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Thermal conduction through porous and dispersed three-phase systems

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Abstract. A loose three-phase system made of metal, non-metal and air is considered resulting from small successive dispersions in effective continuous medium (ECM). The effective thermal conductivity of loose three-phase systems is estimated by extending the ECM approach to multi-phase systems. The unsteady state line source (needle) method is employed to determine the effective thermal conductivity of some selected three-phase materials. The calculated and observed values show good agreement suggesting that the continuous medium approach can be applied to estimate effective thermal conductivity of multi-phase systems.

Keywords. Continuous phase; effective continuous medium; successive dispersion; unsteady state.

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1. Introduction

A knowledge of heat-transfer in static multi-phase systems is useful for studies in thermal engineering, meteorology, soil physics and geophysics. While considering the insulating ability of a natural or fabricated system, the thermal parameters are often estimated through an expression. In this paper, we present an expression which can work on samples where a conductor is dispersed in an insulator and by varying the corresponding parameters it can be used to systems where the dispersed phase has become dispersing. One finds in literature experimental measurements (De Veries and Peck 1958; Sugawara and Yoshizawa 1961; Chaudhary *et al* 1969; Law 1971; Cheng *et al* 1972; Singh *et al* 1985) of effective thermal conductivity λ_e of loose and heterogeneous two and three-phase systems of different nature. Also the resistor model (Wiener 1904; De Veries 1952; Woodside and Messmer 1961; Cheng and Vachon 1969; Chaudhary and Bhandari 1968; Kumar and Chaudhary 1980; Singh *et al* 1984) and the flux concept (Maxwell 1904; Meredith and Tobias 1960; McPhedran and McKenzie 1978) used for theoretical estimation of λ_e of two-phase media have mostly been extended to moist soils. Influence of loose metallic and insulative dispersions on heat conduction through granular two-phase systems has been investigated here experimentally using the thermal probe method. A theoretical approach using effective continuous media (ECM) has been developed to determine λ_e of loose and granular three-phase systems. As most of the loose materials lie in porosity range 0·3–0·7, the ECM approach (Pande *et al* 1984) for loose three-phase systems should be suitable. Although there is no dearth of data from literature there are insufficient information about the specimens. We have taken measurements on some composites of engineering interest which have loose metallic and non-metallic dispersions.

2. Theory

2.1 *Reduction of loose two-phase systems to effective continuous media*

Considering small cubic dispersions in the continuous phase, Pande *et al* (1984) derived an expression for the λ_e of two-phase systems when $\lambda_g/\lambda_s \to 0$. When one phase is solid (s) and the other is gas (g) the expressions are

$$\lambda_e = \lambda_g\{1 + 3{\cdot}844\ \psi_s^{2/3}\}\ (\text{for } 0{\cdot}5 > \psi_s > 0), \tag{1a}$$

and

$$\lambda_e = \lambda_s\{1 - 1{\cdot}545\ \psi_g^{2/3}\}\ (\text{for } 0{\cdot}5 > \psi_g > 0), \tag{1b}$$

where ψ_g and ψ_s stand for the volume fractions of the gas and the solid phases. Expressions (1a) and (1b) are valid for small dispersions. As the loose materials have a large range of density, Pande *et al* (1984) modified (1a) and (1b) for the porosity variation in the range 0·3–0·7. In loose two-phase systems, as none of the phases provides continuous matrix, (1a) and (1b) do not correspond to actual loose two-phase systems. The continuous matrix at large porosity values ($\psi_s = \psi_g = 0{\cdot}5$) is contributed by both the phases. When equal solid-void value occurs we call it ECM. Thus we obtain a real loose two-phase system by making small dispersions (ξ) of solid or gas phase in ECM. Therefore the λ_e of loose and granular two-phase systems for small dispersions become

$$\lambda_e = \lambda_{ec}\{1 + 3{\cdot}844\ \xi_s^{2/3}\} \text{ for } \xi_s > 0, \tag{2a}$$

and

$$\lambda_e = \lambda_{ec}\{1 - 1{\cdot}545\ \xi_g^{2/3}\} \text{ for } \xi_g > 0, \tag{2b}$$

where $\xi_g = \psi_g - 0{\cdot}5$, $\xi_s = \psi_s - 0{\cdot}5$ and λ_{ec} is the thermal conductivity of ECM. Evaluation of λ_e requires the estimation of λ_{ec}. The resulting matrix as ECM can be obtained by considering n successive small dispersions, each of value $\delta\psi$ in continuous solid or gas phases in the limit $n\delta\psi_s = n\delta\psi_g = n\delta\psi = 0{\cdot}5$. The thermal conductivity of ECM has thus been evaluated (Pande *et al* 1984) for $n\delta\psi = 0{\cdot}5$ and $n \geqslant 1$ as

$$\lambda_{ec} \geqslant 0{\cdot}6132\ (\lambda_s\lambda_g)^{1/2}. \tag{3}$$

2.2 *Influence of the dispersion of another solid phase in* ECM

Dispersion of solid (2) in a loose material replaces solid (1) and a few void spaces of the system. When the volume fraction of solid (2) is zero, the sample is a completely two-phase system composed by solid (1) and air. Gradual addition of solid (2) in the sample changes the thermal conductivity of solid-phase. When the volume fraction ψ_{s2} of solid phase (2) is unity, solid (1) is completely replaced by solid (2). Then the sample is again a two-phase system composed of solid (2) and air. Between porosity values of ψ_{s2}, we have three phases: solid (1), solid (2) and air. The thermal conductivity of this three-phase system lies between the thermal conductivity values of the sample at low extreme compositions ($\lambda_{s1}\ \psi_{s1}$) and ($\lambda_{s2}\ \psi_{s2}$). As the thermal conductivity of ECM changes with the values of λ_s (equation 3) and λ_s alters at each value of ψ_{s2}, the thermal conductivity of ECM also varies with the amount of dispersion ψ_{s2} of solid phase (2). When the dispersion ψ_{s2} in λ_{s1} is small, the new thermal conductivity of solid phase λ_s' is given through (1a) {as $\lambda_{s1}/\lambda_{s2}$ does not tend to zero}

$$\lambda_s' = \lambda_{s1}\left\{1 + 3\cdot844\left(\frac{\lambda_{s2}-\lambda_{s1}}{\lambda_{s2}+2\lambda_{s1}}\right)\psi_{s2}^{2/3}\right\}, \tag{4}$$

where ψ_{s2} is the volume fraction of solid (2). Thus the thermal conductivity of the new ECM would be

$$\lambda_{nec} \geqslant 0\cdot6132\ (\lambda_s'\lambda_g)^{1/2}. \tag{5}$$

2.3 *Effective thermal conductivity of loose three-phase systems*

The actual loose three-phase system is formed by making small dispersions of previous solid-phase (1) or air-phase in new ECM. Therefore for $\psi_{s1} = \xi_{s1} + 0\cdot5$ or $\psi_g = \xi_g + 0\cdot5$, (2a) and (2b) lead to λ_e such that

$$\lambda_e \geqslant \lambda_{nec}\{1 + 3\cdot844\ \xi_s^{2/3}\}, \tag{6a}$$

and

$$\lambda_e \geqslant \lambda_{nec}\{1 - 1\cdot545\ \xi_g^{2/3}\}. \tag{6b}$$

Substituting the values of λ_{nec} from expression (5), we get

$$\lambda_e = 0\cdot6132(\lambda_{s1}\lambda_g)^{1/2}\{1 + 3\cdot844\ \xi_s^{2/3}\}\left\{1 + 3\cdot844\left(\frac{\lambda_{s2}-\lambda_{s1}}{\lambda_{s2}+2\lambda_{s1}}\right)\psi_{s2}^{2/3}\right\}^{1/2}, \tag{7a}$$

and

$$\lambda_e = 0\cdot6132\ (\lambda_{s1}\lambda_g)^{1/2}\{1 - 1\cdot545\ \xi_g^{2/3}\}\left\{1 + 3\cdot844\left(\frac{\lambda_{s2}-\lambda_{s1}}{\lambda_{s2}+2\lambda_{s1}}\right)\psi_{s2}^{2/3}\right\}^{1/2}. \tag{7b}$$

3. Measurements

3.1 *Material preparation*

The physical properties of the loose materials in which various combinations were made for verification are as given in table 1.

These materials were sieved and oven-dried at 50°C for 24 hr. A known amount (m) of the metal or insulative material was mixed and randomized homogeneously in the

Table 1. Physical properties of the materials.

Materials	Average grain size (μ)	Bulk porosity	Solid density (g/cc)
Cement (i)	40	0·68	2·66
(ii)	100	0·514	2·66
Dune sand	170	0·485	2·67
River base sand	140	0·485	2·76
Aluminium fine	40	0·62	2·70
Iron fine	40	0·56	7·46

Table 2. Measured and calculated values of λ_e ($Wm^{-1}K^{-1}$) of metal and insulation dispersed loose and granular three-phase systems.

System	ψ_{s1}	ψ_{s2}	ψ_g	λ_e Experimental	λ_e using Lichtnecker (1926) formula	λ_e using Brailsford and Major (1964) formula	λ_e using Present expression
1. Loose cement dispersed with iron fine	0·315	0·018	0·667	0·091	0·129	0·064	0·095
	0·307	0·039	0·654	0·095	0·147	0·067	0·105
	0·295	0·064	0·641	0·103	0·169	0·069	0·116
λ_g (air) = 0·026	0·277	0·095	0·628	0·114	0·198	0·072	0·128
	0·251	0·134	0·615	0·121	0·238	0·074	0·140
λ_{s1} (cement) = 2·60	0·227	0·171	0·602	0·135	0·284	0·077	0·152
	0·184	0·227	0·589	0·156	0·362	0·080	0·168
λ_{s2} (iron) = 65·406	0·138	0·286	0·576	0·178	0·464	0·084	0·183
	0·086	0·352	0·562	0·210	0·609	0·087	0·199
2. Loose cement dispersed with aluminium fine	0·290	0·036	0·674	0·091	0·138	0·063	0·097
	0·260	0·072	0·668	0·098	0·166	0·064	0·109
	0·229	0·109	0·662	0·101	0·200	0·066	0·118
λ_g (air) = 0·026	0·195	0·149	0·656	0·111	0·245	0·067	0·120
	0·163	0·187	0·650	0·120	0·298	0·068	0·135
λ_{s1} (cement) = 2·60	0·131	0·225	0·644	0·126	0·361	0·069	0·142
	0·098	0·264	0·638	0·130	0·440	0·070	0·150
λ_{s2} (aluminium) = 204·25	0·065	0·303	0·632	0·142	0·536	0·072	0·157
	0·0324	0·342	0·626	0·166	0·652	0·074	0·165
3. Loose cement dispersed with river base sand	0·449	0·047	0·504	0·180	0·258	0·099	0·158
	0·406	0·096	0·498	0·186	0·269	0·100	0·176
	0·364	0·146	0·490	0·189	0·283	0·103	0·187

λ_g (air) = 0·026	0·317	0·198	0·485	0·203	0·293	0·104	0·200
	0·266	0·249	0·485	0·207	0·297	0·104	0·211
λ_{s1} (cement) = 2·60	0·214	0·301	0·485	0·212	0·301	0·105	0·213
	0·161	0·354	0·485	0·215	0·305	0·105	0·214
λ_{s2} (river base sand) = 3·35	0·108	0·407	0·485	0·216	0·309	0·105	0·216
	0·055	0·460	0·485	0·218	0·313	0·105	0·217
4. Loose dune sand dispersed with aluminium fine	0·472	0·043	0·485	0·227	0·379	0·105	0·269
	0·447	0·071	0·482	0·259	0·434	0·107	0·292
	0·377	0·143	0·480	0·283	0·588	0·108	0·328
λ_g (air) = 0·0260	0·332	0·182	0·486	0·271	0·669	0·106	0·326
	0·279	0·227	0·494	0·255	0·777	0·104	0·313
λ_{s1} (dune sand) = 3·35	0·229	0·267	0·504	0·255	0·882	0·101	0·275
	0·164	0·301	0·535	0·239	0·864	0·093	0·246
λ_{s2} (aluminium) = 204·25	0·099	0·337	0·564	0·218	0·869	0·086	0·227
	0·042	0·360	0·598	0·202	0·812	0·078	0·205
5. Loose cement dispersed with dune sand	0·286	0·049	0·665	0·090	0·123	0·063	0·085
	0·280	0·099	0·621	0·108	0·153	0·072	0·104
	0·269	0·151	0·580	0·136	0·187	0·080	0·120
λ_g (air) = 0·026	0·254	0·206	0·540	0·157	0·228	0·089	0·139
	0·222	0·273	0·505	0·178	0·272	0·099	0·164
λ_{s1} (cement) = 2·60	0·189	0·346	0·465	0·224	0·334	0·111	0·245
	0·117	0·418	0·465	0·264	0·340	0·111	0·247
λ_{s2} (dune sand) = 3·35	0·059	0·471	0·470	0·256	0·336	0·110	0·242
	0·009	0·516	0·475	0·248	0·332	0·108	0·235

given amount of loose two-phase material. Except for aluminium-dispersed dune sand and cement-dispersed dune sand systems, the grain size of the dispersed phase and loose material was nearly the same.

3.2 *Apparatus and method*

The method adopted here is the unsteady-state line source method, also called the thermal probe or needle method. The thermal conductivity of the infinite material sample surrounding the thermal probe is given by the relation (De Veries and Peck 1958; Carslaw and Jaeger 1959)

$$\lambda_e = \frac{Q}{4\pi(\theta_2 - \theta_1)} \ln (t_2/t_1), \tag{8}$$

where θ_2 and θ_1 are two values of the temperature of the probe surface at t_1 and t_2. Q is the power per unit length supplied to the probe. The experimental arrangement is similar to that described by Chaudhary *et al* (1969). The various appliances employed in this investigation give an experimental error of 5% through equation (8).

4. Comparison and discussion

The measured and estimated values of λ_e for metal and insulative dispersed three-phase loose and granular systems are given in table 2. The λ_e values have been calculated and compared using the relations given by Lichtnecker (1926), Brailsford and Major (1964) and the present expressions (7a) and (7b). The calculated values of λ_e using the present expressions are much closer to the measured results. The agreement is better particularly at high and low metallic or insulative dispersions (systems 1, 2 and 3, table 2). The average deviation in this case lies between 0 and 6·3%.

For systems of dissimilar grain sizes (systems 4 and 5, table 2) the values of λ_e calculated through (7a) and (7b) show better agreement at low and high metallic and insulative dispersions but the average deviation is larger (10·18%). This is perhaps due to the inhomogeneity in the distribution of dispersed phase particles, developed by greater difference in the grain size.

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References

Brailsford A D and Major K G 1964 *Br. J. Appl. Phys.* **15** 313
Carslaw H S and Jaeger J C 1959 *Conduction of heat in solids* (Oxford: Clarendon Press) 2nd edn pp. 261
Chaudhary D R and Bhandari R C 1968 *Br. J. Appl. Phys.* (*J. Phys. D*) **1** 815
Chaudhary D R, Agarwal M P and Bhandari R C 1969 *Indian J. Pure Appl. Phys.* **7** 252
Cheng S C and Vachon R I 1969 *Int. J. Heat Mass Transfer* **12** 249

Cheng S C, Law Y S and Kwan C C Y 1972 *Int. J. Heat Mass Transfer* **15** 355

De Veries D A 1952 *Thesis, Meded. Landbouhogeschool*, Wageningen, Vol. 52, No. 1, pp. 1–73

De Veries D A and Peck A J 1958 *Aust. J. Pure Appl. Phys.* **11** 255

Kumar V and Chaudhary D R 1980 *Indian J. Pure Appl. Phys.* **18** 984

Lichtnecker K 1926 *Physik Z.* **27** 115

Law Y S 1971 *Thermal conductivity of two-phase and three-phase heterogeneous solid mixtures* M.Sc. thesis, Mech. Engg., Univ. of Ottawa

Maxwell J C 1904 *A treatise on electricity and magnetism* (Oxford: Clarendon Press) III ed Vol. I, p. 440

Meredith R E and Tobias C W 1960 *J. Appl. Phys.* **31** 1270

McPhedran R C and McKenzie D R 1978 *Proc. R. Soc.* (*London*) **A259** 45

Pande R N, Kumar V and Chaudhary D R 1984 *Pramana* **22** 63

Sugawara A and Yoshizawa Y 1961 *Aust. J. Phys.* **14** 469

Singh R, Beniwal R S and Kumar V 1984 *Indian J. Pure Appl. Phys.* **22** 556

Singh R, Saxena N S and Chaudhary D R 1985 *J. Phys.* **D18** 1

Wiener O 1904 *Phys. Z.* (*Germany*) **5** 332

Woodside and Messmer J H 1961 *J. Appl. Phys.* **32** 1688