Pramāṇa, Vol. 24, No. 6, June 1985, pp. 813–816. © Printed in India.

Photon emission from non-oriented spin systems

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MS received 1 December 1984; revised 9 February 1985

Abstract. A method is suggested to determine experimentally whether the state of a spin system is oriented or non-oriented by measuring the angular distribution, I and the circular polarization asymmetry, A_c of the photons emitted by the system. These also provide enough data to determine the density matrix completely.

Keywords. γ-decay; radiative decays; oriented spin states; non-oriented spin states; polarized nuclei; photon circular polarization asymmetry; Stokes parameters.

PACS No. 13·40; 14·80; 23·20; 24·70

A statistical assembly of quantum systems with spin j is usually described by giving the probabilities $W(m)$ of finding the system in a state $|jm\rangle$, where m denotes the magnetic quantum number. Such an assembly is said to be 'oriented' w.r.t. the z-axis along which m is defined. As Blin-Stoyle and Grace (1957) have long ago emphasised, "such ordering is restricted in the sense that it is referred to a well defined axis in space. In general, it is possible to have more complicated orderings which cannot be described solely in terms of the probabilities $W(m)$ of having a component of angular momentum m along a given axis." Subsequently, Ramachandran and Umerjee (1964) found that the recoil deuterons in $d(\gamma, \pi^0)d$ provide an example of such an ordering which is not 'oriented'. To fix the nomenclature, we note that Bernardini (1966) uses the terms 'non-oriented' and 'un-oriented' to denote the assemblies where the spins are oriented at random, for which, the probability $W(m)$ of finding the system in a substate $|jm\rangle$ is $1/(2j+1)$. However, Ramachandran and Murthy (1978, 1979, 1980) use the terminology 'non-oriented' to refer to any ordering other than the oriented and the random. We adopt this usage. The density matrix ρ of such an assembly (when set up in the basis $|jm\rangle$) cannot be diagonalised by means of a rotation, and this has been emphasised by Bourrelly *et al* (1980). The well-known homomorphism between the two groups SU(2) and SO(3) ensures however that any statistical assembly of spin $\frac{1}{2}$ particles can only be either oriented or random. Therefore, when we talk of non-oriented systems, we are necessarily considering spin $j \geqslant 1$, and, it is of considerable interest to examine how non-oriented systems can be detected experimentally. We suggest in this paper that photon-emission from such systems can be used as a reliable tool for this purpose.

It is well known in the context of γ-decay from oriented nuclei (DeGroot *et al* 1974) that the angular distribution of the intensity I of 2^L-multipole radiation from an oriented system with spin j is of the form

$$I(\Theta) \propto \sum_{k=0}^{2L} G_k W(LLjj; kj') C(LLk; 1, -1, 0) \times \\ \{1 + (-1)^k\} p_k(\cos\Theta), \tag{1}$$

where G_k denote the Fano statistical tensors of the oriented system, j' the spin of the final system and Θ is the angle of emission w.r.t. the axis of orientation. In general, if W_i and $|\psi_i^0\rangle$, $i = 1, \ldots, 2j+1$, denote respectively the eigenvalues and eigenstates of the density matrix ρ, the intensity of radiation with polarization μ is given by

$$I = C \sum_{i=1}^{2j+1} W_i \sum_f |\langle \psi_f | \mathbf{A}\cdot\mathbf{p} | \psi_i^0 \rangle|^2, \tag{2}$$

where C is a constant of proportionality and $\mathbf{A}$ is given by (Rose 1957)

$$\mathbf{A} = (2\pi)^{1/2} \sum_{L=1}^{\infty} \sum_{M=-L}^{L} i^L (2L+1)^{1/2} D^L_{M\mu}(\phi, \theta, 0) \mathbf{A}_{LM}, \tag{3}$$

where $\mathbf{A}_{LM} = (\mathbf{A}^m_{LM} + i\mu \mathbf{A}^e_{LM})$ in terms of the electric and magnetic multipoles.

In non-oriented systems, the eigenstates $|\psi_i^0\rangle$ of ρ cannot all be identified as eigenstates $|jm\rangle$ of J_z. (If so, the system would be oriented.) We can however express the $|\psi_i^0\rangle$ (of the non-oriented system) in the form

$$|\psi_i^0\rangle = \sum_m C^i_m |jm\rangle, \tag{4}$$

where the states $|jm\rangle$ are defined conveniently w.r.t. a given laboratory z-axis. The same z-axis is used to define the final spin states $|j'm'\rangle$. It is easily seen that the angular distribution of the intensity I for a 2^L-multipole radiation is now of the form

$$\begin{aligned} I(\theta, \phi) = {} & C(-1)^{j'-j+1} 2\pi (2L+1)(2j'+1) |\langle j' || \mathbf{A}_L \cdot \mathbf{p} || j \rangle|^2 \\ & \times \sum_{k=0}^{2L} \sum_{q=-k}^{+k} \sum_{m=-j}^{+j} (-1)^{j-m+q} C(jjk; m, q-m, q) \\ & \times \sum_{i=1}^{2j+1} W_i C^i_m C^{i*}_{m-q} W(LLjj; kj') C(LLk; 1, -1, 0) \\ & \times \{1 + (-1)^k\} (4\pi)^{1/2} (2k+1)^{-1/2} Y_{k,q}(\theta, \phi), \end{aligned} \tag{5}$$

which readily specialises to (1) for oriented systems if the z-axis is chosen parallel to the axis of orientation instead of the arbitrary laboratory axis; in such a case $C^i_m = \delta_{i,\, j+m+1}$ and $\theta = \Theta$. Since such a natural choice of z-axis does not exist for a non-oriented system, the dependence of I on azimuthal angle ϕ persists in all frames.

This circumstance is of considerable importance from the practical point of view, where the following interesting question arises. Since one does not know *a priori* whether the system undergoing γ-decay is oriented, and if so, which is the axis of orientation, it would primarily be desirable to get this information from measurements made in the laboratory. Suppose the system is oriented and (θ_0, ϕ_0) denote the polar angles of the axis of orientation w.r.t. the laboratory frame, it follows that

$$I(\theta, \phi - \phi_0) = I(\theta, \phi_0 - \phi), \tag{6}$$

since Θ in (1) is given by

$$\cos\Theta = \cos\theta \cos\theta_0 + \sin\theta \sin\theta_0 \cos(\phi - \phi_0). \tag{7}$$

This implies that if $I(\theta, \phi)$ given by (5) is studied as a function of ϕ for any given θ, I would be the same for *all* pairs of angles ϕ_1, ϕ_2 such that

$$\phi_1 + \phi_2 - 2\phi_0 = 2\pi. \tag{8}$$

if the system is oriented. This readily leads to the determination of ϕ_0. After such a symmetry is established, one can then study $I(\theta, \phi)$ as a function of θ at $\phi = \phi_0$. Noting that only even k contribute to (1), it follows that

$$I(\theta-\theta_0, \phi_0) = I(\pi-(\theta-\theta_0), \phi_0), \tag{9}$$

for oriented systems, which implies that I at $\phi = \phi_0$ is the same for *all* pairs of angles θ_1, θ_2 such that

$$\theta_1 + \theta_2 - 2\theta_0 = \pi, \tag{10}$$

which readily fixes θ_0. If we now choose the z-axis (let us call it z_0) parallel to the direction (θ_0, ϕ_0), I should obviously be cylindrically symmetric w.r.t. rotations about z_0 for oriented systems. If (6) and (9) are not satisfied, it is clear that the system is non-oriented. However, if the measured distribution I satisfies (6) and (9), that does not by itself guarantee that the system is oriented since I is insensitive to the spherical tensor parameters t_{kq} with odd k, characterising the initial spin system. It is therefore necessary to measure some other observable which is complementary to I to decide this question. Trumpy (1957) has long ago pointed out that, when polarized neutrons are captured, the capture γ-rays are in general circularly polarized (see also Daniels 1965). The most suitable choice is to look at the photon polarization asymmetry A_c defined through

$$A_c = (I^+ - I^-)/I \tag{11}$$

where $I^\pm$ denote the intensities with right and left circular polarization respectively. A_c is essentially the Stokes parameter s_3 while I is s_0 (Ramachandran *et al* 1980). The product $I \cdot A_c$ is given by an expression identical with (5) except that the factor $\{1 + (-1)^k\}$ is replaced by $\{1-(-1)^k\}$, which means that only odd k contribute to it. If, in addition to (6) and (9), the asymmetry A_c satisfies

$$A_c(\theta, \phi-\phi_0) = A_c(\theta, \phi_0-\phi), \tag{12}$$

and

$$A_c(\theta-\theta_0, \phi_0) = -A_c(\pi-(\theta-\theta_0), \phi_0) \tag{13}$$

i.e., if A_c is also independent of the azimuthal angle when the z-axis is chosen parallel to z_0, we can decide conclusively that the system is oriented and the axis of orientation is parallel to z_0; otherwise it is non-oriented.

We may note in paranthesis that Blum (1981) and Blum and Kleinpoppen (1979) who have studied γ-emission from polarized atoms have also obtained expressions for the three Stokes parameters in the particular case of dipole emission. They do not, however, discuss how the study of the Stokes parameters could be exploited to determine whether or not the decaying spin system is oriented, although they discuss how to use them to determine the tensor parameters t_1 and t_2 which are the only tensor parameters in the case of dipole emission. It is pertinent to point out here that in general, for any arbitrary order L of multipole emission, the t_{kq}, $k = 1, \ldots, 2L$, could in fact be determined by studying only two, *viz* s_0 and s_3, of the four parameters. One can compute

$$i_{kq} = \int I(\theta, \phi) Y_{k,q}(\theta, \phi) \mathrm{d}\Omega \tag{14}$$

and

$$a_{kq} = \int A_c(\theta, \phi) I(\theta, \phi) Y_{kq}(\theta, \phi) \mathrm{d}\Omega \tag{15}$$

for even and odd values of k respectively and note that the spherical tensor parameters t_{kq} satisfying the Madison convention (Satchler *et al* 1971) are obtained readily by multiplying i_{kq} and a_{kq} by a factor K, given by

$$K = (-1)^{j-j'+1}(1/4\pi)^{3/2}\frac{(2j+1)^{1/2}(2k+1)^{1/2}}{C(2j'+1)(2L+1)|\langle j'||\mathbf{A}_L\cdot\mathbf{p}||j\rangle|^2} \times \frac{1}{W(LLjj;kj')C(LLk;1,-1,0)}. \tag{16}$$

Thus the full density matrix, ρ is determined. One can in fact use the t_{kq} thus obtained to check if the system is non-oriented or oriented by applying the constraints obtained recently by Ramachandran and Mallesh (1984). However, the elegant procedure outlined in this paper obviates the need for such a cumbersome procedure.

In conclusion, it may also be pointed out that the elegant method suggested here may find application to analyse the spin state of not only atomic and nuclear systems that decay by γ-emission (Ramachandran and Ravishankar 1983), but also in analysing the resonance states (Ramachandran *et al* 1984) such as $\omega(783)$, $\chi(3510)$, $\chi(3555)$, $D^{*0}(2010)$, $\psi(3685)$ and J/ψ, which have respectable branching ratios for the decay mode.

Acknowledgement

One of the authors (VR) thanks the CSIR (India) for financial assistance.

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