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Charmed baryon magnetic moments in the covariant oscillator quark model with isospin symmetry breaking

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Abstract. The magnetic moments of charmed baryons are studied in the covariant oscillator quark model including isospin symmetry breaking effect. In the uncharmed sector, the results differ from those obtained using conventional non-relativistic quark model (NRQM). But in the charmed sector the present values are much nearer to the NRQM results than those calculated using models with hadron mass dependence.

Keywords. Quark model; magnetic moments; charmed baryons.

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1. Introduction

The baryon magnetic moments have been a popular testing ground for symmetry schemes and quark ideas. Since the time of proposal of non-relativistic quark model [NRQM] (Rujula *et al* 1975) many attempts (Lipkin 1978; Franklin 1979) applying it to the octet baryon moments have been made. Initially the predicted moments were found to be in approximate agreement with the data available at that time but now from the recent precise measurements it is clear that the simple picture of mass broken NRQM (Rujula *et al* 1975) cannot explain these moments. To obtain best fit various attempts (Isgur and Karl 1980; Singh *et al* 1979; Goffen and Wilson 1980; Lipkin 1981) to include additional symmetry breaking of strong interaction in the wave-functions have been made but 1 % level of accuracy cannot be obtained. Bohm *et al* (1982) calculated the baryon magnetic moments using second order perturbation theory taking into account the effect of spin dependent interaction arising from QCD but failed to obtain an improved fit. On the other hand some authors (Teese and Settles 1979; Tomozawa 1979; Singh 1981a) have shown that the situation improves if one brings in baryon mass dependence in the computation of baryon magnetic moments. This clearly indicates that since all our observations are made through hadrons and not through quarks, it is necessary to have all observables represented in terms of hadron variables. Especially in the case of electromagnetic processes, to get meaningful qualitative results in any model, it is indispensable to have the covariant and conserved effective current expressed in terms of hadrons themselves. When NRQM is unable to explain baryon moments, an approach of the covariant oscillator quark model (COQM) (Feynman *et al* 1971; Ishida and Oda 1979) is still important as being complementary to quantum chromodynamics. In COQM, hadrons themselves are unambiguously defined as Fierz components of multilocal fields, which satisfy a covariant equation heurestically obtained and play the role of wave-functions of composite hadrons. There, the effective hadron currents are explicitly given which are conserved with hadron masses (squared) determined by the four dimensional oscillator. Recently Ishida and co-workers have used the extended COQM (Ishida and Sonoda 1983) without isospin symmetry breaking for the analyses of octet baryon magnetic moments (Ishida *et al* 1983a), and radiative decays of vector mesons (Ishida *et al* 1983b) and found remarkable improvement over simple NRQM.

The magnetic moments of charmed baryons have already been studied in symmetry schemes (Singh *et al* 1979), quark model (Singh 1977; Lichtenberg 1977) and bag model (Chatley 1983; Tiwari and Singh 1984). In this paper we calculate magnetic moments of charmed baryons in the extended covariant oscillator quark model particularly including the isospin symmetry breaking effect. Since charmed baryons have been experimentally observed and in the light of the possible mass spectrum some of the $1/2^+$ baryons are expected to be stable against strong interactions, such studies may be important.

2. Effective baryon currents and magnetic moments

In the COQM baryons are described as

$$\phi_{(p)\alpha_1\alpha_2\alpha_3}(x_1, x_2, x_3), \tag{1}$$

where the α_i's represent spinor indices, the x_i's are Lorentz four vectors which represent space time coordinates of the constituents. The spin and space wave-functions can be covariantly extended (Ishida *et al* 1977) to finally obtain the action for EM interaction up to the first order in EM coupling constant

$$\begin{aligned} L_I &= \prod_{J=1}^{3} \mathrm{d}^4 x_j \sum_{i=1}^{3} J_{i\mu}(x_1, x_2, x_3) A_\mu(x_i) \\ &\equiv \int \mathrm{d}X\, J_\mu(X)\, A_\mu(X), \end{aligned} \tag{2}$$

$$J_{i\mu} = -i\bar{\phi} e_i \left[\frac{d}{2m_i} \frac{\bar{\partial}}{\partial x_{i\mu}} + g_M \frac{d}{2m_i} i\sigma^{(i)}_{\mu\nu} \left[\frac{\vec{\partial}}{\partial x_{i\nu}} + \frac{\overleftarrow{\partial}}{\partial x_{i\nu}} \right] \right], \tag{3}$$

where e_i is the charge of the ith quark and g_M is a parameter related with FKR (1971) prescription. Equation (2) systematically describes all the EM interactions of three quark systems. With a little effort one can obtain the current $J_\mu(x)$ in the momentum representation as

$$J_\mu = Q_{fi}(p' + p)_\mu \bar{\psi}_f(p')\psi_i(p) + \mu_{fi} i\psi_f(p')\sigma_{\mu\nu}\psi_i(p)q_\nu \tag{4}$$

where $\psi_i(p)\,[\psi_f(p')]$ being the Dirac spinor [its adjoint] having momentum $p[p']$ representing the initial [final] baryon and $q = p - p'$ is the momentum of the EM field. In (4)

$$Q_{fi} = \delta_{fi} Q_i, \quad Q_i = e_1 + e_2 + e_3$$

is the charge of the relevant baryon and μ_{fi} are constants obtained by calculating expectation values

$$\mu_{fi} \equiv \langle f|\hat{\mu}_B|i\rangle \tag{5}$$

of the operator

$$\hat{\mu}_B = g_M \sum_{i=1}^{3} \frac{d}{2m_i} e_i \sigma_3^{(i)}. \tag{6}$$

Using the effective current (4) one can obtain the magnetic moment and transition moment, respectively, as

$$\mu_B(ii) = \left(\frac{1}{2M_B}\right)\mu_{ii} \tag{7}$$

$$\mu_T(fi) = \left(\frac{1}{2M_f 2M_i}\right)^{1/2} \mu_{fi} \tag{8}$$

where M_f and M_i are the corresponding baryon masses. Here the obtained values for μ_B and μ_T will be in intrinsic particle magnetrons $1/2M_i$ unit.

3. Estimation of magnetic moments

The magnetic moments of baryons can be calculated using (6)–(8) and spin unitary spin wave-functions. The corresponding matrix elements for each baryon are given in table 1. They are described in terms of four parameter *viz* g_M, $\lambda = m_u/m_d$, $x = m_u/m_s$, and $y = m_u/m_c$. The parameters g_M, λ and x are determined by taking $\mu(p) = 2{\cdot}79$ n.m., $\mu(n) = -1{\cdot}91$ n.m. and $\mu(\Lambda) = -0{\cdot}614$ n.m. as input which give $g_M = 0{\cdot}954$, $\lambda = 1{\cdot}125$ and $x = 0{\cdot}683$. For the parameter y as there is no charmed baryon whose magnetic moment is experimentally known, following Rujula *et al* (1975) we determine its value using the known baryon masses in the expression

$$y = \frac{2(M_{\Sigma_c^*} - M_{\Sigma_c})}{2M_{\Sigma_c^*} + M_{\Sigma_c} - 3M_{\Lambda_c}} = 0{\cdot}23. \tag{9}$$

Once the parameter values are known the magnetic moments of the baryons can easily be calculated and are given in table 2. For calculation, the values of the masses of unknown charmed baryons are taken from Singh (1981b).

4. Conclusions

From our analysis on the magnetic moments of charmed baryons in covariant oscillator quark model we noticed that for uncharmed baryons the values obtained here significantly differ from those of the non-relativistic quark model. Present values agree well with the experimental data and also compare favourably with the models involving hadron mass dependence. The slight variation in the results may be due to an extra term d appearing in the operator of the present analysis. But as we go to charm sector it is observed that the present values are much nearer to the NRQM values than those of other

Table 1. Magnetic moments of baryons.

Particle	$\langle \sum_{i=1}^{3} \frac{m}{m_i} e_i \sigma_3(i) \rangle$	$\sum_{i=1}^{3} \frac{m_i}{m}$
$J^P = \frac{1}{2}^+$		
p	$\frac{1}{9}(8+\lambda)$	$2+\frac{1}{\lambda}$
n	$-\frac{2}{9}(1+2\lambda)$	$1+\frac{2}{\lambda}$
Λ	$-\frac{1}{3}x$	$1+\frac{1}{\lambda}+\frac{1}{x}$
Σ^+	$\frac{1}{9}(8+x)$	$2+\frac{1}{x}$
Σ^0	$\frac{1}{9}(4-2\lambda+x)$	$1+\frac{1}{\lambda}+\frac{1}{x}$
Σ^-	$-\frac{1}{9}(4\lambda-x)$	$\frac{2}{\lambda}+\frac{1}{x}$
Ξ^0	$-\frac{2}{9}(2x+1)$	$1+\frac{2}{x}$
Ξ^-	$-\frac{1}{9}(4x-\lambda)$	$\frac{1}{\lambda}+\frac{2}{x}$
$\langle \Lambda \| \Sigma^0 \rangle$	$-\frac{1}{3\sqrt{3}}(\lambda+2)$	$1+\frac{1}{\lambda}+\frac{1}{x}$
Σ_c^{++}	$\frac{2}{9}(4-y)$	$2+\frac{1}{y}$
Σ_c^+	$\frac{2}{9}(2-\lambda-y)$	$1+\frac{1}{\lambda}+\frac{1}{y}$
Σ_c^0	$-\frac{2}{9}(2\lambda+y)$	$\frac{2}{\lambda}+\frac{1}{y}$
Ξ_c^+	$\frac{2}{9}(2-x-y)$	$1+\frac{1}{x}+\frac{1}{y}$
Ξ_c^0	$-\frac{2}{9}(\lambda+x+y)$	$\frac{1}{\lambda}+\frac{1}{x}+\frac{1}{y}$
Ω_c^0	$-\frac{2}{9}(2x+y)$	$\frac{2}{x}+\frac{1}{y}$
Ξ_{cc}^{++}	$\frac{2}{9}(4y-1)$	$1+\frac{2}{y}$
Ξ_{cc}^+	$\frac{1}{9}(8y+\lambda)$	$\frac{1}{\lambda}+\frac{2}{y}$
Ω_{cc}^+	$\frac{1}{9}(8y+x)$	$\frac{1}{x}+\frac{2}{y}$
$\Lambda_c'^+$	$\frac{2}{3}y$	$1+\frac{1}{\lambda}+\frac{1}{y}$
$\Xi_c'^+$	$\frac{2}{3}y$	$1+\frac{1}{x}+\frac{1}{y}$
$\Xi_c'^0$	$\frac{2}{3}y$	$\frac{1}{\lambda}+\frac{1}{x}+\frac{1}{y}$
$\langle \Lambda_c'^+ \| \Sigma_c^+ \rangle$	$\frac{1}{3\sqrt{3}}(\lambda+2)$	$1+\frac{1}{\lambda}+\frac{1}{y}$

Table 1. (*Contd.*)

Particle	$\langle \sum_{i=1}^{3} \frac{m}{m_i} e_i \sigma_3(i) \rangle$	$\sum_{i=1}^{3} \frac{m_i}{m}$
$\langle \Xi_c'^{+} \| \Xi_c^{+} \rangle$	$\frac{1}{3\sqrt{3}}(x+2)$	$1+\frac{1}{x}+\frac{1}{y}$
$\langle \Xi_c'^{0} \| \Xi_c^{0} \rangle$	$\frac{1}{3\sqrt{3}}(1-x)$	$\frac{1}{\lambda}+\frac{1}{x}+\frac{1}{y}$
$J^P = \frac{3}{2}^{+}$		
Δ^{++}	2	3
Δ^{+}	$\frac{1}{3}(4-\lambda)$	$2+\frac{1}{\lambda}$
Δ^{0}	$\frac{2}{3}(1-\lambda)$	$1+\frac{2}{\lambda}$
Δ^{-}	$-\lambda$	$\frac{3}{\lambda}$
Σ^{*+}	$\frac{1}{3}(4-x)$	$2+\frac{1}{x}$
Σ^{*0}	$\frac{1}{3}(2-\lambda-x)$	$1+\frac{1}{\lambda}+\frac{1}{x}$
Σ^{*-}	$-\frac{1}{3}(2\lambda-x)$	$\frac{2}{\lambda}+\frac{1}{x}$
Ξ^{*0}	$\frac{2}{3}(1-x)$	$1+\frac{2}{x}$
Ξ^{*-}	$-\frac{1}{3}(\lambda+2x)$	$\frac{1}{\lambda}+\frac{2}{x}$
Ω^{-}	$-x$	$\frac{3}{x}$
Σ_c^{*++}	$\frac{1}{3}(4+2y)$	$2+\frac{1}{y}$
Σ_c^{*0}	$\frac{1}{3}(-2\lambda+2y)$	$\frac{2}{\lambda}+\frac{1}{y}$
Ξ_c^{*+}	$\frac{1}{3}(2-x+2y)$	$1+\frac{1}{x}+\frac{1}{y}$
Ξ_c^{*0}	$\frac{1}{3}(-\lambda-x+2y)$	$\frac{1}{\lambda}+\frac{1}{x}+\frac{1}{y}$
Ω_c^{*0}	$\frac{1}{3}(-2x+2y)$	$\frac{2}{x}+\frac{1}{y}$
Ξ_{cc}^{*++}	$\frac{1}{3}(2+4y)$	$1+\frac{2}{y}$
Ξ_{cc}^{*+}	$\frac{1}{3}(-\lambda+4y)$	$\frac{1}{\lambda}+\frac{2}{y}$
Ω_{cc}^{*+}	$\frac{1}{3}(-x+4y)$	$\frac{1}{x}+\frac{2}{y}$
Ω_{ccc}^{*++}	$2y$	$\frac{3}{y}$

The expectation value of μ_B is given by the product of columns 2 and 3.

Table 2. Estimated magnetic moments of baryons. Input values are underlined.

			Mass corrections with			
Particle	Present analysis	Simple quark model	$1/\sqrt{M_B}$ (Teese and Settles 1979)	$1/\sqrt{M_B}$ (Singh 1981a)	$1/\Sigma m_i$ (Tomozawa 1982; Tewari and Singh 1983)	Experimental values (Particle data 1984)
$J^P = 1/2^+$						
p	2·79	2·79	2·79	2·79	2·79	2·79
n	−1·91	−1·86	−1·91	−1·91	−1·86	−1·913
Λ	−0·614	−0·614	−0·61	−0·608	−0·614	−0·614 ± 0·004
Σ^+	2·51	2·69	2·39	2·37	2·42	2·368 ± 0·013
						2·38 ± 0·02
Σ^0	0·68	0·82	0·95	0·69	0·76	—
Σ^-	−1·02	−1·04	−0·90	−0·99	−0·91	−1·15 ± 0·0027
						−0·89 ± 0·14
Ξ^0	−1·37	−1·44	−1·27	−1·22	−1·24	−1·25 ± 0·014
Ξ^-	−0·46	−0·51	−0·49	−0·48	−0·49	−0·75 ± 0·06
						−0·69 ± 0·04
$\langle\Lambda\|\Sigma\rangle$	−1·57	−1·61	−1·45	−1·50	−1·44	−1·82 ± 0·22
Σ_c^{++}	1·65	2·36	1·44	1·45	1·72	—
Σ_c^+	0·33	0·46	0·29	0·30	0·34	—
Σ_c^0	−1·25	−1·43	−0·86	−0·85	−1·07	—
Ξ_c^+	0·58	0·73	0·37	0·40	0·42	—
Ξ_c^0	−1·07	−1·16	−0·76	−0·72	−0·81	—
Ω_c^0	−0·85	−0·89	−0·65	−0·59	−0·58	—
Ξ_{cc}^{++}	−0·04	−0·12	−0·00	−0·03	−0·02	—
Ξ_{cc}^+	−0·769	−0·82	−0·47	−0·45	−0·44	—
Ω_{cc}^+	−0·659	−0·69	−0·42	0·39	0·31	—
$\Lambda_c'^+$	0·38	0·37	0·30	0·27	0·32	—
$\Xi_c'^+$	0·37	0·37	0·29	0·26	0·27	—
$\Xi_c'^0$	0·36	0·37	0·29	0·26	0·28	—
$\langle\Lambda_c'^+\|\Sigma_c^+\rangle$	1·43	1·61	0·99	0·99	1·21	—
$\langle\Xi_c'^+\|\Xi_c^+\rangle$	1·25	1·41	0·90	0·87	0·94	—
$\langle\Xi_c'^0\|\Xi_c^0\rangle$	0·14	0·20	0·07	0·06	0·09	—
$J^P = 3/2^+$						
Δ^{++}	4·36	5·58	4·87	4·87	—	4·7–6·4
Δ^+	2·014	2·79	2·44	2·44	—	—
Δ^0	0·08	0	0	0	—	—
Δ^-	−2·18	−2·79	−2·44	−2·44	—	—
Σ^{*+}	2·48	3·14	2·47	2·53	—	—
Σ^{*0}	0·14	0·35	0·17	0·23	—	—
Σ^{*-}	−2·05	−2·48	−2·13	−2·06	—	—
Ξ^{*0}	0·49	0·71	0·34	0·45	—	—
Ξ^{*-}	1·85	−2·08	−1·85	−1·73	—	—
Ω^-	−1·61	−1·73	−1·61	−1·44	−1·44	—
Σ_c^{*++}	3·41	4·09	—	2·55	—	—
Σ_c^{*+}	1·01	1·30	—	0·84	—	—
Σ_c^{*0}	−1·32	−1·49	—	−0·88	—	—
Ξ_c^{*+}	1·38	1·65	—	0·98	—	—
Ξ_c^{*0}	−1·03	−1·13	—	−0·68	—	—
Ω_c^{*0}	−0·71	−0·78	—	−0·50	—	—
Ξ_{cc}^{*++}	2·267	2·60	—	1·36	—	—
Ξ_{cc}^{*+}	0·16	0·19	—	−0·04	—	—
Ω_{cc}^{*+}	0·34	0·167	—	0·105	—	—
Ω_{ccc}^{*++}	1·07	1·12	—	0·56	—	—

models. This probably may be because of the increasing masses of charmed baryons. However, the assumptions involved in the present work will be tested only when new data on charmed baryons will be available.

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