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Some comments on the Coriolis attenuation problem

A K JAIN and KIRAN JAIN
Department of Physics, University of Roorkee, Roorkee 247 667, India

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Abstract. To explore the Coriolis attenuation problem we have carried out a schematic $i_{13/2}$ rotor plus single quasi-particle band-mixing calculation. The results reveal that the calculations are largely insensitive towards the location of the Fermi energy near the low-K single particle states only, and therefore are incapable of taking into account the transition from 'full' decoupling to 'partial' decoupling as the Fermi level is increased. We trace the possible reasons for this insensitivity and find that this may be primarily due to the BCS approximation for calculating the quasiparticle energies.

Keywords. Nuclear structure; quasi-particle plus rotor band-mixing calculations; $i_{13/2}$ neutron orbital; Coriolis attenuation.

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1. Introduction

The seemingly arbitrary attenuation of the non-diagonal Coriolis matrix elements in the band-mixing calculations, commonly known as the 'Coriolis attenuation' problem, has recently been discusssed by several workers (Neergard 1979; Chen 1980; Rekstad and Engeland 1980). This problem crops up when one tries to reproduce the strongly perturbed bands (SPB) by mixing of bands based on mainly the high-j unique parity orbitals (Hjorth *et al* 1970; Rekstad and Engeland 1980). In fact, most of the experimental evidence for Coriolis attenuation comes from the positive parity bands (PPB) based on the $i_{13/2}$ neutron orbital in the rare earth region. The novel and physically meaningful interpretation of these bands as decoupled bands (Stephens and Simon 1972) has kept them in the limelight during the past decade and their occurrence is now known to be a common phenomenon (Stephens 1975). However, the conditions for full decoupling viz large-j, low-Ω and small deformation are easily changed if one increases the Fermi energy in going from one nucleus to another, say by the addition of a neutron, as is the case in odd-mass odd-neutron rare-earth nuclei. The alignment therefore deteriorates quickly (Bengtsson and Frauendorf 1979). This decrease in alignment is however not reproduced by the band-mixing calculations and hence the need for attenuation. This is often stated as the overestimation of the Coriolis term by the band-mixing calculations.

Whether the Coriolis attenuation is really not needed in the cranking model calculations has become debatable (Almberger *et al* 1980; Muller and Neergard 1983) and is a separate subject. However, the pairing-plus-recoil model (Rekstad *et al* 1979) which claims to have overcome the attenuation problem shows mainly the inadequacy of the BCS approximation. This model which employs an exact diagonalization of the pairing plus full recoil term has been applied with some success to several nuclei (Engeland *et al* 1983; Henriquez *et al* 1983a, b).

In the present paper we show, by analysing the results of a standard particle plus rotor model, that the Coriolis attenuation is indeed related to the BCS approximation. The BCS quasiparticle plus rotor band-mixing calculations are shown to be intrinsically insensitive towards the location of the Fermi energy near the low-K single particle levels (Jain and Jain 1983). These calculations therefore cannot reproduce the decrease in alignment as the Fermi energy increases. The Coriolis attenuation therefore becomes indispensable. This attenuation is in addition to that coming from the pairing correlations. The attenuation due to pairing, represented by the $uu+vv$ factor is marginal and is excluded from our discussion as it does not affect our conclusions. Further, we find that the insensitivity of the band-mixing calculations seems to diminish as we increase the Fermi energy near the $K = 11/2$ and $13/2$ levels and therefore may not be needed there.

2. Theory and calculations

2.1 *The Hamiltonian*

The Hamiltonian for an axially symmetric core plus particle is usually written as

$$H = H_{qp} + A(R_X^2 + R_Y^2), \tag{1}$$

where A is the moment of inertia parameter. This is rewritten in terms of the total angular momentum I and the quasiparticle angular momentum j as

$$H = H_{qp} + A[I(I+1)] + A[\ \langle j(j+1) - 2K^2 \rangle\ ] + H_c, \tag{2}$$

where $I_z = K = \Omega = j_z$ and the Coriolis term H_c is given by

$$H_c = -A(I_+ j_- + I_- j_+)\kappa. \tag{3}$$

The off-diagonal matrix elements of H_c are usually attenuated by a factor κ in the range 0·5 to 0·8 when a least square fitting of SPB data is carried out. We have explicitly included an attenuation factor κ in the Coriolis term with the understanding that only off-diagonal terms are to be attenuated. We have used only the $i_{13/2}$ Nilsson model energies assuming pure-j states. This is a very good approximation for unique parity bands and does not affect our conclusions. The moment of inertia parameter $A = 14$ keV and the pairing gap $\Delta = 1{\cdot}0$ MeV was used.

2.2 *Measure of aligned angular momentum*

To obtain a quantitative estimate of the alignment, we have determined the projection of the quasiparticle spin $\mathbf{j}$ on the component of total angular momentum $\mathbf{I}$ along the rotation axis. This quantity denoted as $J_\perp$ is given by

$$J_\perp = \left\langle \frac{\mathbf{I}\cdot\mathbf{j} - K^2}{|\mathbf{I} - I_3|} \right\rangle = \frac{1}{2}\left\langle \frac{I_+ j_- + I_- j_+}{[I(I+1) - K^2]^{1/2}} \cdot \right\rangle \tag{4}$$

How a decrease in the 'aligned' spin follows from the attenuation may be seen from the following consideration (Flaum and Cline 1976). We rewrite (2) in terms of $J_\perp$ as

$$H = A[I(I+1) + \langle j(j+1) - 2K^2 \rangle] - 2A\kappa J'_\perp (I(I+1) - K^2)^{1/2} + \delta_{K,1/2} A(-1)^{I+1/2} a(I+\tfrac{1}{2}). \quad (5)$$

Here we have omitted the quasiparticle energy and separated out the diagonal Coriolis term (which is not attenuated) from $J_\perp$. The non-diagonal Coriolis term has been denoted by $J'_\perp$. For $I \gg K$, we obtain

$$H = A[I(I+1) + \langle j(j+1) - 2K^2 \rangle] - 2A\kappa J'_\perp [I(I+1)]^{1/2} - \delta_{K,1/2} A(-1)^{I-1/2} a(I+\tfrac{1}{2}) + A\kappa \frac{\langle K^2 \rangle J'_\perp}{[I(I+1)]^{1/2}}. \quad (6)$$

To extract the effective aligned angular momentum, let us consider a classical rotor with angular momentum R and a decoupled particle with an aligned angular momentum i, i.e. $I = R + i$. Such a rotor will exhibit a spectrum given by

$$H_{\text{decoupled}} = AR(R+1) = A[I(I+1) - 2iI + i(i-1)]. \quad (7)$$

Comparison of (6) and (7) immediately gives us the effective alignment as

$$i = \kappa J'_\perp [I(I+1)/I^2]^{1/2} + a(-1)^{I-\frac{1}{2}}. \quad (8)$$

provided the last term in (6) is constant. Thus i gives us a measure of the aligned angular momentum in the classical sense. This quantity for $\kappa = 1{\cdot}0$, gives us $J_\perp$. For $\kappa < 1{\cdot}0$

$$J_\perp = i \approx \kappa J'_\perp + a(-1)^{I-\frac{1}{2}}.$$

3. Results and discussion

In figure 1, we show the favoured level energies from our calculation for a deformation $\delta = 0{\cdot}27$, no Coriolis attenuation ($\kappa = 1{\cdot}0$) and three different locations of the Fermi energy at three deformations of $\delta = 0{\cdot}19, 0{\cdot}23$ and $0{\cdot}27$. All the level energies have been measured from the $I = 13/2$ level. It is interesting to note that in all the cases, the level energies hardly change although the Fermi energy rises from $K = 1/2$ to $3/2$ and then $5/2$. At $\delta = 0{\cdot}19$, the change is as little as $< 1\,\%$ while at $\delta = 0{\cdot}23$ and $0{\cdot}27$, the changes are never more than 20 to 30 keV, representing a 2 to 3 % variation. Moreover, all the level energies (except the $I = 13/2$) change in the same direction, and therefore, the transition energies are even less affected. Thus, a change in the location of the Fermi energy has almost no effect on the energy levels of the favoured band as long as the Fermi energy is kept near the low-K states. This is surprising in view of the fact that the alignment is supposed to deteriorate as one moves from $K = 1/2$ to $3/2$ and then $5/2$. It is interesting to note that the insensitivity of the energy levels is confined to the lower lying Fermi energies only. As soon as we cross the $K = 5/2$ Fermi energy the nature of the results changes dramatically. This becomes clear from figure 2, where we plot the ratio

$$\Gamma(I) = (E_I - E_{13/2})/\Delta E(17/2 \to 13/2),$$

as a function of different locations of the Fermi energy for a deformation $\delta = 0{\cdot}19$. The dashed lines represent the ratios expected for the usual $i_{13/2}$ decoupled band. Very little

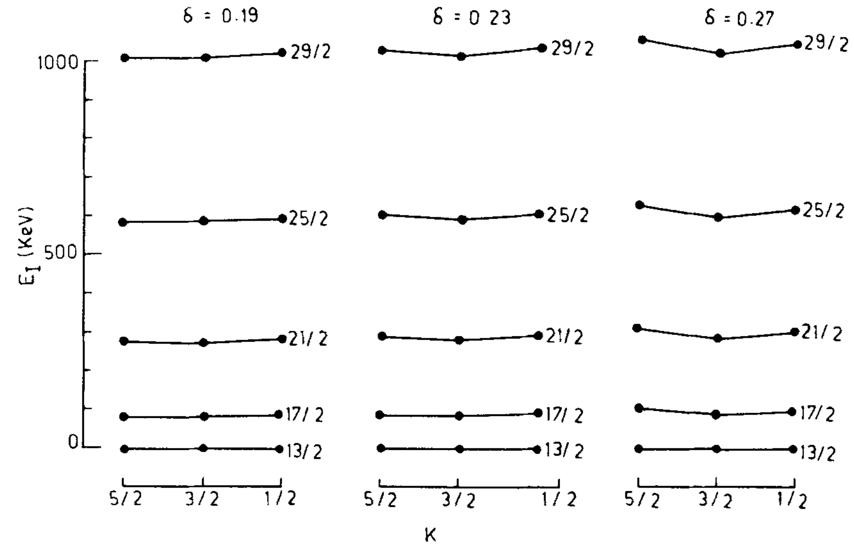

Figure 1. Energies of the favoured band levels for the Fermi energy lying near the $K = 1/2$, 3/2 and 5/2 single quasiparticle states at three values of the deformation $\delta = 0{\cdot}19, 0{\cdot}23$ and $0{\cdot}27$.

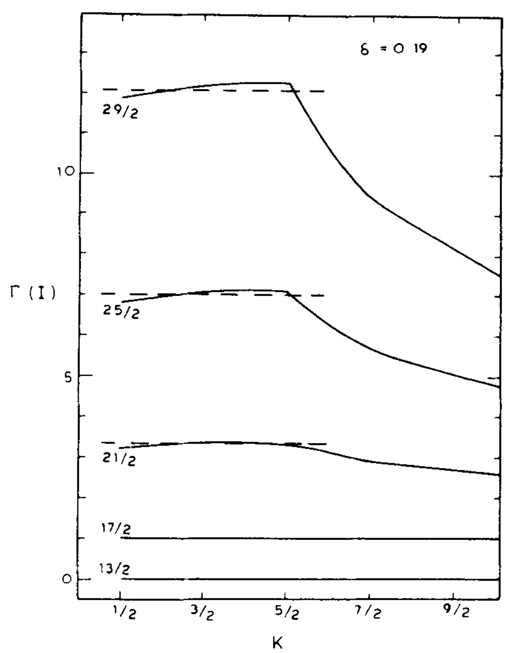

Figure 2. The ratio $\Gamma(I) = (E_I - E_{13/2})/\Delta E(17/2 \rightarrow 13/2)$ plotted for the Fermi energy lying near the $K = 1/2$ to $K = 5/2$ levels. The dashed lines show the ratio for usual decoupled band (i.e. rigid rotor spectrum).

deviation is seen from the dashed lines as long as the Fermi energy lies near the $K = 1/2$ to $5/2$ single particle states. However, in going from $K = 5/2$ to $K = 7/2$ the ratios drop dramatically indicating a sudden change in the nature of the results and further calculations reveal that the need for Coriolis attenuation diminishes at higher Fermi energies. In fact we have found that the results become almost independent of the Coriolis attenuation for the Fermi energy lying near the $K = 11/2$ and $13/2$ levels (Jain and Jain 1984).

This insensitivity of band-mixing calculations may be understood if we look at the simpler two band-mixing calculation between the bands based on say $K = 1/2$ and $K = 3/2$ quasiparticle states. If we choose to ignore the diagonal Coriolis contribution for the $K = 1/2$ bands (which mainly shifts the unfavoured levels with respect to the favoured levels), then it becomes clear that the diagonalisation of the 2×2 matrix will yield the same yrast level energies whether we choose the Fermi energy near the $K = 1/2$ state or, the $K = 3/2$ state because in either case the $K = 1/2$ and the $K = 3/2$ quasiparticle states maintain the same spacing between them. The wavefunctions will also be identical in both the cases. These features are reflected very well in the $i_{13/2}$ seven band-mixing calculations also. For the Fermi energy lying near the $K = 1/2$ or the $K = 3/2$ states, the seven band-mixing calculation behaves as if mainly these two bands alone were being mixed. The structure of the wavefunction is nearly the same for both the cases and almost 60 % of the strength is concentrated in the $K = 1/2$ state and 30 % in the $K = 3/2$ state. Hence, the degree of alignment is also expected to be the same in both the cases.

The quantity $J_\perp$, given by (4) is just the renormalized Coriolis term and is a measure of decoupling when no attenuation is done ($\kappa = 1{\cdot}0$). In figure 3, we have plotted $J_\perp$ vs spin I for the three different locations of the Fermi energy and $\delta = 0{\cdot}27$. It is clear that for $\kappa = 1{\cdot}0$ (no attenuation), the alignment is same for both the $\varepsilon_F = |K = 1/2\rangle$ and $\varepsilon_F = |K = 3/2\rangle$ cases. This intrinsic insensitivity of band-mixing calculations persists even for the Fermi energy $\varepsilon_F = |K = 5/2\rangle$ leading to almost no change in the level energies

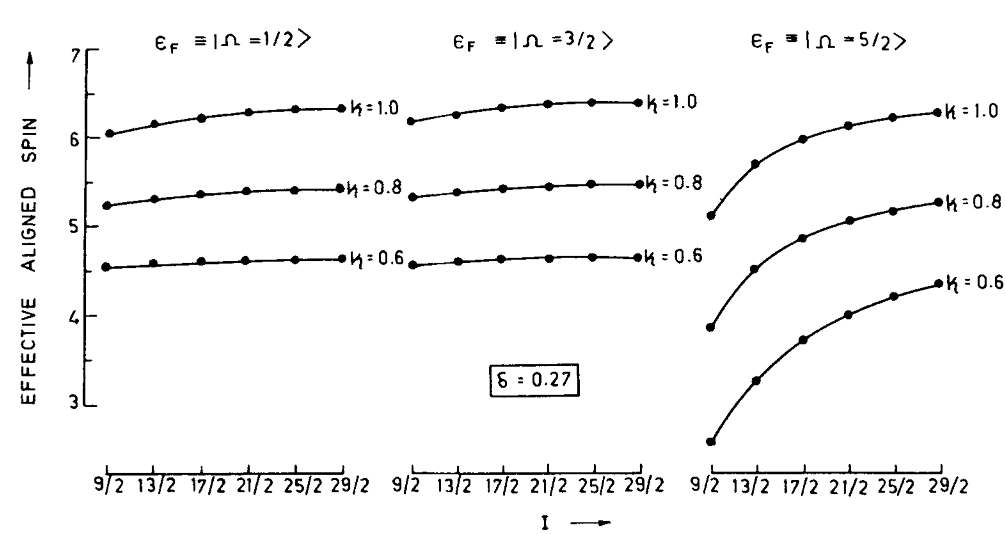

Figure 3. Effective aligned spin i as a function of I for three different locations of the Fermi energy and $\delta = 0{\cdot}27$ for $\kappa = 1{\cdot}0$ the aligned spin $i = J_\perp$.

(figure 2). The wavefunctions are however not the same since the $K = 5/2$ band now mixes on both the sides with $K = 3/2$ and $K = 7/2$ bands. The alignment is therefore smaller at lower spins but it is seen to recover quickly at higher spins and attains almost the same value at $I = 29/2$ as at the lower Fermi energy. In other words, although the Fermi level has been increased from $K = 1/2$ to $K = 3/2$ and then to $K = 5/2$, there is almost no change in the energies as well as the alignment (considered at relatively high spins). If we now introduce an attenuation factor κ in the Coriolis term and attenuate it by a factor $\kappa = 0{\cdot}8$ and $0{\cdot}6$, the results obtained are shown in figure 3. We find that the effective alignment i decreases very quickly. It is clear that the alignment decreases considerably as the attenuation is brought in. Moreover, the decrease in the alignment occurs in a manner which matches well with the occurrence of 'effectively' decoupled bands (Jain and Jain 1984).

That the alignment should decrease considerably is well accepted (Bengtsson and Frauendorf 1979) and is also borne out by our recent interpretation of the decoupled bands in the rare earth region as 'effectively' decoupled bands, wherein the effective alignment for $i_{13/2}$ bands has been found to be 9/2 or even 5/2 and not the usual 13/2 (Jain 1983, 1984; Jain and Jain 1984).

4. Conclusions

We find that the band-mixing calculations are largely insensitive towards the location of the Fermi energy near the low-K states only and are therefore incapable of reflecting the decrease in the alignment as the Fermi energy is raised. This insensitivity is built up into these calculations and is mainly due to the BCS approximation of calculating the quasiparticle energy coupled to the band-mixing calculation. Perhaps it is important to consider the effects of blocking the single particle states lying below the Fermi energy.

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