	4·64	4·064	−0·121	0·018	—	3·961	3·795	—	—
Ω_{ccc}^*	4·21	5·308	−0·229	0·011	—	5·09	4·827	—	—

4. Conclusions

Taking pionic correction as a perturbative contribution to the MIT bag model, we have estimated its effect on the heavier hadron masses in the CBM, the size of the pionic correction E_p is of the order of gluonic contribution E_M. From the tables it is noticed that the resulting fit in case of baryons is quite good whereas in case of mesons, the results are not very close to the experimental results. This suggests that we need a more sophisticated treatment of the pionic corrections to the mesons. On comparing the present values with those of the MIT bag model results (DeGround *et al* 1975; Jaffe and Kiskis 1976) we observed an overall improvement. In the present analysis the mass of the strange quark is reduced from the MIT value of 0·279 GeV to 0·215 GeV which is closer to the 0·150 GeV, a value preferred by current algebra. The value of colour coupling constant α_s(1·37) found here is slightly better (lower) than those obtained by DeTar [1·7 −1·5] (1981); and Myhrer *et al* [1·55] (1981). The value of the phenomenological parameter $p^{1/2}$ found here (1·63 GeV) agrees with the value computed on the basis of chiral symmetry (Jaffe 1979; Thomas 1982) *i.e.*

$$p^{1/2} = \left(\frac{400\pi}{3}\right)^{1/2} \frac{f\pi}{g_A}$$
$$= 1{\cdot}52 \text{ GeV}$$

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