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Tachyons in general relativity

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Abstract. The tachyonic version of the Schwarzschild (bradyonic) gravitational field within the framework of extended relativity is considered. The metric of a tachyonic black hole is obtained through superluminal transformations from a bradyonic metric. The extended space-time manifold of this geometry which includes both black and white tachyonic holes is analysed, and the differences between the tachyonic and bradyonic versions are noted. It is shown that the meanings of black holes, tachyons and bradyons depend on the character of the reference frame and are not absolute.

Keywords. Black hole; tachyon; extended manifold.

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1. Introduction

In the framework of contemporary general relativity a certain number of asymptotically flat exact solutions of Einstein and Einstein-Maxwell equations are known which have pseudosingular horizons. These theoretical models describe the field of a gravitationally collapsed object—a black hole (also white and grey holes).

It is now widely accepted (Misner *et al* 1973) and there is enough evidence (Carter 1975) that black holes can have a mass (obligatory), an electrical or magnetic charge and an angular momentum with respect to the symmetry axis. Exact solutions of Einstein-Yang-Mills equations give also black holes with a colour (Perry 1977).

Over the last two decades a generalization of special relativity to superluminal reference frames and faster-than-light objects—tachyons—has been obtained. However, if tachyons did appear in the context of special relativity (more explicitly, extended relativity) (Recami and Mignani 1974), then naturally one would like to investigate them in the context of general relativity also. The simplest topic concerning that is the consideration of tachyons on the background of gravitational fields as test particles: they should move along space-like geodesics in space-time. But it does not give any new concepts in relativity (see bibliography in the survey by Sharp 1976).

The more fundamental problem is the investigation of tachyons as sources of a gravitational field, *i.e.* finding out the exact solutions of the general relativistic equations with a tachyonic space-time singularity. Such asymptotically flat solutions with horizons are, obviously, tachyonic analogues of black holes. It is interesting to study the extension of the tachyon theory to strong gravitational fields in connection with the peculiarities of faster-than-light objects.

It is possible that tachyons will help the experimental observation of black holes in the Universe, and that black holes, in turn will help the observation of tachyons (Narlikar and Dhurandhar 1978).

It is reasonable to apply to these aims the theory of extended relativity considering tachyons on an equal footing and in a symmetric manner with bradyons (Recami and Mignani 1974). The essence of the theory consists in the exchange of sign of the space-time line element, $ds^2 \to -ds^2$, by a transformation between subluminal and superluminal objects.

This paper investigates the tachyonic version of the spherically symmetric Schwarzschild metric and its extended manifold. Section 4 discusses the consequences of this unified approach for tachyonic and bradyonic gravitational fields in different regions of the extended manifolds, and for their representation on a more-than-four dimensional space-time structure.

2. The geometry of a tachyonic Schwarzschild field

Peres (1970) first defined the geometry of the gravitational field near an uncharged non-rotating tachyon. This problem was later considered by Schulman (1971), Gavrilina and Efremov (1982), and Gott (1974); in the last paper a more complete analysis has been carried out. However, there are several solutions which are valid for "different regions" of the space-time. We shall consider Gott's procedure and separate the solution which is true within the framework of extended relativity.

Let us derive the vacuum field metric for the tachyon associated with a space-like worldline. Firstly we divide the space-time near the tachyon into three regions according to the work of Gott (1974), as shown in figure 1. The metrics for the different regions will be slightly different in the background coordinate system (T, X, Y, Z). We assume that the tachyonic configuration moves along the Z-axis and regardless of the movement it is completely spherically symmetric. The centre of the sphere, the point $X = Y = Z = 0$ and the line $X = Y = 0$ are the space-time singularities.

The most general line element must be a quadratic form only of form-invariants with respect to Lorentz transformations, which are:
In regions I and II

$$X^2 + Y^2 - T^2 \equiv -\tau^2,\ dX^2 + dY^2 - dT^2,$$

$$d\tau,\ ZdZ,\ XdX + YdY - TdT = -\tau d\tau;$$

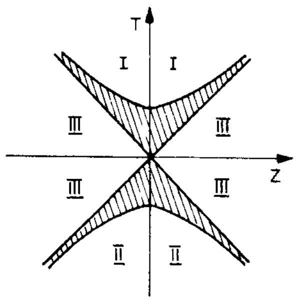

Figure 1. Coordinate regions for the search of tachyonic metric.

In region III

$$X^2+Y^2-T^2 \equiv \sigma^2,\ \mathrm{d}X^2+\mathrm{d}Y^2-\mathrm{d}T^2,\ \mathrm{d}\sigma,$$
$$Z\mathrm{d}Z,\ X\mathrm{d}X+Y\mathrm{d}Y-T\mathrm{d}T=\sigma\mathrm{d}\sigma;$$

We adopt here the following coordinates:

In region I: Z, τ, θ, ϕ

where $\quad T=\tau\,\mathrm{ch}\,\theta,\ X=\tau\,\mathrm{sh}\,\theta\cos\phi,\ Y=\tau\,\mathrm{sh}\,\theta\sin\phi;$ (1a)

In region II: $Z, \tau, \theta, \phi,$

where $\quad T=-\tau\,\mathrm{ch}\,\theta,\ X=\tau\,\mathrm{sh}\,\theta\cos\phi,\ Y=\tau\,\mathrm{sh}\,\theta\sin\phi;$ (1b)

In region III: $Z, \sigma, \alpha, \phi,$

where $\quad T=\sigma\,\mathrm{sh}\,\alpha,\ X=\sigma\,\mathrm{ch}\,\alpha\cos\phi, Y=\sigma\,\mathrm{ch}\,\alpha\sin\phi.$ (1c)

They have the ranges: $-\infty<Z<+\infty$, $0<\tau<\infty$, $0<\sigma<\infty$, $-\infty<\alpha<+\infty$, $0<\theta<\infty$, $0<\phi<2\pi$.

As usual (for example, by a solution of the Schwarzschild problem), with suitable coordinate transformations we may represent the metric obeying the tachyon symmetry conditions in the following forms:

$$\text{I, II: } \mathrm{d}s^2=\exp[\lambda_1(\tau)]\,\mathrm{d}z^2-\exp[-\lambda_2(\tau)]\,\mathrm{d}\tau^2-\tau^2(\mathrm{d}\theta^2+\mathrm{sh}^2\theta\,\mathrm{d}\phi^2), \quad (2a)$$

$$\text{III: } \mathrm{d}s^2=\exp[\gamma_1(\sigma)]\,\mathrm{d}z^2+\exp[-\gamma_2(\sigma)]\,\mathrm{d}\sigma^2+\sigma^2(\mathrm{ch}^2\alpha\,\mathrm{d}\phi^2-\mathrm{d}\alpha^2). \quad (2b)$$

(Note, here that the coordinate z is not a Cartesian one).

If now we use the vacuum Einstein field equations, $R_{\mu\nu}=0$, we obtain:

$$\text{I, II: } \mathrm{d}s^2=(1-2M/\tau)\mathrm{d}z^2-(1-2M/\tau)^{-1}\mathrm{d}\tau^2-\tau^2(\mathrm{d}\theta^2+\mathrm{sh}^2\theta\,\mathrm{d}\phi^2), \quad (3a)$$

$$\text{III: } \mathrm{d}s^2=(1-2M/\sigma)\mathrm{d}z^2+(1-2M/\sigma)^{-1}\mathrm{d}\sigma^2+\sigma^2(\mathrm{ch}^2\alpha\,\mathrm{d}\phi^2-\mathrm{d}\alpha^2). \quad (3b)$$

Here and in the following the geometrized units are used where c, the speed of light, and G, the Newton gravitational constant, are equalled to 1. In these metrics M is an arbitrary constant which has the physical sense of absolute value of the mass of the gravitational field source.

Before proceeding further, let us recall the information following the consequences of extended relativity drawn by Recami and Maccarrone (1980), Caldirola *et al* (1980) and Barut *et al* (1982). If the considered superluminal transformations are applied to a bradyonic spherical body, we finally obtain a body moving along the boost motion-line with superluminal velocity. Such a tachyon will appear (with respect to a bradyonic frame) to occupy the spatial region confined between a two-sheeted hyperboloid and a double cone, both having as symmetry axis the boost motion-line. Thus, in our case such a structure travels in regions I and II. Consequently, in region III tachyon is unobservable (for a bradyonic observer).

From such a point of view it is reasonable to consider the tachyonic gravitational field in regions I and II (figure 1), *i.e.* the geometry of the tachyonic Schwarzschild counterpart is described by the metrics (2a) and (3a) (Gurin 1984).

The metric (3a) is asymptotically flat, it has the singularities: $\tau=0$, a physical curvature singularity, and $\tau=2M$, a coordinate Schwarzschild-like singularity (pseudosingularity) which can be removed by suitable coordinate transformations (see below). It should be noted that the z-coordinate is timelike, and the τ-coordinate plays the role of a spacelike radial when the factor $(1 - 2M/\tau)$ is positive, *i.e.* above the pseudosingularity $\tau = 2M$. While this factor changes its sign, under the surface $\tau = 2M$, as well as in case of the bradyonic Schwarzschild space-time the coordinates z and τ become space-like and time-like, respectively.

The truth of the tachyonic character for the metric in question can be proved by obtaining it straightforwardly from the Schwarzschild metric through the superluminal Lorentz transformation, without directly solving the Einstein field equations, but according to the "tachyonization rule" of extended relativity (Recami and Mignani 1974). This is easy to do for an isotropic form of the Schwarzschild line element; when $r = (1 + M/2R)^2 R$; $R^2 = x^2 + y^2 + z^2$:

$$ds^2 = (1 - M/2R)^2 (1 + M/2R)^{-2} dt^2 - (1 + M/2R)^4 (dx^2 + dy^2 + dz^2). \quad (4)$$

Let us perform the superluminal boost along the z-axis:

$$z = (Z - VT)(1 - V^2)^{-1/2}, \; t = (T - VZ)(1 - V^2)^{-1/2}, \; ds^2 \to -ds^2, \quad \text{for } V > 1. \quad (5)$$

It should be noted that this boost is distinguished from that in the paper by Peres (1970) by the additional transformation $ds^2 \to -ds^2$ in the sense of extended relativity. We find that

$$ds^2 = \left(\frac{1 - M/2R}{1 + M/2R}\right)^2 \frac{(dT - VdZ)^2}{V^2 - 1} + (1 + M/2R)^4 \left[dx^2 + dy^2 - \frac{(dZ - VdT)^2}{V^2 - 1}\right]. \quad (6)$$

A conversion from (6) to the tachyonic metric in curvature coordinates (3a) is carried out by the formulae (Peres 1970):

$$(T - VZ)(V^2 - 1)^{-1/2} = R \operatorname{ch} \theta, \quad (7a)$$

$$x + iy = R \operatorname{sh} \theta \exp(i\phi), \quad (7b)$$

$$\tau = R(1 + M/2R)^2. \quad (7c)$$

Thus, the heuristic Peres's procedure and the ideas of extended relativity allow one to perform the "tachyonization" of general relativity, *i. e.* the last can be extended to faster-than-light objects analogously to special relativity. It can be suggested that the unified theories of space-time and matter, such as supergravity and quantum gravity, also admit some similar extension.

3. Extended manifold of the tachyonic metric

Let us now consider a geodesically complete space-time manifold of the tachyonic field. This approach enables one to remove the unphysical singularities and helps to understand better the space-time structure.

We replace the coordinates (τ, z) by the Kruskal-type coordinates (u, v) (Misner *et al* 1973), in which case the metric takes the form

$$ds^2 = (32M^3/\tau) \exp(-\tau/2M)(du^2 - dv^2) - \tau^2 (d\theta^2 + \operatorname{sh}^2\theta \, d\phi^2). \quad (8)$$

The coordinates (u, v) cover the complete manifold by four charts (coordinate neighbourhoods) forming the Kruskal-like diagram by setting $\theta = \text{const}$, $\phi = \text{const}$, the regions of which (A_1, A_2, A_3, A_4) are put in correspondence to each chart:

$$A_1, \tau > 2M\colon u = (\tau/2M - 1)^{1/2}\,\text{ashb};\; v = (\tau/2M - 1)^{1/2}\,\text{achb}; \tag{9a}$$

$$A_2, \tau > 2M\colon u = -(\tau/2M - 1)^{1/2}\,\text{ashb},\; v = -(\tau/2M - 1)^{1/2}\,\text{achb}; \tag{9b}$$

$$A_3, \tau < 2M\colon u = (1 - \tau/2M)^{1/2}\,\text{achb}, \quad v = (1 - \tau/2M)^{1/2}\,\text{ashb}; \tag{9c}$$

$$A_4, \tau < 2M\colon u = -(1 - \tau/2M)^{1/2}\,\text{achb},\; v = -(1 - \tau/2M)^{1/2}\,\text{ashb}, \tag{9d}$$

where the notations $a = \exp(\tau/4M)$ and $b = z/4M$ are introduced for brevity. Here (u, v)-coordinates have the ranges:

$$-\infty < u < +\infty; \quad -\infty < v < +\infty. \tag{10}$$

This diagram is shown in figure 2. It looks exactly like the usual Kruskal diagram for the Schwarzschild black hole except that it is rotated by 90°. The rotation changes the essential properties of the space-time structure. This reflects the fact that tachyons behave opposite to bradyons: here the singularity $\tau = 0$ is time-like, in contrast to the space-like singularity $r = 0$ in the Schwarzschild metric; the external region $\tau > 2M$ $(\tau \to \infty)$ is dynamical (tachyons cannot be at rest with respect to a bradyonic reference frame) opposite to the static region $r > 2M$ of the Schwarzschild black hole.

Such a character of the tachyonic field leads to certain consequences in the features of a particle motion. We shall consider these qualitatively from the properties of Kruskal-like diagrams (Misner *et al* 1973). The following three classes of geodesics are observed (figure 2): (i) time-like ones (B) which can pass from the region A_2 to region A_1 through a "throat of wormhole" and cannot reach the singularity; (ii) null ones (L) moving along the horizons and (iii) space-like ones (T) which can go out from the past singularity (to

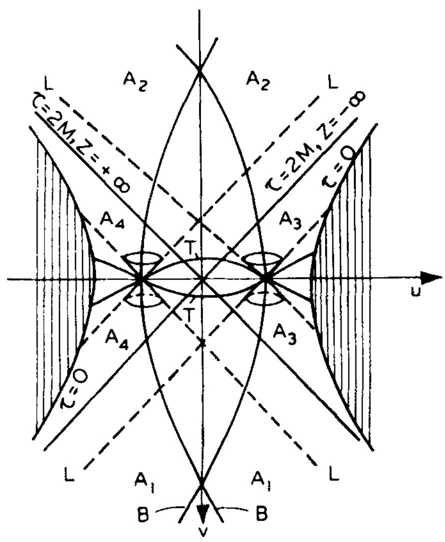

Figure 2. The complete extended manifold of a tachyonic Schwarzschild metric.

the right in figure 2), then it crosses the past horizon ($z = -\infty$, $\tau = 2M$), and obligatory after the passage *via* the future horizon ($z = +\infty$, $\tau = 2M$) it reaches the future singularity (to the left in figure 2). In the case of Schwarzschild space-time that is not so. But in this extended manifold as well as in the extended Schwarzschild manifold both black and white holes appear (Misner *et al* 1973).

It should be concluded that a tachyonic black hole is a black hole only for tachyons, but it does not have black hole features for bradyons. Similarly a bradyonic black hole is a black hole only for bradyons. Both tachyons in the bradyonic field and bradyons in the tachyonic field do not perceive the singularities: curvature singularities and horizons. Evidently, with the help of the tachyonic analogues of black and white holes it is possible to get the passage of the usual (bradyonic) matter between different "sheets" of a topologically non-trivial space-time. In this connection it can be noted that some possible astrophysical consequences of such non-trivial topology of the space-time are linked with superluminal objects (Trofimenko 1984; Recami and Shah 1979).

4. General "duality principle"

In the extended special relativity, by the unified consideration of bradyonic and tachyonic frames, a "duality principle" occurs, expressing the tachyon-bradyon symmetry of the theory (Recami and Mignani 1974). This principle may be briefly put in the form: "bradyons, tachyons and superluminal frames do not have an absolute meaning, but only a relative one". If we choose the inertial frames K, having a speed $V_K < 1$, and K', having a speed $V_{K'} > 1$, with respect to a particular inertial frame K_0, then it can be deduced that a bradyon in reference to K' will be just a tachyon in reference to any K, and vice versa.

From the preceeding consideration in §3, it is clear that this fundamental principle of relativistic physics can be generalized to gravitational fields, *i.e.* to general relativity.

As deduced above, a tachyonic black hole with respect to bradyonic objects does not appear to possess the peculiarities which occur for the usual black holes. On the other hand it is known that a bradyonic black hole with respect to tachyonic objects does not also have the black hole features (Hettel and Helliwell 1973; Honig *et al* 1974): tachyons cannot reach the curvature singularity of the Schwarzschild space-time, but can pass through "the throat of wormhole". However, black hole-peculiarities occur for a bradyonic black hole with respect to bradyons, and for a tachyonic black hole with respect to tachyons. Thus the concept of black hole does not have an absolute meaning, but it depends on the considered frame.

Hence, in general relativity a general "duality principle" is valid: The terms: black(white) hole, tachyon, bradyon and event horizon are not absolute and depend on the frame with respect to which they are considered. Also, the topological non-triviality of space-time connected with black and white holes depends on the kind of observer. This principle can be expressed as follows:

$$T-BH(K') = B-BH(K), T-BH(K) = B\text{-}BH(K') \tag{11}$$

where *T–BH* and *B–BH* represent tachyonic and bradyonic black holes, respectively.

Before concluding, a remark about the dimensionality of space-time by the unified consideration of tachyons and bradyons and tachyonic and bradyonic gravitational field with horizons is in order. This interesting and complex space-time structure can be represented by the six-dimensional manifold with three time-axes (Pavšič 1981a,b; Pavšič and Recami 1977 and references therein), by the six-dimensional manifold with imaginary space and time (Pavšič and Recami 1977; Gurin 1984) and by the eight-dimensional manifold—*i.e.* the complex-four-dimensional—where real and imaginary parts of the complex coordinates relate to objects separated by a horizon or the light barrier (Gurin 1984).

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