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Phase transition in gauge theories

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Abstract. In this short review we present the consequences of the spontaneously broken gauge theories will lead to when describing matter at high temperature and density. It appears various phase transitions should occur leading to the restoration of symmetry at high temperature of the originally broken one. Symmetry behaviour in external magnetic fields and in the early universe has been briefly mentioned.

Keywords. Symmetry restoration; phase transition.

I will summarize briefly the nature of phase transition in spontaneously broken gauge theories (SBGT) of fields and its application to particle phenomena and cosmology. Phase transition is a process with a change of symmetry which before or after the phase transition is spontaneously broken. Phase transition in SBGT is similar to those of Bose condensation or superconductivity. In particular one expects in spontaneous symmetry breaking gauge theories a phase transition with restoration of symmetry with the increase of temperature (Kirzhnits 1972) as in superconductivity where heating destroys superconductivity. Symmetry behaviour in gauge theories under the influence of external factors such as external fields and currents and density increase, etc. have been extensively studied during the past few years (Kirzhnits and Linde 1972, 1976). Before we go into it let us see the connection of SBGT to Bose condensation and other phenomenon.

Consider a Bose gas with occupation number, n_p given by

$$n_p = \langle a_{\mathbf{p}}^+ a_{\mathbf{p}} \rangle,$$

when $a_{\mathbf{p}}$ and $a_{\mathbf{p}}^+$ are the standard annihilation and creation operators for a scalar field operator.

Bose condensation implies nearly all particles occupy the state characterized by $\bar{p} = 0$, i.e.

$$\langle a_0^+ a_0 \rangle \simeq N,$$

where N is very large. Thus

$$a_0 |\Phi_0 (N)\rangle = \sqrt{N} |\Phi_0 (N-1)\rangle,$$

and

$$a_0^+ |\Phi_0 (N)\rangle = \sqrt{N+1} |\Phi_0 (N+1)\rangle.$$

That is a_0 and a_0^+ multiply Φ_0 by $\sqrt{N}$ and $\sqrt{N+1}$ which are very large and indistinguishable. Let us define new operators

$$\xi_0 = V^{-1/2} a_0, \quad \xi_0^+ = V^{-1/2} a_0^+.$$

Then

$$\xi_0|\Phi_0(N)\rangle = \sqrt{N/V}|\Phi_0(N-1)\rangle,$$

and $$\xi_0^+|\Phi(N)\rangle = [(N+1)/V]^{1/2}|\Phi_0(N+1)\rangle.$$

For $N \to \infty$, $V \to \infty$ with $N/V \to$ constant, ξ_0 and ξ_0^+ now multiply $\Phi_0(N)$ by a fixed number. But

$$[\xi_0, \xi_0^+] = 1/V \to 0.$$

Thus ξ_0 and ξ_0^+ behave as C-number quantities. Hence in the expansion of the scalar field $\phi(x)$ we have

$$\phi(x) = \xi_0 + \sigma$$

where ξ_0 stands for the classical part of the field $\phi(x)$ and the operator part is contained in σ. This is similar to spontaneously broken gauge theory where the symmetry breaking occurs through the non-vanishing vacuum expectation value of the Higgs scalar field $\phi(x)$ with $\phi(x) = u_0 + \sigma$, u_0 being $\langle 0|\phi|0\rangle$. This classical part u_0 of $\phi(x)$ in SBGT can be treated as the Bose condensation of the particles of the field ϕ at four momenta $p = 0$. Here in Higgs field Bose-condensation takes place because of the interaction of the scalar fields since $\lambda\phi^4$ term is essential to get the non-vanishing minimum u_0. The analogy is particularly interesting in the theory of superconductivity which according to Ginzburg and Landau (1950) is characterised by the energy E where

$$E = E_0 + \frac{1}{2}|H|^2 + \frac{|(\nabla - 2ie\mathbf{A})\psi|^2}{2m} - \alpha|\psi|^2 + \beta|\psi|^4.$$

Here E_0 is the energy of the normal metal without the magnetic field H, $\frac{1}{2}|H|^2$ is the magnetic energy and ψ stands for the classical Cooper field with mass m. This Lagrangian can be compared to the Higgs (1964) Lagrangian,

$$|(\partial_\mu - ieA_\mu)\phi|^2 + \mu^2\phi^+\phi - \lambda(\phi^+\phi)^2; \quad \lambda > 0,$$

which is essentially a covariant generalization of the previous Lagrangian.

The sign of the mass term is "wrong" as in the previous equation where the wrong sign of α leads to instability in the vacuum corresponding to the minimum $\psi = 0$.

It is well known that superconductivity is destroyed by the presence of external current or magnetic field. Salam and Strathdee (1974) suggested that ordinary magnetic field can lead to symmetry restoration in SBGT as in superconductivity. To see this let us consider the following Higgs model with the Lagrangian, given by

$$\mathscr{L} = -\frac{1}{4}\mathbf{F}_{\mu\nu}^2 + (D_\mu\boldsymbol{\phi})^2 + \frac{\mu^2}{2}(\boldsymbol{\phi})^2 - \frac{\lambda}{4}(\boldsymbol{\phi}^2)^2, \tag{1}$$

where $\mathbf{F}_{\mu\nu} = \partial\mu\,\mathbf{A}_\nu - \partial_\nu\mathbf{A}_\mu + 2e\mathbf{A}_\mu \times \mathbf{A}_\nu$; $D_\mu\boldsymbol{\phi} = \partial_\mu\boldsymbol{\phi} + e\mathbf{A}_\mu \times \boldsymbol{\phi}$,

and $\mathbf{A}_\mu$, $\boldsymbol{\phi}$ are $O(3)$ triplets:

In the tree approximation when the external field is absent the $O(3)$ symmetry of the Lagrangian (1) is spontaneously broken down to $O(2)$ by giving a non-vanishing vacuum expectation value to the third component of the Higgs scalar field $\boldsymbol{\phi}$, namely:

$$\langle 0|\phi^3|0\rangle = u_0,$$

where $u_0 = \mu\lambda^{-1/2}$ in tree approximation. As a result $A_\mu^{1,2}$ get masses but A_μ^3 remain mass-less. We identify this with external electromagnetic field. In the tree approximation the electromagnetic field has no effect on $\langle 0|\phi^3|0\rangle$ i.e. on u_0. But if quantum fluctuations due to the electromagnetic field (we consider here a uniform external magnetic field of strength H) is taken then this is no longer true. Let $\langle 0|\phi^3|0\rangle$ acquire a new non-vanishing value ψ. To find this new minimum we calculate one-loop-correction to the effective potential $V'(\psi, H)$. The contribution from the charged scalar field in the presence of the magnetic field of strength H is given by

$$V' = (2\pi)^{-4} \int d^4k \log G^{-1}, \tag{2}$$

where G is the propagator of the charged scalar field ϕ in presence of H with a mass $\lambda\psi^2 - \mu^2 \equiv m^2$. (If we put $\psi = u_0$, then $m = 0$ showing that in the tree approximation charged scalar field is massless). The expression for G is given by

$$G = (k_0^2 - k_H^2 + (2n+1)eH + m^2)^{-1},$$

where k_H is the momentum component along the magnetic field H [$A_2^3 = 1/2\, Hx$, $A_1^3 = -1/2\, Hy$ and $(2n+1)\, eH$ denotes the energy eigenvalues of the charged scalar field in presence of H; here $n = 0, 1, 2, 3, \ldots$.] Thus

$$V' = 2\pi eH \int dk_0\, dk_H \sum_{n=0}^{\infty} \log G^{-1}. \tag{3}$$

From (3) we deduce

$$\frac{dV'}{d\psi} = -\frac{\lambda e\psi}{8\pi^2} H.$$

Thus the minimum of the total Higgs potential is obtained from

$$\frac{dV}{d\psi} = 0 = \psi[\lambda\psi^2 - \mu^2 - (8\pi^2)^{-1} \lambda eH].$$

If the scalar ϕ^3 contribution is added then one obtains

$$\psi[\lambda\psi^2 - \mu^2 + a(8\pi^2)^{-1} \lambda^2\mu^2 - (8\pi^2)^{-1} \lambda eH] = 0 \tag{4}$$

with $a = 2{\cdot}5$ approximately.

Thus symmetry restoration is possible for the following critical value H_c of H given by

$$\frac{\lambda eH_c}{8\pi^2} = a\frac{\lambda^2\mu^2}{8\pi^2} - \mu^2.$$

Thus for $a\lambda^2 > 8\pi^2$ we find

$$H_c \simeq \frac{8\pi^2}{\lambda e}\left(\frac{a\lambda^2\mu^2}{8\pi^2} - \mu^2\right).$$

We thus see that the presence of external magnetic field can restore the symmetry. If one includes fermions in the Lagrangian (1) and consider the H-dependent term in V' due to fermion loop contribution then symmetry breaking increases with the strength of H but vector-meson loop contribution however restores the symmetry. For a reasonable value of the symmetry breaking parameter, u_0, the value of H_e turns out to be of the order of 10^{16} Gauss. This value stimulated some attempts to discover the phase transition in theories with spontaneous symmetry violation. For example Salam and Strathdee (1974) proposed that CP-violation in the K_L decay to the charged modes $\pi^{\pm} + e^{\mp} + \nu_e$ may disappear due to external magnetic field $H_c \simeq 10^{16}$ Gauss in theories where CP-violation may occur through spontaneous symmetry breaking. But the maximum strength of H_c that can be present in a nucleus is only of the order of 10^{11} Gauss; thus the hope to discover phase-transition in a laboratory in theories with spontaneous symmetry violation seems a little remote.

Before we take up the case of symmetry restoration in SBGT at high temperature let us consider an important case of symmetry restoration in a non-gauge theory due to density increase originally considered by Lee and Wick (1974). These authors considered a simplified non-gauge model with a scalar field ϕ interacting with a massless spinor field ψ. The Lagrangian considered is

$$\mathscr{L} = -\tfrac{1}{2}(\partial_\mu \phi)^2 + \tfrac{1}{2}\mu^2\phi^2 - \tfrac{1}{4}\lambda\phi^4 - \bar{\psi}(\gamma_\mu \partial_\mu + g\phi)\psi.$$

Lee and Wick (1974) showed that in this model symmetry restoration takes place when the fermion density

$$j_0 = \langle \psi^+ \psi \rangle$$

increases.

To see this one calculates the one-loop contribution to the effective potential due to the fermions in the presence of dense gas. Thus the fermion propagator is changed to take into this effect by simply adding $i\alpha$ to the energy k_0 of the fermion where α is the chemical potential of the fermion (Wallecka 1974). If u_0 denotes the non-vanishing vacuum expectation value $\langle 0|\phi|0\rangle$ the equation determining the minimum of the classical total effective potential is

$$u_0\left[u_0^2\lambda - \mu^2 + \frac{1}{2}g^2\left(\frac{9j_0^2}{\pi^2}\right)^{1/3}\right] = 0.$$

Thus the symmetry restoration takes place when j_0 takes the critical value j_0^c given by

$$j_0^c = 2\sqrt{2}\cdot\frac{\pi}{3}(\mu/g)^3.$$

This is the critical fermion density beyond which symmetry is restored (see figure 1). We thus see that there is a second-order phase transition for the symmetry-breaking parameter, u_0. In the language of Lee and Wick this transition implies that above j_0^c the normal nuclear matter turns into an abnormal nuclear state. The possibility of such transition may be noted in heavy-ion-collision processes. It is worth noting that in a

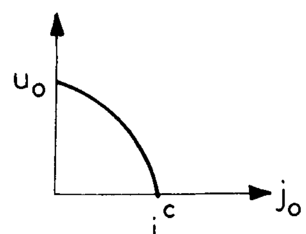

Figure 1. Phase transition of the symmetry breaking parameter u_0 showing the existence of abnormal matter.

gauge theory, however, such a phase transition due to density increase does not take place. This is what was first pointed out by Linde (1976).

In the model of Lee and Wick the current $j_\mu = \langle \bar{\psi}\gamma_\mu\psi \rangle$ does not interact with vector fields whereas in gauge model it interacts with a neutral vector field. In theories when j_μ interacts with vector fields an increase in the fermion charge-density, j_0, actually leads to symmetry breaking with j_0 increasing whereas j increasing leads to symmetry restoration. Linde considered the original Higgs model extended by the inclusion of a fermion and a scalar interaction characterized by a coupling constant g_σ.

For instance the Lagrangian considered has the structure,

$$\mathscr{L} = \frac{1}{4} F_{\mu\nu}^2 - (\partial_\mu + ig A_\mu)\phi^+ (\partial_\mu - ig A_\mu)\phi + \frac{1}{2}\mu^2\phi^+\phi - \frac{\lambda}{4}(\phi^+\phi)^2$$
$$- \bar{\psi}(\gamma_\mu\partial_\mu)\psi + ie\,\bar{\psi}\gamma_\mu A_\mu\psi - \frac{g_\sigma}{\sqrt{2}}\bar{\psi}(\phi^+ + \phi)\psi. \qquad \text{[I]}$$

Notice the "wrong sign" of the mass-term of the Higgs field. We suppose the system has non-vanishing chemical potential so $\langle \bar{\psi}\gamma_\mu\psi \rangle$ = a constant $\neq 0$. We can therefore assume the classical part of the A_μ field is non-vanishing as we have no reason to expect translational invariance to be broken as the current is constant.

To understand the spontaneous breaking of symmetry we shift the fields and assume the classical parts of ϕ and A_μ be constant in space and time;

$$\phi(x) \to \frac{1}{\sqrt{2}}(u_0 + \sigma)\exp\{i[\xi(x)/u_0]\},$$

and

$$A_\mu \to \omega_\mu + \frac{1}{gu_0}\partial_\mu\xi(x),$$

where now $\langle 0|\phi|0 \rangle = u_0/\sqrt{2}$, $u_0^2 = 2\mu^2/\lambda$, $\langle \sigma \rangle = 0$,

and $\langle \omega_\mu \rangle = c_\mu \neq 0$.

After the shift the Lagrangian becomes

$$\mathscr{L}' = -\frac{1}{4}F_{\mu\nu}^2 - \bar{\psi}(\gamma_\mu\partial_\mu + g_\sigma u_0 - ig\omega_\mu\gamma_\mu)\psi - g_\sigma\bar{\psi}\psi\sigma$$
$$-\frac{1}{2}(\partial_\mu\sigma)^2 - \left(-\frac{\mu^2}{4} + \frac{\lambda}{4}\frac{3}{2}u_0^2\right)\sigma^2 - \left(-\frac{\mu^2}{2} + \frac{\lambda}{4}u_0^2\right)u_0\sigma$$
$$-\frac{\lambda}{16}\sigma^4 - \frac{\lambda}{4}u_0\sigma^3 + \frac{\mu^2}{4}u_0^2 - \frac{\lambda}{16}\mu_0^4. \qquad \text{[II]}$$

Calculating the functional derivatives of $\mathscr{L}'$ with respect to the σ-field, ω_μ-field and ψ field respectively and equating them to zero with

$$\langle \sigma \rangle = 0; \quad \langle \omega_\mu \rangle = c_\mu; \quad \text{and} \quad \langle \bar{\psi}\psi \rangle = \rho_S$$

and further neglecting the fluctuations of the higher powers of σ and ω_μ we get in the tree-approximation

$$\left\langle \frac{\delta\mathscr{L}}{\delta\sigma} \right\rangle = 0 = -g^2 C_\mu^2 u_0 - g_S\rho_S - \left[-\frac{\mu^2}{2} + \frac{\lambda}{4}u_0^2\right]u_0, \qquad \text{(a)}$$

$$\left\langle \frac{\delta \mathscr{L}}{\delta \omega_\mu} \right\rangle = 0 = -u_0^2 g^2 C_\mu + g j_\mu; \quad j_\mu = \langle \bar{\psi} \gamma_\mu \psi \rangle, \tag{b}$$

and

$$\left\langle \frac{\delta \mathscr{L}}{\delta \psi} \right\rangle = 0 = (\gamma_\mu \partial_\mu + g_\sigma u_0 - ig \gamma_\mu \omega_\mu) \psi. \tag{c}$$

Consider equations (a) and (b) and for the moment assume $\rho_S = 0$. We get from (b)

$$C_\mu = g \frac{j_\mu}{u_0^2 g^2} = \frac{j_\mu}{u_0^2 g}$$

and from (a)

$$-\frac{g^2 j_\mu^2 u_0}{u_0^4 g^2} - \left[-\frac{\mu^2}{2} + \frac{\lambda}{4} u_0^2 \right] u_0 = 0,$$

or

$$u_0 \left[\frac{\lambda}{4} u_0^2 - \frac{\mu^2}{2} \right] = \left(j_0^2 - \mathbf{j}^2 \right) \Big/ u_0^3,$$

or

$$u_0^4 \left[\frac{\lambda}{4} u_0^2 - \frac{\mu^2}{2} \right] = \left(j_0^2 - \mathbf{j}^2 \right). \tag{d}$$

Hence from (d) we see when $\mathbf{j} = 0, j_0 \neq 0$ and j_0 increases, $u_0^2 \gg 2\mu^2/\lambda$

Thus the vacuum experimental value $\langle \phi \rangle$ increases.

For $j_0 = 0, \mathbf{j} \neq 0$ we see that when $\mathbf{j}$ increases, $u_0^2 \ll 2\mu^2/\lambda$ or vacuum expectation value u_0 decreases.

Thus we see that with increase of fermion charge density symmetry breaking increases whereas the increase of fermion current density leads to restoration of symmetry.

To see the temperature-dependence (Linde 1976) of the symmetry-breaking parameter we solve the Lagrangian [I] by writing,

$$\phi(x) \to \frac{1}{\sqrt{2}} (\phi_1 + u_0 + i\phi_2).$$

and

$$A_\mu \to \omega_\mu + c_\mu.$$

Shifting the Lagrangian and calculating the $\langle \delta\mathscr{L}/\delta\phi_i \rangle = 0$ and $\langle \delta\mathscr{L}/\delta\omega_\mu \rangle = 0$ we get assuming $\langle \phi_i \rangle = \langle \phi_i^3 \rangle = 0$ and that at small temperature one can also assume $\langle \omega_\mu \phi_i \rangle = 0$ and $\langle \phi_1 \partial_\mu \phi_2 - \phi_2 \partial_\mu \phi_1 \rangle = 0$, the following two equations instead of (a) and (b) as obtained before namely;

$$-u_0 \left[\frac{\lambda}{4} u_0^2 - \frac{\mu^2}{2} + \frac{\lambda}{4} \left\{ 3 \langle \phi_1^2 \rangle + \langle \phi_2^2 \rangle \right\} - g^2 C_\mu^2 - g^2 \langle \omega_\mu^2 \rangle \right] = 0 \tag{a'}$$

and

$$g^2 c_\mu u_0^2 + g^2 C_\mu \left[\langle \phi_1^2 \rangle + \langle \phi_2^2 \rangle \right] - g j_\mu = 0. \tag{b'}$$

The temperature dependence can be easily calculated from the thermodynamic averages of $\langle \phi_i^2 \rangle$ etc. For example,

$$\langle \phi^2 \rangle = (2\pi)^{-3} \int d^3 p (2 \langle a_p^+ a_p \rangle + 1)(2\omega_p)^{-1}.$$

Discarding the temperature-dependent effect in $\int d^3p(2\omega_p)^{-1}$, which can be eliminated by mass-renormalization at $T = 0$, one obtains

$$\langle \phi^2 \rangle = (2\pi^2)^{-1} \int_0^{\infty} p^2 \, dp \, (p^2+m^2)^{-1/2} [\exp(p^2+m^2)^{1/2}/T - 1]^{-1}.$$

Thus for $T > m^2$, we evaluate the above integral and obtain,

$$\langle \phi^2 \rangle = \frac{T^2}{12} = -\frac{1}{3}\langle \omega_\mu^2 \rangle.$$

Thus (a′) and (b′) are replaced by

$$u_0\left[\frac{\lambda}{4}u_0^2 - \frac{\mu^2}{2} - g^2 C_\mu^2 + \frac{T^2}{12}(3g^2+\lambda)\right] = 0,$$

$$j_\mu - gC_\mu\left(u_0^2 + \frac{T^2}{6}\right) = 0,$$

or,
$$u_0\left[\frac{\lambda}{4}u_0^2 - \frac{\mu^2}{2} - j_\mu^2 \Big/ \left[u_0^2 + \frac{T^2}{6}\right]^2 + (3g^2+\lambda)T^2/12\right] = 0,$$

which gives at $T = T_c$, $u_0 = 0$ and in particular for $j_\mu = 0$ one gets

$$T_c^2 = 12\cdot(\mu^2/2)/[3g^2+\lambda].$$

Hence we have now,

$$u_0\left[\frac{\lambda}{4}u_0^2 + \frac{\mu^2}{2}\left(\frac{T^2}{T_c^2} - 1\right)\right] = 0$$

which shows for $T > T_c$ the symmetry is restored below which symmetry is broken, showing a phase transition of second order in the symmetry-breaking parameter.

One can easily extend the calculation for the Lagrangian of the Salam and Weinberg model of weak interaction. The equation determining the critical temperature T_c is given by

$$u_0\left[\frac{\lambda}{4}u_0^2 - \frac{\mu^2}{2} - j_\mu^2 \Big/ (u_0^2 + aT^2)^2 + bT^2/12\right] = 0,$$

where now

$$a = \tfrac{1}{6}(1 + 8\cos^4\theta_w)$$

and
$$b = 3g^2(1 + 2\cos^2\theta w)/\sin^2\theta_w.$$

For $sw^2\theta_w \simeq 0{\cdot}24$ one gets

$$T_c^6 = j_0^2/(\lambda + 3g^2),$$

and
$$j_0 = n_\nu - n_{\bar{\nu}},$$

which is the excess of number of neutrinos over the anti-neutrinos. If we express the temperature in terms of photon-density n_γ then

$$n_\gamma = 0{\cdot}244\, T^3.$$

Thus symmetry restoration takes place for the critical photon density n_γ^c even at large j_0 if

$$n_\gamma^c \simeq \frac{j_0}{4\,(\lambda + 3g^2)^{1/2}} = (n_\nu - n_\nu)/4\;(\lambda + (3g^2)^{1/2}.$$

For a definite relationship between densities of photons and the neutrinos phase transition takes place with the restoration of symmetry. This has very interesting cosmological consequences.

Assuming what we are talking now was also true in the infinite past, at the early stage of the universe, when temperature was large as also the density then the last expression implies that symmetry restoration did take place if

$$n_\gamma > n_\gamma^c = j_0/4\;(\lambda + 3g^2)^{1/2}.$$

For a value of $\lambda = g^2 = 10^{-1}$ we find symmetry restoration

at $\qquad 2{\cdot}5\, n_\gamma > j_0.$

Thus there were no symmetry restoration if we have

$$j_0 > 2{\cdot}5\, n_\gamma.$$

Using $n_\gamma = 10^8\, n_B$ where n_B denotes baryon density of the universe then symmetry restoration could not occur at the early days if

$$j_0 > 2{\cdot}5 \times 10^8\, n_B,$$

i.e. $\qquad j_0 > 2{\cdot}5 \times 10^3\ \mathrm{cm}^{-3},$

where we have used $n_B = 1 \times 10^{-5}\ \mathrm{cm}^{-3}$. Hence symmetry restoration did not occur for

$$n_\nu - n_{\bar\nu} > 2{\cdot}5 \times 10^3\ \mathrm{cm}^{-3}.$$

Thus if symmetry was not existing at the early stage then there would be a large excess of neutrinos over the anti-neutrinos. This is very unlikely and we assume that there was a symmetry restoration at the early universe and all masses were zero except the super heavy Higgs which destroys the symmetry and generates the masses.

If, on the other hand, symmetry restoration was not there at the early days then all masses were different from zero and as these are temperature-dependent they were growing with temperature with time, $t \to 0$. Photon is mass-less now probably because of some symmetry restoration which was broken in the early days.

If one extends this idea to strong interaction symmetry one may assume that probably colour symmetry was also broken then the quarks were also not confined. They were free from the early days and we should see them. So far, however, we have not seen any. One possibility is that quarks immediately decay and have not enough time to meet each other and as a result only a small fraction forms stable baryons. Again recall the temperature $T \sim j_0^{1/3}$ which is very large and the masses of all objects including the quarks are proportional to the symmetry-breaking parameter in gauge theories so $m \sim u_0(T)$; thus assuming $m/T \gg 1$ we see that the probability of finding a quark of a given mass will be inhibited $\exp(-m/T)$.

So long we have not noticed the temperature dependent effect in the fermion densities.

To see this we consider the equation (c) namely

$$(\gamma_\mu \partial_\mu + u_0 g_\sigma - ig\,\omega_\mu \gamma_\mu)\psi = 0, \tag{i}$$

where $\langle \omega_\mu \rangle = C_\mu$

and consider for simplicity $C_\mu = i\delta_{\mu 4} C_0$. Hence from (b)

$$C_0 = \rho_v / u_0^2 g, \tag{ii}$$

$$\rho_v = \langle \psi^+ \psi \rangle, \tag{iii}$$

and

$$\rho_S = \langle \bar{\psi} \psi \rangle. \tag{iv}$$

Equation (i) is therefore

$$(\gamma_\mu \partial_\mu + g C_0 \gamma_4 + M^*)\psi = 0. \tag{v}$$

We consider bare-mass $m = 0$ for the fermion and $M^* = g_\sigma u_0$. Put

$$M^* = Mx = g_\sigma u_0,$$

where M denotes a characteristic fermion-mass-scale ($M \simeq 1$ GeV).

Our equation (a) now becomes

$$\rho_v^2 / u_0^3 - g_\sigma \rho_S - \left[\frac{\lambda}{4} u_0^2 - \frac{\mu^2}{2}\right] u_0 = 0, \tag{vi}$$

where $\rho_v = \langle \psi^+ \psi \rangle = \dfrac{\gamma M^3}{(2\pi)^3} \int \mathrm{d}^3\kappa\, [n|\theta) - \bar{n}\,|\theta)]$ and ρ_S

$$= \gamma M^3 (2\pi)^{-3} \int \mathrm{d}^3\kappa\, [n(\theta) + \bar{n}(\theta)]\, x\, [\kappa^2 + x^2]^{-1/2},$$

where $\gamma = 4$ (degeneracy factor for fermion-antifermion system)

$\kappa_i = k_i/M$ (k_i = 3-momentum component)

$x = g_\sigma u_0 / M$

$\theta = kT/M$, T = temperature, k = Boltzman constant,

$n(\theta) = 1/[\exp[(E^+ - \varepsilon_F)/kT] + 1]$,

$\bar{n}(\theta) = 1/[\exp[(E^- - \varepsilon_F)/kT] + 1]$,

$$\frac{E^\pm}{m} = \pm (k^2 + x^2)^{1/2} + \frac{g}{M} c_0.$$

If we choose ε_F, the chemical potential as zero then ρ_v vanishes and so also C_0, then, for $T/M \gg 1$ we get the following equation for the phase-transition of the symmetry breaking parameter u_0, namely:

$$u_0 \left[\frac{\lambda}{4} u_0^2 - \frac{\mu^2}{2} + \frac{1}{3} T^2 g_\sigma^2\right] = 0. \tag{vii}$$

Thus for high enough T and a suitable value of $g_\sigma > 0$ and when

$$\frac{1}{3} T^2 g_\sigma^2 > \mu^2/2$$

the symmetry is restored at the critical temperature T_c given by

$$T_c^2 = \frac{3}{2} \mu^2 / g_\sigma^2.$$

We have seen that the presence of ρ_v increases the symmetry breaking hence when both ρ_v and ρ_S are present symmetry restoration at high temperature may be compensated by ρ_V.

To see how these ideas can be reflected in actual experiment one may consider the fermion system characterized by the Lagrangian given by equation (I) and calculate the energy-density and specific heat capacity of the system. We find (Anand *et al* 1975) that the specific heat curve shows a phase transition at $T_c \sim 200^\circ$ absolute for a given set of coupling parameter and masses relevant for nuclear matter (Theis *et al* 1983). These results will be reported elsewhere.

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