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Total scattering cross-section and extraction of low energy parameters of $\Sigma^{\pm}p$ scattering

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Abstract. The lowest momentum at which the total scattering cross-section data are available for $\Sigma^+ p$ and $\Sigma^- p$ scattering is 145 MeV/c and 142·5 MeV/c respectively. Thus extracting low energy parameters amounts to extrapolating the data to still lower energies. Using the analytic structure of foward scattering amplitude to advantage a parameterization of the σ_T is presented which is hoped to be more reliable and stable for deriving results through extrapolation. The scattering lengths and effective ranges for the $\Sigma^+ p$ and $\Sigma^- p$ are also estimated.

Keywords. Total cross-section; cuts; analyticity; circular mapping; low energy parameters.

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1. Introduction

The mean lifetime of $\Sigma^{\pm}$ particles being very short ($\approx 10^{-10}$ sec) (Hauptman *et al* 1977) the experimental investigation of the $\Sigma^{\pm} p$ interaction is severely handicapped. With the scanty and error-affected data a few theoretical studies (deSwart *et al* 1962; Nagels *et al* 1973, 1977, 1979; Lettessier and Tounsi 1971) have been attempted. However in the analysis of total cross-section data of hyperon nucleon scattering the major efforts (Fast *et al* 1969; Nagels *et al* 1973, 1977, 1979) have been to extract estimates of the low energy parameters *i.e.* the scattering lengths and the effective ranges. One does this by using the relation

$$k\cot\delta = -\frac{1}{a}+\frac{rk^2}{2}, \tag{1}$$

where k is the centre of mass momentum, a, scattering length r, effective range, and δ, the phase shift.

As the low energy scattering receives significant contribution only from the S-states, one uses the parameters of the singlet and triplet states to obtain the expression for the total cross-section

$$\sigma_T = \frac{3\pi}{|k\cot\sigma_t - ik|^2}+\frac{\pi}{|k\cot\sigma_s - ik|^2}. \tag{2}$$

One then tries to obtain estimates of the four parameters a_s, a_t, r_s, r_t by making a least

χ^2 fit to the data on σ_T.The earlier workers were unanimous about the sign of these parameters but the magnitudes differed. The uncertainties regarding the magnitudes of these parameters could be due to the fact that the effective range calculations are expected to yield reliable results (Nishijima 1964) when only very low energy data are used for the analysis. And the lowest momentum, at which the total cross-section data of Σ^+p and Σ^-p scattering are available, are around 145 MeV/c and 142·5 MeV/c respectively. Thus for all practical purposes, when one tries to obtain estimates of those parameters one performs extrapolation through analytic continuation of the data to regions where experimental information is not available. In attempting such an extrapolation one has to consider a procedure which has greater information storage capacity and is thus likely to lead to more stable and reliable results. However, in the absence of an universal alogrithm for such an analytic extrapolation, one hopes that optimal exploitation of the analytic structure could be a better tool (Ciulli *et al* 1975). There have been prescriptions by Cutkosky and Deo (1968) and Ciulli (1969) for providing a relatively stable extrapolation procedure, by optimally exploiting the analytic structure of the scattering amplitude in the energy plane. This technique has been successfully tested by many authors (Cutkosky and Deo 1970; Miller *et al* 1972; Mohanty and Mohapatra 1984, 1985). In this paper we have tried to store the available physical information in the coefficients of an accelerated convergent expansion of σ_T. We then extrapolate the function, σ_T thus constructed to momentum range, 50 MeV/c and use these values as our data to obtain the low energy parameters of $\Sigma^\pm p$ scattering.

2. Scheme of parametrization of σ_T.

The optical theorem states

$$\sigma_T = \frac{4\pi}{k}\,\mathrm{Im}\,f(S,0), \tag{3}$$

where S is the square of the centre of mass energy. To parametrize $\mathrm{Im}\,f(S,0)$ we note that it is holomorphic in the S-plane except for the cuts $S_R \leqslant S \leqslant \infty$ and $-\infty \leqslant S \leqslant S_L$ (figure 1a) where for Σ^+p scattering

$$S_R^{\Sigma^+} = (M_\Lambda + M_p)^2, \tag{4}$$

$$S_L^{\Sigma^+} = 2(M_{\Sigma^-}^2 - M_p^2), \tag{5}$$

and for Σ^-p

$$S_R^{\Sigma^-} = (M_\Lambda + M_p^2), \tag{6}$$

$$S_L^{\Sigma^-} = 2(M_{\Sigma^-}^2 - M_p^2). \tag{7}$$

S_R and S_L correspond to the opening up of two particle thresholds in the direct and crossed channels respectively. Since data for σ_T are available at S values greater than S_R, for our analysis we treat the region $0 \leqslant S \leqslant \infty$ as the physical domain.

As a first step we symmetrize the cuts on the real axis in the X-plane by the mapping (figure 1b).

$$X = S - \tfrac{1}{2}(S_R + S_L). \tag{8}$$

Taking note of the fact that for our analysis negative S will be unphysical hence, we

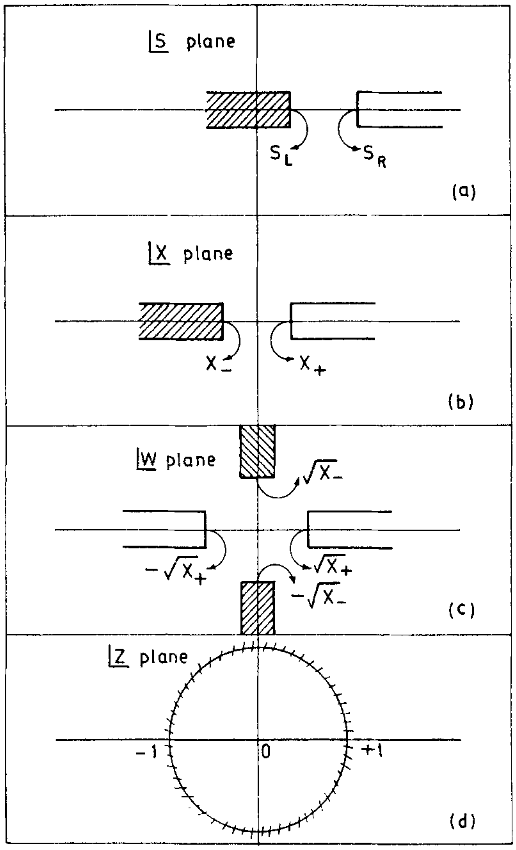

Figure 1a. Analytic structure of Im $f(S, 0)$ in the $\lfloor S$ plane. S_R and S_L are the positions of the right hand and left hand cuts respectively. **b.** X_+, X_- are the positions of the cuts in the symmetrized $\lfloor X$ plane being symmetrized. **c.** Square root mapping of the cuts in the $\lfloor W = \lfloor \sqrt{X}$ plane. **d.** Unit circle of convergence in the mapped $\lfloor Z$ plane with the radius $a = \pm 1$.

do the following mapping

$$W = \sqrt{X}. \tag{9}$$

This symmetrizes separately the left hand and right hand cuts respectively, on the imaginary and real axes of the complex $\lfloor W$ plane, on both sides. In the W-plane the cuts on the real axis, representing the cut $S_R \leqslant S \leqslant \infty$, run from $-\infty \leqslant W \leqslant -W_0$ and $W_0 \leqslant W \leqslant \infty$ (figure 1c).

The optimal convergence can now be obtained (Cutkosky and Deo 1968; Ciulli 1969) by mapping the symmetrical cuts of the $\lfloor W$ plane to form the boundary of an unifocal ellipse with $W = \pm 1$ as foci. But the size of the ellipse is quite large. Therefore the following circular mapping was preferred

$$Z = \frac{(W_0 + W^2)^{1/2} - W_0^{1/2}}{W}. \tag{10}$$

This maps the entire domain of analyticity of Im $f(S, 0)$ into the interior of the unit circle in the Z-plane with its radius $a = \pm 1$ (figure 1d). Further $W = 0$ is mapped onto $Z = 0$, $W = \pm\infty$ to $Z = \pm 1$, and the cuts on the imaginary axis of W form the boundary of the unit circle. Hoping that no region of analyticity is left out of the region of convergence by this mapping, the expansion of $\mathrm{Im}\, f(S, 0)$ in the mapped variable, Z, will converge optimally. So we construct $\mathrm{Im}\, f(S, 0)$ as polynomials in Z

$$\mathrm{Im}\, f(S, 0) = \sum_{n=0}^{\infty} a_n Z^n, \tag{11}$$

and

$$\sigma_T = \frac{4\pi}{k} \sum_{n=0}^{\infty} a_n Z^n. \tag{12}$$

3. Results and conclusion

Using (12) we tried to fit all the $\Sigma^+ p$ and $\Sigma^- p$ scattering total cross-section data (Nagels *et al* 1979, and the references therein). After several attempts it was found that just two terms in the expansion (12) for both the scatterings are enough to give a good fit to the experimental data *i.e.* for both the analysis $\Sigma^+ p$ and $\Sigma^- p$,

$$\sigma_T = \frac{4\pi}{k} (a_0 + a_1 Z). \tag{13}$$

However, the truncated series still gives a faithful representation of the actual σ_T in so far as the fit to the σ_T data is concerned.

For $\Sigma^+ p$ scattering we get the best fit with $\chi^2/\text{NDF} = 0{\cdot}42$. The values of the coefficients are $a_0^{\Sigma^+} = 0{\cdot}6$, $a_1^{\Sigma^+} = 1{\cdot}23$. For $\Sigma^- p$ the best fit is also obtained with $\chi^2/\text{NDF} = 0{\cdot}42$. The coefficient values for this scattering are $a_0^{\Sigma^-} = 1{\cdot}05$ and $a_1^{\Sigma^-} = 1{\cdot}50$. Comparison of the theoretical and experimental values for these scattering is shown in table 1.

To obtain estimates for the low energy parameters we extrapolate our theoretical curves for σ_T to lower incident hyperon momentum regions and compute values of σ_T for $0{\cdot}05$ GeV/c $\leqslant P_Y \leqslant 0{\cdot}16$ GeV/c at a bit of 5 MeV/c. Here Y designates Σ^+ or Σ^-. These 23 values (theoretically extrapolated) of σ_T at corresponding P_Y then serve the

Table 1. Comparison of the theoretical and experimental values for the selected set of data (Nagels *et al* 1973, 1977, 1979). The lab moments are given in MeV/c and the cross sections in mb.

$\Sigma^+ p - \Sigma^+ p$, $\chi^2/\text{NDF} = 0{\cdot}42$			$\Sigma^- p - \Sigma^- p$, $\chi^2/\text{NDF} = 0{\cdot}42$		
P_{Σ^+} (MeV/c)	σ_{exp} (mb)	σ_{th} (mb)	P_{Σ^-} (MeV/c)	σ_{exp} (mb)	σ_{th} (mb)
145·0	123 ± 62	99·8	142·5	152 ± 38	151·6
155·0	104 ± 30	93·5	147·5	146 ± 30	146·5
165·0	92 ± 18	87·9	152·5	142 ± 25	141·7
175·0	81 ± 12	83·0	157·5	164 ± 32	137·3
			162·5	138 ± 19	133·1
			167·5	113 ± 16	129·2

purpose of data points to determine the low energy parameters. Although at high P_Y values the errors in the experimental values are as high as 20% to 30%, we, in the extrapolated theoretical values, assume an error of only 10% at each point so as to induce some amount of stability into the computed values of a_s, r_s, a_t and r_t. We then make a least χ^2 fit to these 23 values of σ_T using equations (1) and (2). Our best values for the low energy parameters thus obtained, are given in tables 2 and 3. Results for $\Sigma^+ p$ scattering along with the values of other workers (Fast *et al* 1969; deSwart 1970; Nagels *et al* 1973, 1977, 1979), in table 2 give a clear idea that there is unanimity regarding the sign of these parameters except for deSwart (1970). They have a positive value for r_t. As regards the results of Fast *et al* (1969), although their a_s and r_t match in sign the values are of a high order. However our values more or less match well with recent efforts by Nagels *et al* (1973, 1977, 1979) except that our a_t value is slightly high. The low a_t values by Nagels *et al* (1977, 1979) are primarily for two reasons; (i) they have used data around the momentum value of 150 MeV/c and equations (1) and (2) are not very accurate at such high values of the momentum; (ii) even at this momentum their theoretical values of σ_T are greater than the experimental σ_T values because of the strong SU(3) constraint in their model. Our analysis is independent of such constraints.

Table 2. Values of the scattering lengths and effective ranges of $\Sigma^+ p$ scattering in fermi from the present and earlier analysis (Fast *et al* 1969; de Swart 1970; Nagels *et al* 1973, 1977, 1979).

		Analysis of other workers					
Parameters	Present analysis	Fast *et al* 1969	de Swart 1970	Nagels *et al* (1973)	Nagels *et al* (1977)	Nagels *et al* (1979)
a_s	-2.52 ± 0.33	-6 ± 1	-2.6	-2.42 ± 0.3	-3.66 ± 0.33	-3.84
r_s	3.34 ± 0.5	2.1 ± 0.3	1.9	3.41 ± 0.3	3.52 ± 0.25	4.03
a_t	1.54 ± 0.02	-0.2 ± 0.05	0.7	0.71	0.34 ± 0.01	0.62
r_t	-0.61 ± 0.05	-40	2.3	-0.78	-7.31 ± 0.2	-1.91

Table 3. Values of scattering lengths and effective ranges of $\Sigma^- p$ scattering in fermi from present and earlier analysis (Nagels *et al* 1973, 1977).

Parameters	Present analysis	Analysis of Nagels *et al* (1973)	Analysis of Nagels *et al* (1977)
a_s	-2.99 ± 0.06	-2.77 ± 0.45	-4.6 ± 0.6
r_s	3.39 ± 0.07	3.55 ± 0.32	3.69 ± 0.27
a_t	2.31 ± 0.04	0.63	0.32 ± 0.01
r_t	-0.63 ± 0.01	-0.76	-6.01 ± 0.12

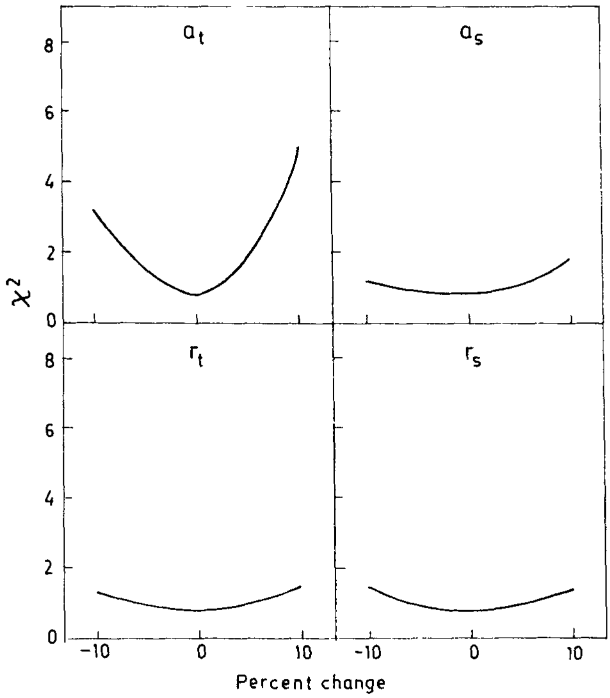

Figure 2. Effect of changes in a_s, r_s, a_t and r_t for Σ^+p scattering.

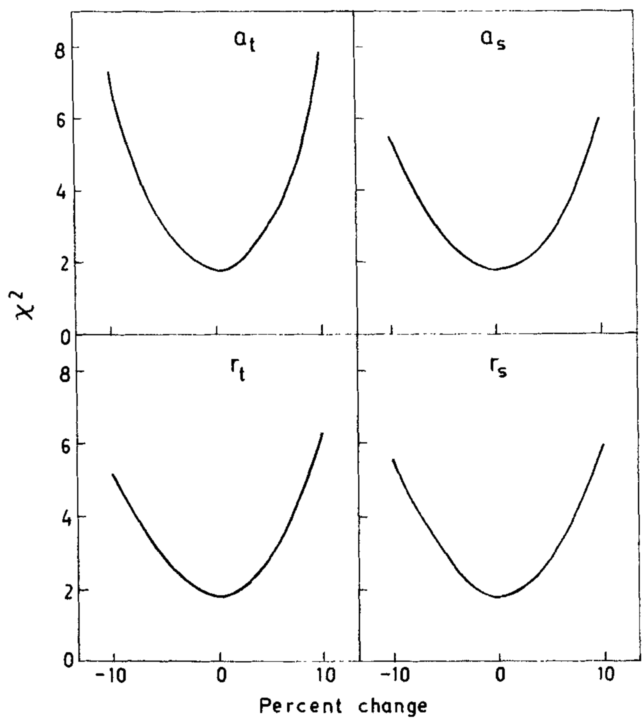

Figure 3. Effect of changes in a_s, r_s, a_t and r_t for Σ^-p scattering.

Further we note that because of this SU(3) constraint, and consequent small a_t values, their zero energy σ_T is much smaller than our zero energy value for σ_T.

Similarly for $\Sigma^- p$ scattering, analysing the results given in table 3 we see that our values match well with that of earlier workers (Nagels *et al* 1973, 1977) in magnitude and sign except that our values are slightly, high for a_t.

We have presented a parametrization which faithfully reflects the analytic structure of σ_T and hopefully best represents σ_T. As the fit is quite stable in the sense that small changes in the parameters produce large noises in χ^2 as shown in figures 2 and 3, we hope that our values are reliable and stable. Also, the sensitivity, of these fits towards changes in a_t for both the scatterings are much sharper than those in a_s, r_s and r_t.

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