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Bosonic loops and gluon condensate

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Abstract. Contributions to the vacuum polarisation in QCD are calculated separately with fermion as well as boson loops to have an idea of results expected for possible supersymmetric extension. It is found that the results are not altered in any significant way.

Keywords. Bosonic loops; gluon condensate; supersymmetry.

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1. Introduction

Supersymmetry treats both bosons and fermions on the same footing. As virtual objects in closed loops, both fermions and bosons are likely to be equally important. Vacuum polarisation and non-perturbative effects for quantum chromodynamics with quark loops have been calculated by Shifman *et al* (1979). Bell and Bertlmann (1981) have also analysed the exponential moments of vacuum polarisation function and have obtained a non-relativistic potential which is confining in nature. However, non-perturbative analysis with spin zero objects like s-quarks in the loops has not yet been carried out.

Since the lowest order vacuum polarisation function with spin zero particles has already been calculated by several authors (Akhiezer and Berestetskii 1965), in the present work we shall calculate the non-perturbative higher order contribution with scalar quark loops or bosonic loops. Our calculation shows that in the non-relativistic limit the potential is still the superposition of a coulomb and a confining term with coefficients which do not differ appreciably from those calculated with fermionic loops alone.

One begins by calculating the vacuum expectation value of two heavy, quark or s-quark currents, $j_\mu(x)$. Its Fourier transform, the polarisation tensor in the true QCD vacuum is

$$i\int d^4x \exp(iqx)\langle\Omega|T(j_\mu(x)j_\nu(o))|\Omega\rangle = (q_\mu q_\nu - q^2 g_{\mu\nu})\pi(Q^2)$$
$$Q^2 = -q^2 \tag{1}$$

$\pi(Q^2)$ has an operator product expansion (Wilson 1969)

$$\pi(Q^2) = \sum_n C_n O_n \simeq C_{\text{pert}}(Q^2)I + C_G(Q^2)\langle\Omega|G^a_{\mu\nu}G^a_{\mu\nu}|\Omega\rangle + \ldots . \tag{2}$$

where I is the identity operator and $G^2 = \langle G^a_{\mu\nu}G^a_{\mu\nu}\rangle$ is the local operator constructed from gluon fields. The functions $C_{\text{pert}}(Q^2)$ and $C_G(Q^2)$ can be calculated perturbatively. The gluon condensate $\langle GG\rangle$ represents the confinement mechanism and is important in determining the properties of the bound states. The polarisation function $\pi(Q^2)$ satisfies a dispersion relation

$$\pi(Q^2)=\frac{1}{\pi}\int\frac{\mathrm{Im}\,\pi(s)\,ds}{s+Q^2}, \tag{3}$$

where the resonances in the physical Im $\pi(s)$ are related to the QCD calculations in (2). Differentiating (3) n times with respect to $(-Q^2)$ one obtains the power moments (Shifman *et al* 1981),

$$M_n(Q^2)=\frac{1}{\pi}\int\frac{\mathrm{Im}\,\pi(s)ds}{(s+Q^2)^{n+1}}. \tag{4}$$

The increase in n emphasizes the low energy region and probes large distances. Moments at arbitrary Q^2 have been calculated for various currents by Reinders *et al* (1981). Their application to charmonium produced beautiful results.

The exponential moment

$$M(\sigma)=\int ds\exp(-\sigma s)\,\mathrm{Im}\,\pi(s)$$
$$Q^2=\frac{n}{\sigma},\ n\to\infty \tag{5}$$

analysed by Bell and Bertlmann (1981) was an improvement on the inverse power moments. The exponential weight cuts off the large S contribution in Im $\pi(s)$ more sharply than a power weight and enhance the low energy contribution relative to the high energy one. Moreover they are directly related to imaginary time Green's function in the non-relativistic limit. Calculated within potential models, the results have always been closer to the exact results than the corresponding power moments (Bell and Bertlmann 1981).

2. Calculation of exponential moments

2.1 *Quark loops*

As noted earlier Bell and Bertlmann (1981) have calculated the exponential moments for quarks. We denote the moment by

$$M_F(\sigma)=\int ds\exp(-\sigma s)\,\mathrm{Im}\,\pi_F(s). \tag{6}$$

This exponential moment can be written as a sum of several terms

$$M_F(\sigma)=\exp(-4m^2\sigma)\left[\pi A_F(\sigma)\{1+\alpha_s a_F(\sigma)+\phi b_F(\sigma)\}\right]. \tag{7}$$

The first term is the simple single loop result with

$$\pi A_F(\sigma)=\frac{3}{16}\frac{1}{\sqrt{\pi}}\frac{1}{\sigma}G\left(\frac{1}{2},\frac{5}{2},w\right) \tag{8}$$

the Whittaker function

$$G(b,c,w)=\int dt\exp(-t)t^{c-1}(w+t)^{-b} \tag{9}$$

The second term is the contribution from the single fermion loop corrected by one virtual gluon exchanges. This has been calculated by Schwinger (1973). The extrapolated term is

$$\operatorname{Im}\pi_F^1(s) = \operatorname{Im}\pi_F^0(s)\left[1+\frac{4\alpha_s}{3}\left\{\frac{\pi}{2v}-\frac{3+v}{4}\left(\frac{\pi}{2}-\frac{3}{4\pi}\right)\right\}\right]$$

$$v=\left(1-\frac{4m^2}{s}\right)^{1/2} \tag{10}$$

where v is the velocity of the quark of mass m. Substituting (10) in (6) one obtains the quantity $a_F(\sigma)$ of the second term of (7) as

$$a_F(\sigma)=\frac{4}{3\sqrt{\pi}\,G(\frac{1}{2},\frac{5}{2},w)}\left[\pi-\left(\frac{7\pi}{12}-\frac{3}{8\pi}\right)G(1,2,w)+\left(\frac{\pi}{6}-\frac{1}{4\pi}\right)G(2,3,w)\right]-C. \tag{11}$$

where $\quad C=\frac{\pi}{2}-\frac{3}{4\pi}.$

The third term $\phi\, b_F(\sigma)$ is the non-perturbative contribution. This is obtained by calculating the box diagrams, with fermions in the loop, which yields the term $G^a_{\mu\nu}G^a_{\mu\nu}$ in the operator expansion. Here the quantity $b_F(\sigma)$ is

$$b_F(\sigma) = -\frac{w^2}{2}\frac{G(-\frac{1}{2},\frac{3}{2},w)}{G(\frac{1}{2},\frac{5}{2},w)}, \tag{12}$$

where $w=4m^2\sigma$ is a dimensionless variable and

$\phi=\dfrac{\frac{4\pi}{9}\langle\alpha_s G^2\rangle}{(4m^2)^2}$ is the gluon condensate parameter.

The ratio of moments calculated by Bertlmann (1981) for the above case is

$$R_F(\sigma) = -\frac{\mathrm{d}}{\mathrm{d}\sigma}\log M_F(\sigma),$$

with $\quad R_F(\sigma)=4m^2-\frac{\mathrm{d}}{\mathrm{d}\sigma}\left[\log A_F(\sigma)+\alpha_s a_F(\sigma)+\phi\, b_F(\sigma)\right].\qquad (13)$

With a power potential $V(r)=\Sigma\,\lambda_s r^s$, the ratio of moments calculated from the inverse moments of svz (Bertlmann 1981) is

$$R(\tau)=\frac{3}{2\tau}+\sum_s\lambda_s\Gamma\left(2+\frac{S}{2}\right)\left(\frac{\tau}{2\mu}\right)^{S/2}$$

$$\tau=m\sigma;\quad 2\mu=m \tag{14}$$

Taking the large σ (or τ) limit of $R_F(\sigma)$ and comparing it with $R(\tau)$ (equation (14)) one obtains the following non-relativistic potential

$$V(r) = -\frac{4}{3}\frac{\alpha_s}{r}+\frac{\phi_1}{64}mr^4.$$

$$\phi_1=(4m^2)^2\phi. \tag{15}$$

This is the superposition of a coulomb and a quartic potential. The strength of the confining part, being proportional to quark mass, has a strong flavour dependence.

2.2 *Scalar quark loops*

Similar equations as well as expressions for the potential can also be obtained with scalar quarks in the vacuum loops. The scalar quarks are assumed to have similar quantum numbers as ordinary quarks. We define, for a particular flavour but summed over all colours, a polarisation tensor

$$\pi^B_{\mu\nu}(q) = i\int d^4x \exp(iqx)\langle\Omega|T(j^B_\mu(x)j^B_\nu(0)|\Omega\rangle$$
$$= (q_\mu q_\nu - q^2 g_{\mu\nu})\pi^B(Q^2) \tag{16}$$

where the electromagnetic current j^B_μ are due to the S-quarks or bosons

$$j^B_\mu(x) = -i\phi^*(x)\overleftrightarrow{\partial}_\mu \phi(x)$$

As before we can write

$$\pi^B(Q^2) = \langle\Omega|C^B_I(Q^2)I + C^B_G(Q^2)\alpha_s G^2 + \ldots|\Omega\rangle, \tag{17}$$

and the exponential moment as

$$M_B(\sigma) = \int ds \exp(-\sigma s)\,\text{Im}\,\pi_B(s), \tag{18}$$

which can also be written as

$$M_B(\sigma) = \exp(-4m^2\sigma)\,\pi A_B(\sigma)[1 + \alpha_s a_B(\sigma) + \phi\, b_B(\sigma)]. \tag{19}$$

The first term in (19) is obtained from the simple S-quark loop given in figure 1. The imaginary part of the vacuum polarisation function to the lowest order is (Akhiezer and Berestetskii 1965)

$$\text{Im}\,\pi^0_B(s) = \frac{1}{16\pi}\left[\frac{(s-4m^2)^3}{s}\right]^{1/2}\theta(s-4m^2). \tag{20}$$

Substituting this in (18) one obtains

$$\pi A_B(\sigma) = \frac{3}{64}\frac{1}{\sqrt{\pi}}\frac{1}{\sigma}G(\tfrac{3}{2},\tfrac{5}{2},w). \tag{21}$$

The virtual gluon corrections to the simple S-quark loop is given in figure 1b. The corresponding imaginary part has also been calculated by Schwinger for bosons. He has shown that an adequate interpolation for large and small σ is obtained by the

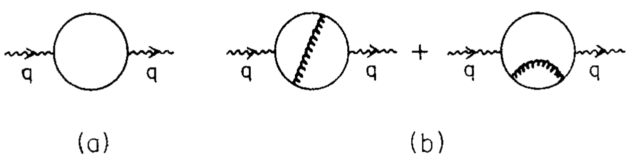


Figure 1. **a.** Simple S-quark loop diagram. **b.** Simple S-quark loops with virtual gluon correction to order α_s.

interpolation formula

$$\operatorname{Im} \pi_B^1(s) = \operatorname{Im} \pi_B^0(s)\left[1 + \frac{4\alpha_s}{3}\left\{\frac{\pi}{2v} - \frac{1+v}{2}\left(\frac{\pi}{2} - \frac{3}{\pi}\right)\right\}\right]. \tag{22}$$

This along with (8) gives

$$a_B(\sigma) = \frac{8}{9\sqrt{\pi}\, G(\frac{3}{2}, \frac{5}{2}, w)}[\pi(G(1, 2, w) - 3C_1 G(2, 3, w)] - C_1, \tag{23}$$

where $\quad C_1 = \frac{\pi}{3} - \frac{2}{\pi}.$

The third term $b_B(\sigma)$ in (19) is the non-perturbative contribution of the scalar quarks which we calculate below.

3. Calculation of fourth order diagrams with boson loops

With respect to heavy quarks the vacuum gluon field can be considered as an external field. Thus any gauge condition is admittable. One thus uses the Schwinger gauge (Dubovikov and Smilga 1981)

$$x_\mu A_\mu^a(x) = 0. \tag{24}$$

Here $A_\mu^a(x)$ can be expressed directly in terms of $G_{\mu\nu}^a(0)$, the gluon field tensor, and its covariant derivatives, namely,

$$A_\mu^a(x) = \frac{1}{2\cdot 0!} x_\rho G_{\rho\mu}^a(0) + \frac{1}{3\cdot 1!} x_\alpha x_\rho D_\alpha G_{\rho\mu}^a(0) + \dots \tag{25}$$

Fourier-transformed this gives

$$A_\mu^a(k) = -i\frac{(2\pi)^4}{2} G_{\rho\mu}^a(0) \frac{\partial}{\partial k_\rho} \delta^4(k) + \frac{(-i)^2 (2\pi)^4}{3} (D_\alpha G_{\rho\mu}^a(0)) \frac{\partial^2}{\partial k_\rho \partial k_\alpha} \delta^4(k) + \dots \tag{26}$$

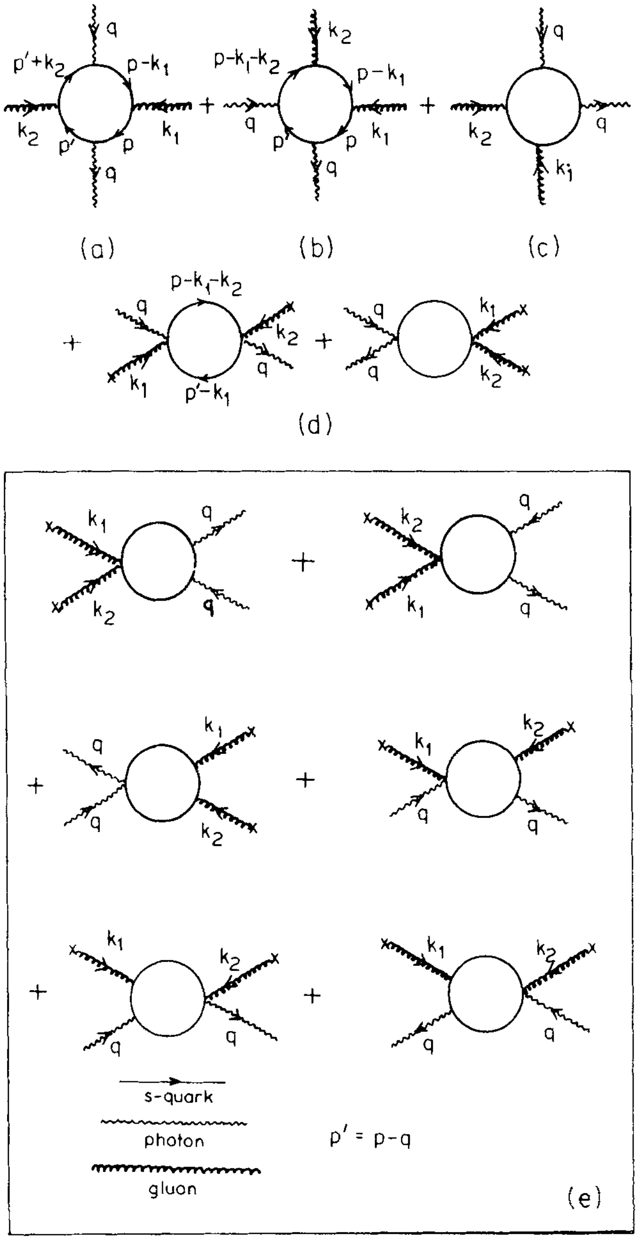
To obtain the G^2 contribution it suffices to keep only the first term in the expansion of $A(k)$ and insert $A(k)$ twice. The diagrams which contribute to G^2 in case of spin zero quarks are given in figures 2a–e. Figure 2e comes from the usual contact terms in the Lagrangian like $\phi^* \phi A_\mu^2$ and $\phi^* \phi A_\mu A_\mu^{em}$.

Using the fact

$$\langle \text{vac}| G_{\mu\nu}^a(0) G_{\alpha\beta}^b(0) |\text{vac}\rangle = \frac{1}{96} \delta^{ab} (g_{\mu\alpha} g_{\nu\beta} - g_{\mu\beta} g_{\nu\alpha}) \langle \text{vac}| G^2 |\text{vac}\rangle, \tag{27}$$

One obtains

$$[\pi_{\mu\nu}^B(q^2)]a = \frac{1}{3\times 96} \langle g^2 G^2\rangle (g_{\rho\sigma} g_{\alpha\beta} - g_{\rho\beta} g_{\alpha\sigma}) \int \frac{d^4 p}{(2\pi)^4} \frac{\partial}{\partial k_{1\rho}} \frac{\partial}{\partial k_{2\sigma}} \left[\frac{(p+p')_\mu (2p - k_1)_\alpha (p + p' + k_2 - k_1)_\nu (2p + k_2)_\beta}{(p^2 - m^2)[(p' + k_2)^2 - m^2][(p - k_1)^2 - m^2](p'^2 - m^2)}\right]_{k_1 = k_2 = 0}. \tag{28}$$



Figures 2a–e. Diagrams to order α_s to determine the coefficient function $C^B_G(Q^2)$ with S-quark loops.

Contracting μ and ν and differentiating with respect to k_1 and k_2 this reduces to zero. Proceeding in the same way

$$[\pi^B_{\mu\mu}(q^2)]_{b+c} = -\frac{i}{12}\langle g^2G^2\rangle \int \frac{\mathrm{d}^4p}{(2\pi)^4}\frac{p^2+p\cdot p'}{(p^2-m^2)^3(p'^2-m^2)}, \tag{29}$$

$$[\pi^B_{\mu\mu}(q^2)]_d = \frac{i}{6}\langle g^2G^2\rangle \int \frac{d^4p}{(2\pi)^4}\left[-\frac{2}{(p^2-m^2)^2(p'^2-m^2)} + \frac{2p^2}{(p^2-m^2)^3(p'^2-m^2)} + \frac{p\cdot p'}{(p^2-m^2)^2(p'^2-m^2)^2} + \frac{8m^2}{(p^2-m^2)^4}\right], \tag{30}$$

$$[\pi^B_{\mu\mu}(q^2)]_e = -\frac{i\langle g^2G^2\rangle}{6}\int \frac{d^4p}{(2\pi)^4}\left[-\frac{p^2+p\cdot p'-m^2}{(p^2-m^2)^3(p'^2-m^2)} + \frac{2m^2(p^2+2p\cdot p'+p'^2)}{(p^2-m^2)^4(p'^2-m^2)} - \frac{p'^2}{2(p^2-m^2)^2(p'^2-m^2)^2}\right]. \tag{31}$$

Thus the net contribution to $\pi^B_{\mu\mu}(q^2)$ with scalar quarks or internal boson state reduces to the following expression

$$\pi^B_{\mu\mu}(Q^2) = \frac{\alpha_s G^2}{8\pi}\int_0^1 dx\left[-\frac{8}{3m^2} + \frac{5x}{2[Q^2x(1-x)+m^2]} - \frac{m^2x(1-2x)}{2[Q^2x(1-x)+m^2]^2} + \frac{16m^4x^3}{3[Q^2x(1-x)+m^2]^3} + \frac{4m^2Q^2x^3}{3[Q^2x(1-x)+m^2]^3}\right]. \tag{32}$$

Hence

$$C^B_G(Q^2) = \frac{\alpha_s G^2}{288\pi Q^4}\left[\frac{15a-3a^2-2}{2a^{3/2}}\ln\frac{\sqrt{a}+1}{\sqrt{a}-1} + \frac{3a^2-17a-2}{a(a-1)}\right] \tag{33}$$

where $\qquad a = 1-\dfrac{4m^2}{s}; \; s = q^2 = -Q^2.$

The non-perturbative part of the exponential moment is then given by

$$M^{\text{nonpert}}_B(\sigma) = -\frac{\alpha_s G^2\sigma\sqrt{\pi}}{576}\left[4G(\tfrac{1}{2},\tfrac{3}{2},w) + 19G(\tfrac{3}{2},\tfrac{3}{2},w) + 24G(\tfrac{5}{2},\tfrac{3}{2},w)\right], \tag{34}$$

and

$$b_B(\sigma) = -\frac{w^2}{3}\left[\frac{G(\tfrac{1}{2},\tfrac{3}{2},w) + \tfrac{19}{4}G(\tfrac{3}{2},\tfrac{3}{2},w) + 6G(\tfrac{5}{2},\tfrac{3}{2},w)}{G(\tfrac{3}{2},\tfrac{5}{2},w)}\right] \tag{35}$$

4. Non-perturbative moments and the potential with scalar quarks

To obtain an equivalent non-relativistic potential in this case we proceed in the same way as Bertlmann and compute the ratio of moment

$$R_B(\sigma) = 4m^2 - \frac{d}{d\sigma}[\log A_B(\sigma) + \alpha_s a_B(\sigma) + \phi b_B(\sigma)]. \tag{36}$$

For heavy quarks one considers the non-relativistic limit, which is obtained for $\sigma \to \infty$ or $w \to \infty$.

In this limit

$$\pi A_B(\sigma) = \frac{3}{64\sqrt{\pi}}\frac{1}{(2m)^3}\sigma^{-5/2}$$

$$a_B(\sigma) = \frac{8\sqrt{\pi}}{9}w^{1/2} - C_1 \tag{37}$$

$$b_B(\sigma) = -\frac{w^3}{3}$$

Thus $$R_B(\sigma) = 4m^2 + \frac{5}{2\sigma} + 4m^2\left(-\alpha_s\frac{4\sqrt{\pi}}{9}w^{-1/2} + \phi w^2\right). \tag{38}$$

To compare this with (14) one makes the following substitution

$$w = m\tau, \quad 2\mu = m$$

$$\exp(4m^2\sigma)\, M(\sigma) = 4m\, M(\tau) \tag{39}$$

$$\tfrac{3}{5}(4m)^{-1}[R(\sigma) - 4m^2] = R(\tau)$$

With these (38) reduces to

$$R(\tau) = \frac{3}{2z} + \left[-\alpha_s\frac{4\sqrt{\pi}}{15}(\tau/m)^{-1/2} + \frac{3\phi_1}{80}m(\tau/m)^2\right] \tag{40}$$

Comparing (40) and (14) one obtains the equivalent non-relativistic potential

$$V(r) = -\frac{8}{15}\frac{\alpha_s}{r} + \frac{\phi_1}{160}mr^4. \tag{41}$$

Equation (41), for the potential with scalar quarks in the loops is new and has not been published before. It is also the superposition of a coulombic and a quartic term, the coefficients of both the terms being smaller than those in (15). The strength of the confining part has flavour dependence also.

5. Possible supersymmetric calculation

We consider here the possibility that both quark and scalar quark currents co-exist. This may for instance happen in a supersymmetric phase. When supersymmetry is broken at high temperatures the scalar becomes heavier and need not be considered any further. In such a case one may consider the calculation of this section to be more of a hypothetical one. Normal quarks at low temperature contribute as usual and there will be no change in the present world, and the values of R and the leptonic width of J/ψ will not be affected.

So for completeness we consider the system in which the fermion and bosons of nearly equal mass are both assumed to be present in the loop diagrams. The exponential moment to first order in α_s and ϕ is then

$$M_u(\sigma) = \exp(-4m^2\sigma)\,\pi A_u(\sigma)[1 + \alpha_s a_u(\sigma) + \phi\, b_u(\sigma)], \tag{42}$$

where the free term is

$$\pi A_u(\sigma) = \pi A_F(\sigma) + \pi A_B(\sigma)$$
$$= \frac{3}{16}\frac{1}{\sqrt{\pi}}\frac{1}{\sigma}G\left(\frac{1}{2},\frac{5}{2},w\right) + \frac{3}{64}\frac{1}{\sqrt{\pi}}\frac{1}{\sigma}G\left(\frac{3}{2},\frac{5}{2},w\right).$$

Using the integral representation of G function

$$G(\tfrac{3}{2},\tfrac{5}{2},w) = \frac{1}{\Gamma(\frac{3}{2})}\int \mathrm{d}t \exp(-t)t^{3/2}(w+t)^{-3/2},$$
$$= -2G(\tfrac{1}{2},\tfrac{5}{2},w) + 2G(\tfrac{1}{2},\tfrac{3}{2},w),$$
$$\pi A_u(\sigma) = \frac{3}{32}\frac{1}{\sqrt{\pi}}\frac{1}{\sigma}\left[G(\tfrac{1}{2},\tfrac{3}{2},w) + G(\tfrac{1}{2},\tfrac{5}{2},w)\right]. \tag{43}$$

and the perturbative gluon correction

$$a_u(\sigma) = \frac{8}{3\sqrt{\pi}\left[G(\frac{1}{2},\frac{3}{2},w) + G(\frac{1}{2},\frac{5}{2},w)\right]}$$
$$\left[\pi - \left(\frac{5\pi}{12} - \frac{3}{8\pi}\right)G(1,2,w) + \frac{3}{4\pi}G(2,3,w)\right]$$
$$-\frac{\left(\frac{2\pi}{3} + \frac{1}{2\pi}\right)G\left(\frac{1}{2},\frac{5}{2},w\right) + C_1 G\left(\frac{1}{2},\frac{3}{2},w\right)}{G\left(\frac{1}{2},\frac{3}{2},w\right) + G\left(\frac{1}{2},\frac{5}{2},w\right)}. \tag{44}$$

The non-perturbative contribution to the momenta in this case is

$$b_u(\sigma) = -\frac{w^2}{24}\left[\frac{4G(\frac{1}{2},\frac{3}{2},w) + 19G(\frac{3}{2},\frac{3}{2},w) + 24G(\frac{5}{2},\frac{3}{2},w)}{G(\frac{1}{2},\frac{5}{2},w) + G(\frac{1}{2},\frac{3}{2},w)}\right]$$
$$-w^2\left[\frac{G(-\frac{1}{2},\frac{3}{2},w)}{G(\frac{1}{2},\frac{5}{2},w) + G(\frac{1}{2},\frac{3}{2},w)}\right]. \tag{45}$$

In the non-relativistic limit $w \to \infty$

$$\pi A_u(\sigma) = \frac{3}{16}\frac{1}{\sqrt{\pi}}\frac{1}{2m}\sigma^{-3/2}$$
$$a_u(\sigma) = \frac{4\sqrt{\pi}}{3}w^{1/2} - C_2 \tag{46}$$
$$b_u(\sigma) = -\frac{11w^2}{24} - \frac{w^3}{2}$$

We have retained the contribution of both bosons and fermions in the non-perturbative part.

Hence we have for the ratio of moment

$$R_u(\sigma) = 4m^2 + \frac{3}{2\sigma} + 4m^2\left(-\alpha_s\frac{2\sqrt{\pi}}{3}w^{-1/2} + \frac{11\phi}{12}w + \frac{3\phi}{2}w^2\right). \tag{47}$$

To compare (47) with (14) we make the following substitution

$$w = m\tau, \quad 2\mu = m$$

$$(4m)^{-1}[R(\sigma) - 4m^2] = R(\tau)$$

Equation (47) then reduces to

$$R(\tau) = \frac{3}{2\tau} + -\alpha_s \frac{2\sqrt{\pi}}{3}(\tau/m)^{-1/2} + \frac{11\phi_1}{192}\frac{1}{m}(\tau/m) + \frac{3\phi_1}{32}m(\tau/m)^2. \quad (48)$$

Comparing (14) and (48) one gets the following non-relativistic potential.

$$V(r) = -\frac{4}{3}\frac{\alpha_s}{r} + \frac{11\phi_1}{384}\frac{1}{m}r^2 + \frac{\phi_1}{64}mr^4$$

This potential contains the usual coulombic term, and the leading term in the confining part is the quartic term. Further the confining part also has the same flavour dependence (equation (15)).

Thus we conclude that including both spin $\frac{1}{2}$ and spin zero object loops contributing to the vacuum polarisation function does not alter the quartic confining nature of the potential.

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