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Some consequences of global horizontal symmetry

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Abstract. It is pointed out that the present $SU(3)_c \times SU(2)_L \times U(1)$ gauge interactions with three families have a global horizontal symmetry (denoted hereby $SU(3)_H$) which is broken only by the weak charged hadron current J_h. Also, with (u, c), (d, s), (ν_e, ν_μ) and (e^-, μ^-) as doublets of $SU(2)_H$ (subgroup of $SU(3)_H$), J_h has simple transformation properties under this subgroup. Amplitude relations, using $SU(2)_H$ symmetry, for hadronic leptonic and semi-leptonic decays are given.

Keywords. Global horizontal symmetry.

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1. Introduction

The present theory does not tell us how many generations or families of quarks and leptons exist nor does it tell us what additional symmetry serves to distinguish them. Number of attempts (Davidson *et al* 1979, 1984, and references therein) have been made to classify the families according to some horizontal symmetry group (HSG), both discrete and continuous. Attempts to gauge the HSG and its attendant problems (like $K_L - K_S$ mass difference, $\mu \to e\gamma$, etc) have also been discussed.

In this note, we consider the consequences of the global horizontal symmetry already present in the accepted $SU(3)_C \times SU(2)_L \times U(1)$ gauge interactions formulated in terms of quarks and leptons. In fact, for three families all the interactions, except for the weak hadronic charged current J_h, are invariant under a group, which we call $SU(3)_H$, if we assume that (u, c, t), (d, s, b), $(\nu_e, \nu_\mu, \nu_\tau)$ and (e^-, μ^-, τ^-) transform as triplets of this group. This is clearly true for the quark-gluon, electromagnetic, neutral current and charged lepton current interactions with all gauge bosons being $SU(3)_H$ singlets. The weak current J_h breaks the global invariance under $SU(3)_H$ due to the Kobayashi-Maskawa (1973) mixing among the quarks. Also, the very large mass differences among the quarks (and leptons) breaks this symmetry. Moreover, exploiting $SU(3)_H$ will give us relations involving decay amplitudes of the t and b quarks for which sufficiently good data is not available. Therefore, instead, we consider the consequences of the invariance under its sub-group $SU(2)_H$ of which

$$U = (u, c),\ D = (d, s),$$
$$L_e = (e^-, \mu^-) \text{ and } L_\nu = (\nu_e, \nu_\mu), \tag{1}$$

are taken to be doublets with $SU(2)_H$ spin $H = 1/2$. Also, u, d, etc. will have $H_3 = +1/2$ while c, s, etc. have $H_3 = -1/2$*. The advantages of considering $SU(2)_H$ are three-fold:

* The usual U-spin $SU(2)_U$ (subgroup of flavour SU(3)) is different from $SU(2)_H$. For that, though (d, s) form a doublet, all the others $(u, c, e^-$, etc.) are singlets.

(a) The mass differences among the members of a doublet are not so large;
(b) The relations obtained among the weak decays of the strange and charmed hadrons would hopefully be amenable to confrontation with data;
(c) The current J_h has simple and well-defined transformation properties under $SU(2)_H$, namely*

$$J_h \simeq \cos\theta\,(\bar{u}d + \bar{c}s) + \sin\theta\,(\bar{u}s - \bar{c}d) + \ldots \tag{2}$$

Here we have neglected the other mixing-parameters as they are found to be small experimentally (Chau and Keung 1984 and references therein). The dots in (2) represent the terms involving the t and b quarks. Further, θ is just the Cabibbo angle. Under $SU(2)_H$, the first term in (2) has $H = 0$, while the coefficient of $\sin\theta$ transforms as the sum of $H = 1, H_3 = +1$ and $H = 1, H_3 = -1$ objects which belong to the same $H = 1$ multiplet. Denoting these by $J(H, H_3)$ one can write the terms involving only the u, d, s and c quarks as

$$J_h = \cos\theta\, J(0,0) + \sin\theta\,\{J(1,1) + J(1,-1)\} + \ldots \tag{3}$$

Thus, the H-spin selection rules obeyed by J_h are

$$|\Delta \mathbf{H}| = 0 \text{ and } |\Delta \mathbf{H}| = 1 \text{ with } \Delta H_3 = \pm 1.$$

Having abstracted the $SU(2)_H$ properties of J_h a H-spin analysis can be done directly for the hadron decay amplitudes to obtain relations among them and test them, in the same spirit as is done for flavour isospin or SU(3) analyses. The new idea, in this paper, is noting the existence of $SU(2)_H$ symmetry and its exploitation.

To proceed further one has to classify hadrons according to $SU(2)_H$. This is done in §2 using (1) and the knowledge of the quark content of the hadrons. In §3 we briefly describe the application to the leptonic and semileptonic hadron decays** and give only such amplitude relations which are amenable to experiment.

2. Classification of hadrons

The members of the U (or D) doublet carry the same electric charge so that hadrons with the same charge Q are likely to belong to multiplets of $SU(2)_H$ although they may carry very different flavour quantum numbers like strangeness (S) and charm (C). Also, as we shall see, in general, a hadron will be a superposition of different H-spin representations.

2.1 *Mesons*

The sixteen low-lying mesons will be formed out of $U\bar{U}$, $D\bar{D}$, $U\bar{D}$ and $D\bar{U}$ with $Q = 0, 0, 1$ and -1 respectively. Each of these contains four states which form $H = 0$ and 1 multiplets. Clearly, the members of a given isospin multiplet will fall into different H-spin multiplets. The four $U\bar{U}$ ($D\bar{D}$) states contain no strange (charm) mesons. As a typical example note that in $D\bar{U}$, $\left(D^-, \frac{F^- - \pi^-}{\sqrt{2}}, K^-\right)$ and $\left(\frac{\pi^- + F^-}{\sqrt{2}}\right)$

* The usual $(V - A)$ structure has not been indicated.

** Application to purely leptonic processes does not give anything new since the leptons are observable (unlike quarks) and all their interactions have $H = 0$.

transform as $H = 1$ and 0 objects. The other H-spin multiplets are easy to write down and one can then obtain the meson states in terms of the H-spin multiplets.

2.2 *Baryons*

The low-lying baryons are obtained from UUU, UUD .... Each of these is a product of three doublets and reduces to eight states which form one $H = 3/2$ and two distinct $H = 1/2$ multiplets. The construction of the baryon states is quite straightforward as their quark content is well-known (Gupta 1976). Instead of writing these out in full detail we indicate briefly where the usual octet and decuplet states occur. Baryons with different charge will clearly belong to distinct sets of H-spin multiplets.

2.2*a* $Q = -1$ *baryons*: These come from DDD and have the simplest H-spin transformation properties since $D = (d, s)$. In fact, $(\Delta^-, \Sigma^{*-}, \Xi^{*-}, \Omega^-)$ has $H = 3/2$ while $(\Sigma^-, -\Xi^-)$ has $H = 1/2$. The remaining $H = 1/2$ doublet does not correspond to any definite baryon states and are not needed for our analysis.

2.2*b* $Q = 0$ *baryons*: These come from products like UDD. The familiar baryons which these contain are n, Λ, Σ°, Ξ°, $\Sigma^{*\circ}$ and $\Xi^{*\circ}$, none of which has simple H-spin properties. All of these (Λ, Σ°, etc.) are superpositions of eight different H-spin multiplets except for n, which is a superposition of 3 distinct H-spin multiplets.

2.2*c* $Q = 1$ *baryons*: These arise from products like UUD. The familiar baryons present in these are $p, \Sigma^+, \Delta^+, \Sigma^{*+}$. The p and Δ^+ are superpositions of 3, while Σ^+ and Σ^{*+} are superpositions of 8 different H-spin multiplets.

2.2*d* $Q = +2$ *baryons*: The only low-lying baryon, which it contains is Δ^{++} which transforms as the $H_3 = 3/2$ component of a $H = 3/2$ multiplet containing baryons which carry charm equal to 1, 2 and 3 units.

Given this brief description of the classification of the hadrons we proceed to a $SU(2)_H$ analysis of their weak decays. In doing this our object is to consider such set of decays where experimental data is available or is soon likely to be available so that the amplitude relations obtained can be confronted with experiment.

3. Consequences of H-spin analysis

The leptonic and semi-leptonic hadron decays arise from the effective interaction ($J_h J_l^\dagger$ + h.c.), where J_l is the charged leptonic current. The H-spin properties of this interaction are determined by J_h since J_l has $H = 0$. Further, the leptonic part of the amplitude factorizes and is explicitly known and would be the same for the amplitude $A(h \to h' + l^- + \bar{\nu}_l)$ for any two hadrons h and h'. So the relevant part for $SU(2)_H$ analysis is the hadronic part $\langle h' | J_h | h \rangle$. Relations obtained among the hadronic part of the decay amplitudes would also be true for the full amplitude for a given lepton pair. These remarks apply to the meson leptonic decay amplitude $A(h^- \to l^- + \bar{\nu}_l)$ where the hadronic part is $\langle 0 | J_h | h \rangle$. For ease of notation, we suppress the lepton pair in the final state and write simply $A(h^-)$ and $A(h \to h')$ for the leptonic and semi-leptonic decay amplitudes.

3.1 *Meson leptonic decays*

There are four $Q = -1$ mesons $h = \pi^-, K^-, F^-, D^-$ which decay into a charged lepton pair (e.g. μ^-, $\bar{\nu}_\mu$). The hadronic part of the amplitudes $\langle 0 | J_h | D\bar{U} \rangle$ depends on two

$SU(2)_H$ invariant amplitudes. We thus obtain two simple testable relations

$$|A(\pi^-)| = |A(F^-)|, \tag{4a}$$

$$|A(K^-)| = |A(D^-)|. \tag{4b}$$

These have been given for modulus of the amplitudes because that is what one extracts from the experimental rate after removing the phase space factors. Corresponding relations will be true for the antiparticle decays, $\pi^+ \to \mu^+ + \nu_\mu$ etc. If we recall the quark content of the mesons then (4) is obvious directly from (2) and probably has been already noted earlier. Less obvious consequences of H-spin arise for the semi-leptonic decays.

3.2 *Meson semi-leptonic decays*

There are 32 possible amplitudes ($h \to h' + l^+ + \nu_l$) arising from eight $Q = 0$ mesons going to four $Q = -1$ mesons plus a lepton pair. Of these 24 are allowed by the usual selection rules, $\Delta Q = \Delta S, \Delta Q = \Delta C = \Delta S, |\Delta S| \leqslant 1, |\Delta C| \leqslant 1$ etc. obeyed by J_h. Of the eight forbidden by these selection rules four (*viz*, $D^\circ \to D^-$, $K^\circ \to K^-$, $\bar{K}^\circ \to D^-$, $\bar{D}^\circ \to K^-$) are also forbidden by H-spin selection rule since J_h obeys $|\Delta H_3| \leqslant 1$. However the other four (*viz*, $K^\circ \to F^-$, $\bar{K}^\circ \to \pi^-$, $\bar{D}^\circ \to \pi^-$, $D^\circ \to F^-$) are non-zero at the level of H-spin analysis and are actually expressed in terms of two invariant amplitudes (say a and b). The reason why this happens is that, unlike isospin multiplets, members of an irreducible representation of $SU(2)_H$ carry different S and C quantum numbers. So it is necessary to impose the usual strangeness and charm selection rules after one has done the H-spin analysis. Imposition of these selection rules merely reduces the number of independent H-spin invariant amplitudes. As a result, putting $a = b = 0$ to remove the four unwanted amplitudes, we find the 24 allowed amplitudes are given in terms of 8 H-spin invariant amplitudes. It is straightforward but tedious to obtain the sixteen amplitude relations. Most of these involve 4 or 5 amplitudes and are difficult to test and we do not record them. The only simple relations obtained are

$$|A(\bar{D}^\circ \to D^-)| = |A(D^\circ \to K^-)|, \tag{5a}$$

$$|A(D^\circ \to \pi^-) = |A(\bar{D}^\circ \to F^-)|, \tag{5b}$$

$$|A(\bar{K}^\circ \to F^-) = |A(K^\circ \to \pi^-)|, \tag{5c}$$

The amplitude modulii involved in (5) can be extracted from the corresponding decay, for example, the right side of (5b) can be obtained from the decay rate for $F^- \to \bar{D}^\circ + l^- + \bar{\nu}_l$. Corresponding relations will hold for the amplitudes for $Q = 1$ meson $\to Q = 0$ meson plus a lepton pair. Since $A(\bar{K}^\circ \to \pi^-) = 0 = A(K^\circ \to F^-)$, (5c) can be written as

$$|A(F^- \to K_{L,S})| = |A(K_{L,S} \to \pi^-)| \tag{5d}$$

Equations (5) are easy to test. However, one has to await data on the charmed meson semi-leptonic decays.

3.3 *Baryon semi-leptonic decays*

There are six $\Delta S = 0$ and six $\Delta S = \Delta Q = 1$ semi-leptonic decays of the usual baryon octet. From the viewpoint of $SU(2)_H$, these 12 decays break up into two different sets of six decays each, depending on the charge Q of the baryons involved. The first set contains transitions between $Q = 0$ and $Q = 1$ baryons (*e.g.* $n \to p$, $\Lambda \to p$, $\Sigma^+ \to \Lambda$, etc.)

while the second set contains the decays of $Q = -1$ baryons (*e.g.* $\Sigma^- \to n$, $\Xi^- \to \Xi^\circ$, etc.). This break up occurs because baryons with different Q belong to different H-spin multiplets and consequently the decays in the two sets will be given in terms of different invariant amplitudes. Thus, $SU(2)_H$ symmetry will give no relations between the decays of the two sets. Among the decay amplitudes in the first set no relation is obtained because the $Q = 0$ and $Q = 1$ baryons do not have simple H-spin properties. However, as pointed earlier, the $Q = -1$ baryons have simple H-spin properties and, in fact, one obtains two relations. In the $\Delta S = 0$ sector, we find

$$\sqrt{2}A(\Xi^- \to \Xi^\circ) + \sqrt{3}A(\Sigma^- \to \Lambda^\circ) = A(\Sigma^- \to \Sigma^\circ), \tag{6}$$

where the lepton pair in the final state is understood. In the $\Delta S = 1$ sector, H-spin analysis gives a four amplitude relation which involves the $\Delta Q = -\Delta S$ amplitude $A(\Sigma^- \to \Xi^\circ)$. As explained above, we can impose the $\Delta Q = \Delta S$ rule and require $A(\Sigma^- \to \Xi^\circ) = 0$, thus reducing the number of H-spin invariant amplitudes by one. The relation among the remaining three $\Delta Q = \Delta S$ amplitudes is

$$\sqrt{2}A(\Sigma^- \to n) = \sqrt{3}A(\Xi^- \to \Lambda) - A(\Xi^- \to \Sigma^\circ), \tag{7}$$

The remarkable thing about (6) and (7) is that they form a sub-set of the many more relations obtained by using the flavour SU(3) transformation properties of J_h (Cabibbo 1963). The latest experimental results (Jarlskog 1983 and references therein) are in conformity with the predictions of flavour SU(3) symmetry. As such (6) and (7) do not provide a definitive test of $SU(2)_H$ symmetry. For predictions which are specific to it one would presumably have to appeal to relations involving charmed baryon semi-leptonic decays.

For the sake of completeness, we give the $SU(2)_H$ predictions for the semi-leptonic decays of $J^P = 1/2^+$ charmed baryons into the $1/2^+$ octet baryons, even though data is not available, at present, to test these. We use the notation of Gaillard *et al* (1975) to denote the $1/2^+$ charmed baryons with the quark content in brackets so that the flavour and H_3 quantum numbers become explicit. The $\Delta C = \Delta S = \Delta Q$ decays (proportional to $\cos\theta$) go through the $J(0,0)$ part of J_h, while the $\Delta C = \Delta Q$, $\Delta S = 0$ decays (proportional to $\sin\theta$) go through the $(J(1,1) + J(1,-1))$ part. In each case, one has to do the $SU(2)_H$ analysis for the $(Q = 1 \to Q = 0)$ and $(Q = 0 \to Q = -1)$ type amplitudes separately. The results for the four cases with brief comments are given below.

(*i*) $A(Q = 1 \to Q = 0)$ *amplitudes*

(a) There are 4 charmed baryon decays ($C_1^+ \to \Sigma^\circ$, $C_0^+ \to \Lambda^\circ$, $A^+ \to \Xi^\circ$ and $S^+ \to \Xi^\circ$) obeying $\Delta Q = \Delta C = \Delta S$. These get related to the 3 usual $\Delta S = 0$ amplitudes ($p \to n$, $\Sigma^+ \to \Sigma^\circ$, $\Sigma^+ \to \Lambda$). Noting that $A(C_1^+ \to \Lambda^\circ) = A(C_0^+ \to \Sigma^\circ) = 0$ by the $|\Delta \mathbf{I}| = 0$ selection rule, since C_1^+ (C_0^+) has isospin 1(0), two relations emerge, viz.,

$$A(C_1^+(udc) \to \Sigma^\circ) = \sqrt{2}A(\Sigma^+ \to \Sigma^\circ) - A(p \to n), \tag{8a}$$

$$\sqrt{3}A(C_0^+(udc) \to \Lambda^\circ) = \sqrt{2}A(\Sigma^+ \to \Lambda^\circ) + \sqrt{3}A(p \to n). \tag{8b}$$

(b) There are six $\Delta C = \Delta Q$, $\Delta S = 0$ and three usual $\Delta S = \Delta Q$ decays. These nine amplitudes are given in terms of eight independent $SU(2)_H$ amplitudes. We do not record the complicated relation obtained as there is no hope of testing it.

(ii) $A(Q = 0 \rightarrow Q = -1)$ *amplitudes*

(a) There are three $\Delta C = \Delta Q = \Delta S$ decays ($C_1^\circ \rightarrow \Sigma^-$, $A^0 \rightarrow \Xi^-$, $S^\circ \rightarrow \Xi^-$) which get related to the 3 usual $\Delta S = 0$ amplitudes considered earlier. In addition to (6), two relations involving charmed baryons are obtained, viz.,

$$A(C_1^0(ddc) \rightarrow \Sigma^-) = -\sqrt{2}A(S^\circ(dsc) \rightarrow \Xi^-), \tag{9a}$$

$$\sqrt{3}A(C_1^0(ddc) \rightarrow \Sigma^-) - \sqrt{2}A(A^\circ(dsc) \rightarrow \Xi^-)$$
$$= \sqrt{6}A(\Sigma^\circ \rightarrow \Sigma^-) - \sqrt{2}A(\Lambda^\circ \rightarrow \Sigma^-) \tag{9b}$$

Note that the amplitudes on the right side in (9b) can be obtained from the corresponding semi-leptonic decay.

(b) There are three $\Delta C = \Delta Q, \Delta S = 0$ decays ($A^\circ \rightarrow \Sigma^-$, $S^\circ \rightarrow \Sigma^-$ and $T^\circ \rightarrow \Xi^-$) which get related to the 3 usual $\Delta Q = \Delta S$ amplitudes. In addition to (7), the two new relations which emerge are

$$A(T^\circ(ssc) \rightarrow \Xi^-) = -\sqrt{2}A(S^\circ(dsc) \rightarrow \Sigma^-) \tag{10a}$$

$$= A(n \rightarrow \Sigma^-) \tag{10b}$$

These relations are particularly simple and would hopefully be tested in the near future.

In summary, we have pointed out that the present gauge interactions have an inbuilt global horizontal symmetry which is broken only by the weak hadronic charged current J_h. A subgroup $SU(2)_H$ of this symmetry is used to classify hadrons and obtain relations between hadron leptonic and semi-leptonic decays. Some of these are simple enough to provide a test of $SU(2)_H$ symmetry. It is interesting that using the $SU(2)_H$ and isospin properties of J_h one obtains relations which are normally obtained by assuming higher flavour symmetry like SU(3).

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