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Bose-Einstein condensation in spin-polarized atomic hydrogen: A new superfluid

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Abstract. We find that in the spin-polarized hydrogen, Bose condensation occurs for certain quantized values of the magnetic field. Once the field is fixed, sweeping of the radio-frequency results in nuclear magnetic resonance so that condensation and NMR occur simultaneously. We have found that nuclear self-induced transparency occurs. A new excitation designated by the present author as superboojum, which is a discontinuity in the hydrodynamic equations in spin-polarized hydrogen having finite nuclear as well as electronic spin is discovered.

Keywords. Bose-Einstein condensation; spin-polarized atomic hydrogen; superfluidity; phase transition.

1. Introduction

The spin-polarized hydrogen may be considered to be a new form of matter, a novel quantum fluid which may exhibit a superfluid phase. Therefore there is considerable effort to investigate the properties of this fascinating new material. The monoatomic hydrogen in a high magnetic field and low temperatures does not become ordinary molecular H_2 gas. In fact it is the only substance that on the basis of diatomic potential is predicted to remain a gas at absolute zero. Its Bose-Einstein condensation temperature is expected to be of the order of 0·2 K at a density of about 10^{19} atoms/cm^3. The superfluid spin-polarized hydrogen H$\downarrow$, would then be next only to the superfluid ^{4}He which is a liquid. The Cooper pairs of electrons form a charged fluid, the ^{3}He and the spin-polarized deuterium, D$\downarrow$, are fermions whereas H$\downarrow$ is a neutral boson. The superfluid ^{4}He, the spin-polarized hydrogen H$\downarrow$, and the spin-polarized tritium T$\downarrow$ would then form a class of materials which have spectacular properties. The mass of hydrogen is one fourth the mass of helium so that the Compton wavelength which measures the size of the atom is at least four times larger for H$\downarrow$ than for ^{4}He. Accordingly, the number density of H$\downarrow$ is much less than that of ^{4}He. The H$\downarrow$ has electronic spin of 1/2 and its nuclear spin is also 1/2 so that it has a finite hyperfine field whereas the electronic as well as the nuclear spin of ^{4}He is zero. Therefore the H$\downarrow$ is likely to exhibit phenomena which are not observable in ^{4}He.

We have found the following results: (i) the existence of boson waves on the spin-polarized atomic hydrogen floating on the surface of helium (Shrivastava 1984a, d); (ii) the phase transition in the spin-polarized hydrogen (Shrivastava 1984b); (iii) the probability of sticking of the spin-polarized hydrogen on the surface of the superfluid helium (Shrivastava 1984c) and (iv) the existence of the nuclear spin waves (Shrivastava 1984e). We have also studied the problem of recombination of the H$\downarrow$ atoms and have calculated the recombination time as a function of temperature (Lakshmi and Shrivastava 1985).

In the present lecture we shall discuss the following: (i) critical fields in Bose condensation in spin-polarized atomic hydrogen; (ii) nuclear self-induced transparency and (iii) the wave structure on the surface of the spin-polarized atomic hydrogen.

2. Critical fields in Bose condensation

We report a phenomenon of nuclear magnetic resonance which occurs simultaneously with Bose condensation in electron spin-polarized atomic hydrogen at fixed quantized magnetic fields. When vorton collapses into a boojum and the nuclear spins precess round a magnetic field, the boojum collapses into the spin-boojum and when the electronic as well as the nuclear spins precess round a magnetic field, we call the resulting pattern as the superboojum.

The boojum first found in ^{3}He, appears as a singularity on the surface in hydrodynamic equations (Mineev 1983). In the case of spin-polarized atomic hydrogen at high magnetic fields, the electronic part of the wavefunction is limited to the $|M_s = -1/2\rangle$ state while the nuclear part may be in either of the two $|M_I = \pm 1/2\rangle$ states. Stwally and Nosanow (1977) had suggested a limitation on the ratio of the field to temperature, $H/T > 10^6$ G/K, for the stability of the spin-polarized atomic hydrogen. Later Kagan *et al* (1982) suggested that the magnetic field must satisfy a condition with the singlet as well as the triplet scattering lengths. When the hydrogen atoms are confined between walls separated by a distance of $2a$, their eigenvalues are subject to the quantization on account of boundary conditions on the Schrödinger equation. If we do the nuclear magnetic resonance on such atoms, the magnetic field is quantized and limited by the condition of Bose condensation so that upon varying the radio-frequency we obtain a phenomenon of NMR-Bose condensation. Since we have proved (Shrivastava 1984d) the occurrence of cooperative behaviour, the experiments have indeed become feasible.

Let us look at the spin-dependent part of the single atom hamiltonian,

$$\mathscr{H} = g\,\mu_B H.S - g_N\mu_N\,H.I + A\,S.I \tag{1}$$

for a hydrogen atom of nuclear spin $I = 1/2$ and the electronic spin of $S = 1/2$ which are coupled by the hyperfine interaction A S.I. The coupled angular momentum is given by $\overline{\mathbf{F}} = \overline{\mathbf{S}} + \overline{\mathbf{I}}$ and the eigen functions $|M_s, m_F\rangle$ and given by

$$\begin{aligned}
|a\rangle &= |0, 0\rangle = (1+\varepsilon^2)^{-1/2}[|-\tfrac{1}{2}, \tfrac{1}{2}\rangle - \varepsilon|\tfrac{1}{2}, -\tfrac{1}{2}\rangle],\\
|b\rangle &= |1, -1\rangle = |-\tfrac{1}{2}, -\tfrac{1}{2}\rangle,\\
|c\rangle &= |1, 0\rangle = (1+\varepsilon^2)^{-1/2}[|\tfrac{1}{2}, -\tfrac{1}{2}\rangle + \varepsilon|-\tfrac{1}{2}, \tfrac{1}{2}\rangle],\\
|d\rangle &= |1, 1\rangle = |\tfrac{1}{2}, \tfrac{1}{2}\rangle,
\end{aligned} \tag{2}$$

with $\varepsilon = A/2\mu_B H \simeq 2\times 10^{-3}$ where μ_B is the Böhr magneton and H is the external magnetic field. The eigenvalues of (1) corresponding to the states (2) in the order of increasing energy are given by,

$$\begin{aligned}
E_a = E_{00} &= -A - [\{\tfrac{1}{2}g\,\mu_B H + \tfrac{1}{2}g_N\mu_N H\}^2 + 4A^2]^{1/2},\\
E_b = E_{1,-1} &= A - \tfrac{1}{2}g\,\mu_B H + \tfrac{1}{2}g_N\mu_N H,\\
E_c = E_{1,0} &= A + \tfrac{1}{2}g\,\mu_B H - \tfrac{1}{2}g_N\mu_N H,\\
E_d = E_{1,1} &= -A + [\{\tfrac{1}{2}g\mu_B H + \tfrac{1}{2}g_N\mu_N H\}^2 + 4A^2]^{1/2}.
\end{aligned} \tag{3}$$

The spin-polarized atoms are in the $|a\rangle$ and the $|b\rangle$ states. The $|a\rangle$ state is a boson while $|b\rangle$ has some spin dependence. If the H$\downarrow$ atom is confined between two walls separated by a distance of $2a$ the energy levels are quantized as

$$E_n = \pi^2\hbar^2 n^2/2m(2a)^2, \tag{4}$$

where m is the mass of the hydrogen atom and n is any integer ($n = 0, 1, 2, \ldots$). The net eigenvalues for the H$\downarrow$ atoms

$$\varepsilon_{n,F,m_F} = E_n + E_{F,m_F} \tag{5}$$

are doubly quantized, firstly by the boundary conditions and secondly by the rules of the angular momentum. The Bose-Einstein condensation occurs (Mullin 1980) in the $|a\rangle$ or in the $|b\rangle$ state. The average number of particles is determined by

$$N = n_0 + [\exp\{(E_n + E_{F,m_F} - \mu)\beta\} - 1]^{-1}, \tag{6}$$

where $\beta = 1/k_B T$, $n_0 = [\exp(-\mu\beta) - 1]^{-1}$ and μ is the chemical potential. When,

$$E_n + E_{F,m_F} - \mu = 0, \tag{7}$$

N diverges which is what we call the Bose-Einstein condensation. In case of the $|a\rangle$ state the condition (7) becomes,

$$\frac{\pi^2\hbar^2 n^2}{2m(2a)^2} - A - [\{\tfrac{1}{2} g\mu_B H_a + \tfrac{1}{2} g_N \mu_N H_a\}^2 + 4A^2]^{1/2} - \mu = 0. \tag{8}$$

If the hyperfine interaction is small, the above has a simple solution

$$H_a = \frac{[\pi^2\hbar^2 n^2/\{2m(2a)^2\}] - A - \mu}{\tfrac{1}{2}(g\mu_B + g_N\mu_N)}. \tag{9}$$

We fix the field at the above value which corresponds to the Bose condensation and turn the radio-frequency till we find the nuclear magnetic resonance from $|m_I = \tfrac{1}{2}\rangle$ to $|m_I = -\tfrac{1}{2}\rangle$ so we obtain a Bose-Einstein -NMR at a frequency of

$$\omega = \gamma H_a. \tag{10}$$

However, the field in (9) is quantized due to n^2 so that there are many resonances which

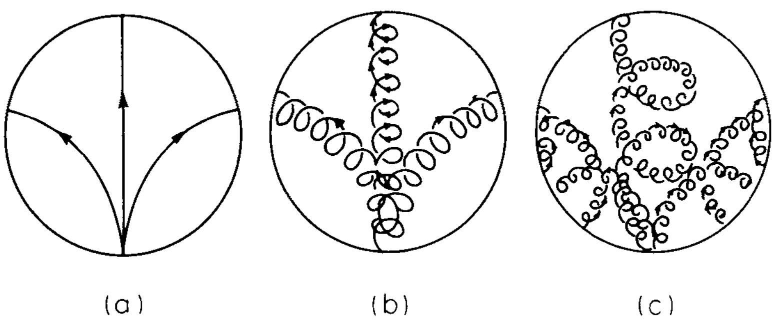

Figure 1. The superflow of the fluid on a spherical surface: (a) Spin zero propagation called the boojum, (b) superboojum with one nuclear spin and (c) the superboojum with one electronic and one nuclear spin as in the spin-polarized hydrogen.

satisfy (10). However, only the $n = 1$ may be observed. Since the expression (9) depends on the distance $2a$ between walls, the excitations are similar to boojums except that they have spins which we call the superboojum. An effort is made to sketch the boojums along with the super boojums in figure 1 (Shrivastava 1985).

We have thus found some new and interesting resonance-condensation effect.

3. Nuclear self-induced transparency

We report that in the spin-polarized atomic hydrogen, there is a new phase in which the nuclear spin reemits the radiation absorbed in a nuclear magnetic resonance so that it appears to be transparent to the radio-frequency wave. This work is of importance for recognizing a new superfluid. The nuclear magnetic moment of the atoms in the state $|a\rangle = |-\frac{1}{2}, \frac{1}{2}\rangle$ where the first index indicates the electronic spin projection M_s of one-hydrogen atom and the second, the nuclear spin projection m_I, is given by $g_N \mu_N I_a$. The number of atoms in this state is N_a. Similarly, the number of atoms in the state $|b\rangle = |-\frac{1}{2}, -\frac{1}{2}\rangle$ is N_b with magnetic moment per atom equal to $g_N \mu_N I_b$. Here g_N is the nuclear gyromagnetic ratio and μ_N is the nuclear magneton. The magnetic field experienced by an atom in the state $|a\rangle$ is $H + H_A$ and that in the state $|b\rangle$ is $H_b = H - H_A$ so that the $m_I = 1/2$ and $m_I = -1/2$ spins precess in opposite directions which follows from the time reversal. As the atoms are coupled by the exchange field H_E, the resonance frequency in the x-direction is $\omega_x = \gamma[H_A + 2H_E(1 - \cos ka)]$ where $\gamma = g_N \mu_N$. The wave of wave vector $\bar{k}$ carries the nuclear spin orientational energy and a is the average distance between atoms. When the external field is larger than $[2H_A H_E + H_A^2]^{1/2}$ there is an instability so that the nuclear spin vectors flop to a position essentially perpendicular to the external magnetic field. The direction of the maximum electronic spin is neither aligned along the field nor along the nuclear spin. We apply a radio-frequency field normal to the external magnetic field to perform a nuclear spin rotation of the ith proton. The life-time of the upper nuclear state is so small that the radiation absorbed is soon emitted so that the NMR lines occur as if the central part of their peaks are cut off and they are bell-shaped. However, the apparent shapes are slightly distorted because of the excitation of analogues of Walker modes. This is what we call as the nuclear self-induced transparency, which was first discussed by Shrivastava (1979). The life-time measurements by Yurke *et al* (1983) support our work on the nuclear self-induced transparency. The self-induced transparency for atoms illuminated by laser was discovered by McCall and Hahn (1969) and its acoustic analog was found by Shiren (1970).

4. Wave structure on the surface of spin-polarized hydrogen on the superfluid helium

The spin-polarized atomic hydrogen may be considered to be a new form of matter, a superfluid quantum gas at high magnetic fields and very low temperatures in which there has been considerable interest in recent years. We find a new wave structure on the spin-polarized atomic hydrogen. We consider the Ginzburg-Landau hamiltonian for the spin-polarized atomic hydrogen in the form (Shrivastava 1984a),

$$\mathscr{H}_q = \frac{\hbar^2 q^2}{2m} \langle n_q \rangle - \frac{1}{2} \frac{d_0 k_1 \langle n_q^2 \rangle q^2}{n_4 m_4 c_3^2 (1 + q^2 \lambda^2)(1 + q^2 \eta^2)} - b \frac{1}{\lambda^4}, \tag{11}$$

where $\langle n_q \rangle$ is the average number density of atomic hydrogen, d_0 is the effective depth of the helium film, n_4 is the number density of helium, m is the mass of hydrogen whereas m_4 is that of helium per atom. Here c_3 is the velocity of the third sound. The Lennard-Jones potential with parameters ε and σ is solved with a parameter $\zeta = z - u(r)$ with $u(r)$ the shape of the dynamical surface (Wilson and Kumar 1983). The fluctuations in the shape of the surface are determined by $\eta^2 = \langle \zeta^{-2} \rangle / 4 \langle \zeta^{-4} \rangle$, and the surface tension β_s determines the distance $\lambda = [\beta_s d_0 / m_4 n_4 c_3^2]^{1/2}$ and the constant k_1 is given by

$$k_1 = [2\pi\varepsilon\sigma^6 n_4 \langle \zeta^{-4} \rangle \eta^2]. \tag{12}$$

The term containing b is most important for the phase transition on the surface. We assume that $q \gg 1$, $q\eta \gg 1$ and minimize (11) with respect to q^2 so that we find a characteristic wave vector q_c given by

$$q_c^4 = -\frac{d_0 k_1 \langle n_{q_c} \rangle m}{n_4 m_4 c_3^2 \hbar^2 \eta^2 \lambda^2}, \tag{13}$$

for slow variation of $\langle n_q \rangle$ with wave vector q. The second term of (11) may be evaluated at q_c as

$$\mathscr{H}'_q = -\frac{1}{2} \frac{d_0 k_1 \langle n_{q_c}^2 \rangle |\psi|^2}{n_4 m_4 c_3^2 \eta^2 q_c^2} = -\frac{k_2 |\psi|^2}{q_c^2}, \tag{14}$$

where we have defined an order parameter from $\psi = 1/\lambda$ which means that we are looking for a surface phase transition. The free energy in the usual Ginzburg-Landau form is given by

$$F = (a - k_2 q_c^{-2})|\psi|^2 - b|\psi|^4. \tag{15}$$

Minimizing with respect to $|\psi|^2$ we find the critical value of the order parameter as

$$\lambda_c^{-2} = |\psi_c|^2 = (a - k_2 q_c^{-2})/2b. \tag{16}$$

Substituting (16) in (13) we obtain

$$q_c^2 (q_c^4 + a k_2 m / b\hbar^2) = k_2^2 m / b\hbar^2 \tag{17}$$

which means that for a given value of a, k_2 and m/b there are three wave vectors,

$$q_1^2 = q_+^2 + q_-^2, \tag{18a}$$

$$q_2^2 = \varepsilon q_+^2 + \varepsilon^2 q_-^2, \tag{18b}$$

$$q_3^2 = \varepsilon^2 q_+^2 + \varepsilon q_-^2, \tag{18c}$$

where

$$q_\pm^2 = (k_2^2 m / 2b\hbar^2)^{1/3} \left[1 \pm \left\{ 1 \pm \frac{4a^3 m}{27 k_2 b \hbar^2} \right\}^{1/2} \right]^{1/3} \tag{19}$$

and ε is a complex root of $x^3 = 1$. We see that one real root occurs. However, three real solutions can also occur so that various wave patterns or textures are found. Our conclusion is that patterns or textures occur over the spin-polarized atomic hydrogen which is floating over the superfluid helium. Our finding of the nuclear spin interaction complements the present result.

5. Conclusions

We have described some results which are useful for the search of a new superfluid, the spin-polarized atomic hydrogen. The nuclear magnetic resonance which avoids the depolarization is preferable to the electron spin resonance for the detection of the superfluid phase of the spin-polarized atomic hydrogen. The electron spin resonance depolarizes the atoms which can then form molecules. A study of the recombination time may also be useful to find the phase transition in the spin-polarized atomic hydrogen. Our calculations suggest the occurrence of a new superfluid which we hope will be found in the experiments in due course of time.

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References

Kagan Yu, Shlyapnikov G V, Vartanyants I A and Glukov N A 1982 *JETP Lett.* **35** 477
Lakshmi T V and Shrivastava K N 1985 *J. Phys.* C (in press)
McCall S L and Hahl E L 1969 *Phys. Rev.* **183** 457
Mineev V P 1983 *Sov. Phys. Usp.* **26** 160
Mullin W J 1980 *Phys. Rev. Lett.* **44** 1420
Shiren N S 1970 *Phys. Rev.* **B2** 2471
Shrivastava K N 1979 Pramana **13** 617
Shrivastava K N 1984a *Solid State Commun.* **51** 635
Shrivastava K N 1984b *Solid State Commun.* **52** 681
Shrivastava K N 1984c *J. Phys.* **C17** L973
Shrivastava K N 1984d *Proc. Int. Conf. Low Temperature Physics* (eds) U Eckern, A Schmid, W Weber and H Wuhl (Amsterdam: Elsevier) p. 535, 537
Shrivastava K N 1985 *Bull. Am. Phys. Soc.* **30** 43
Stwalley W C and Nosanow L H 1977 *Phys. Rev. Lett.* **36** 910
Wilson B G and Kumar P 1983 *Phys. Rev.* **B27** 3076
Yurke B, Denker J S, Johnson B R, Bigelow N, Levy L P, Lee D M and Freed J H 1983 *Phys. Rev. Lett.* **50** 1137