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Effective thermal conductivity of granulated two-phase systems at interstitial air pressures

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Abstract. Considering the thermal conduction through molecular collisions an expression for the effective thermal conductivity λ_e of loose and granular two-phase materials at different interstitial air pressure has been derived. The dependence of λ_e on pore and particle sizes, characteristic pressure and radiative heat transfer is also discussed. Calculated values of λ_e of glass beads and loose building materials are compared with reported results.

Keywords. Characteristic pressure; effective mean free path; molecular collisions; pore diameter.

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1. Introduction

Effective thermal conductivity λ_e of two-phase materials has been much explored in the past. A variety of approaches (Powers 1961; Parrot and Stuckes 1975; Burridge *et al* 1982) have been developed. Although these are highly evolved methods, the utility of these methods to loose and granular two-phase systems which have a large variation in porosity (0·3–0·7) as well as in the ratio of component conductivities ($\lambda_{solid}/\lambda_{gas}$) is yet a matter of investigation (Burridge *et al* 1982). The effective thermal conductivity of granular materials like glass beads (Watson *et al* 1964; Wechsler and Glaser 1964), fibrous insulation material (Pettyjohn 1967; Pratt 1962; Verschoor and Greebler 1952) and lunar materials (Keshock 1972; Wechsler 1972) at different interstitial air pressures have been widely measured. Attempts have also been made to explain the behaviour of λ_e of these materials at interstitial air pressures both qualitatively and quantitatively (Pratt 1962; Verschoor and Greebler 1952; Keshock 1972). Recently with the advent of modern insulation techniques the λ_e of fibrous insulation material at different interstitial air pressures has been estimated (King 1978; Bankvall 1974; Hager and Steere 1960; Schmitt *et al* 1974) considering all the three modes of heat transmission *viz* conduction, convection and radiation. These models lack the generality and are applicable only to fibrous materials.

In the present paper a particle-size-dependent expression for λ_e of loose and granular two-phase materials at an interstitial gas pressure P has been derived using kinetic theory.

2. Theory

2.1 *Dependence of λ_e on P*

Loose and granular two-phase materials are systems made of solid grains and air. If the pore structure is assumed to remain invariant on varying the interstitial air pressure inside the pore, the decisive factors responsible for the variation in λ_e of the sample should be air pressure inside the pore and the pore diameter. In general the effective thermal conductivity of a loose and granular two-phase systems at an interstitial air pressure P should be

$$\lambda_e = f(\lambda_s, \lambda_g, \psi, d \text{ and } P) + \lambda_r + \lambda_v, \tag{1}$$

where λ_s and λ_g are the thermal conductivity values of solid and air phases, ψ is volume fraction of air and d is the pore diameter. λ_r and λ_v are the thermal conductivity contributions due to the radiative and convective heat transfers respectively.

Considering only the conduction part (λ_c) we find from (1)

$$\frac{d\lambda_e}{dP} = \frac{\partial\lambda_e}{\partial\lambda_s}\left(\frac{\partial\lambda_s}{\partial P}\right) + \frac{\partial\lambda_e}{\partial\lambda_g}\left(\frac{\partial\lambda_g}{\partial P}\right) + \frac{\partial\lambda_e}{\partial\psi}\left(\frac{\partial\psi}{\partial P}\right) + \frac{\partial\lambda_e}{\partial d}\left(\frac{\partial d}{\partial P}\right) + \frac{\partial\lambda_e}{\partial P}. \tag{2}$$

Now, as the pore structure does not alter and λ_e and λ_s do not depend explicitly on P, (2) reduces to a simpler form. Thus

$$\frac{\partial\lambda_e}{\partial P} = \frac{\partial\lambda_s}{\partial P} = \frac{\partial d}{\partial P} = \frac{\partial\psi}{\partial P} = 0, \tag{3}$$

and

$$\frac{d\lambda_e}{dP} = \frac{\partial\lambda_e}{\partial\lambda_g}\left(\frac{\partial\lambda_g}{\partial P}\right). \tag{4}$$

2.2 *Dependence of λ_g and P*

Thermal conductivity λ_g of an unconfined gas having mean free path l_f, density ρ, mean velocity of molecules $\bar{u}$, specific heat C_v, pressure P and specific heat ratio γ is given (Eucken 1932) as

$$\lambda_g = \tfrac{1}{4}(9\gamma - 5)\bar{u}C_v\rho l_f. \tag{4a}$$

In a gas-solid two-phase system the interstitial gas is housed inside the pore. When d becomes comparable to l_f, the consideration of total probability of collision of molecules leads to the definition of the effective mean free path l_e (Pratt 1962; Keshock 1972; Sweet 1979; Speil 1964) for a pore confined gas as

$$1/l_e = 1/l_f + 1/d. \tag{5}$$

Here l_f is the mean free path of the same gas at the same pressure P whence the gas molecules have been in free space. In anology to (4a) the thermal conductivity of a pore confined gas at an interstitial air pressure P may be given as

$$\lambda_g = \tfrac{1}{4}(9\gamma - 5)\bar{u}C_v\rho l_e. \tag{6}$$

ρ is invariant for confined and unconfined air at the same pressure P. We also assume that γ, $\bar{u}$ and C_v are independent of interstitial air pressure and behave as a constant at room temperature. Let l_0, ρ_0 and λ_{g0} be the mean free path, gas density and thermal conductivity of free air at the atmospheric pressure P_0, we may then write (6) as

$$\lambda_g = \lambda_{g0}(l_e \rho / l_0 \rho_0). \tag{7}$$

Using kinetic theory we have

$$\rho/\rho_0 = P/P_0 \tag{8}$$

and $$l_0 P_0 = l_f P = \beta (\text{a constant}). \tag{9}$$

Substituting (8) and (9) in (7) we find,

$$\lambda_g = \lambda_{g0}(P l_e / \beta). \tag{10}$$

Now, consider an interstitial air pressure P_c where

$$l_f = d = \beta / P_c. \tag{11}$$

Substituting the value of l_e and β from (5) and (11) respectively in (10) we have

$$\lambda_g = \lambda_{g0} \frac{P l_f}{(l_f + d) P_c}. \tag{12}$$

As $(dP_c/l_f) = P$, we find

$$\lambda_g = \lambda_{g0}[P/(P + P_c)]. \tag{13}$$

2.3 *Thermal conductivity of gas-solid two-phase system*

Substituting the value of $(\partial \lambda_g / \partial P)$ from (13) in (4) and writing $(\partial \lambda_e / \partial \lambda_g)$ as α we find

$$d\lambda_e/dP = \alpha P_c \lambda_{g0} (P + P_c)^{-2}, \tag{14}$$

which on integration yields

$$\lambda_e = -\alpha \lambda_{go} \left(1 + \frac{P}{P_c}\right)^{-1} + K \text{ (constant)}. \tag{15}$$

Using boundary conditions

(i) $P \to 0$, $\lambda_g \to 0$ and $\lambda_e \to \lambda_r + \lambda_v$,

and (ii) $P \to P_0$, $\lambda_g \to \lambda_{g0}$ and $\lambda_e \to \lambda_{e0}$,

in (15) we obtain

$$K = \alpha \lambda_{g0} + \lambda_r + \lambda_v, \; \alpha = (\lambda_{e0}/\lambda_{g0}), \tag{16}$$

and $$\lambda_e = \lambda_{e0}(P/P + P_c) + \lambda_r + \lambda_v. \tag{17}$$

2.4 *Characteristic pressure P_c*

P_c is the value of air pressure (interstitial) when $l_f = d$. Using (5), (13) and (17) together we find that as $P \to P_c$, $l_e = \frac{1}{2} l_f$, $\lambda_g = \frac{1}{2}\lambda_{g0}$ and $\lambda_e = \frac{1}{2}\lambda_{e0}$ *i.e.* P_c is the value of interstitial air pressure when effective thermal conductivity of gas-solid two-phase samples reduces to half its value at the atmospheric pressure.

2.5 *Dependence of λ_c on pore and particle sizes*

Substituting the value of P_c in (17) and considering the conduction part λ_c only we find,

$$\lambda_c = \lambda_{e0}\{1 + (\beta/dP)\}^{-1}. \tag{18}$$

For $P \gg P_c$, $\beta/dP \ll 1$, (18) becomes

$$\lambda_c = \lambda_{e0}(1 - \beta/dP), \tag{19}$$

while for $P \ll P_c$, $\beta/dP \gg 1$, (18) yields

$$\lambda_c = \lambda_{e0}\frac{dP}{\beta}\left(1 - \frac{dP}{\beta}\right). \tag{20}$$

Expressions (19) and (20) exhibit the dependence of λ_e upon the pore diameter d. As the pore diameter in a regular packing is directly proportional to particle diameter D, we have

$$d = aD. \tag{21}$$

(In a simple cubic packing $a = 0{\cdot}4142135$, and in a tetrahedral packing $a = 0{\cdot}1547007$). Thus for $P \gg P_c$

$$\lambda_c = \lambda_{e0}\left\{1 - (\beta/a)\frac{1}{PD}\right\} \tag{22}$$

and for $P \ll P_c$

$$\lambda_c = \lambda_{e0}(a/\beta)(DP)\{1 - (a/\beta)(DP)\}. \tag{23}$$

Expressions (22) and (23) depict D and λ_{e0} as the factors responsible for the differences in λ_c of various two-phase gas solid systems at the same interstitial air pressure P. One notes through (22) and (23) that for a given sample λ_c should decrease as the particle size decreases.

2.6 *Radiative heat transfer at very low pressure*

At very low pressures ($P \to 0$) the effective thermal conductivity is contributed largely by the mechanism of radiation (λ_r). The contribution λ_r has been evaluated for fibrous insulation material (Verschoor and Greebler 1952; King 1978; Schmitt *et al* 1974; Strong *et al* 1960) alone. DeVries (1952) derived an expression for the reduced conductivity, consisting of solid conduction and radiative heat transfer at very low pressure for loose and granular material but so far the existing literature does not have any specific model for radiative heat transfer through loose and granular materials.

It is important to mention here that when $P > P_c$, $l_f < d$. This indicates that a pore consists on an average of more than one air molecule in this region. For the region $P < P_c$, $l_f > d$. This represents a vacated pore. As there are no vacant pores in the region $P > P_c$, the dominant modes of heat transfer above and below $P = P_c$ are conduction and radiation respectively. This consideration leads to the conclusion:

$$P \to P_c,\ \lambda_r \to 0. \tag{24}$$

2.7 *Contribution of convection*

The measurements (Bankvall 1974) of convection at normal pressure indicate that convection contribution λ_v is negligible in fibrous insulations while the present investigations (figure 1) at the same pressure indicate the presence of a small convection in loose building materials.

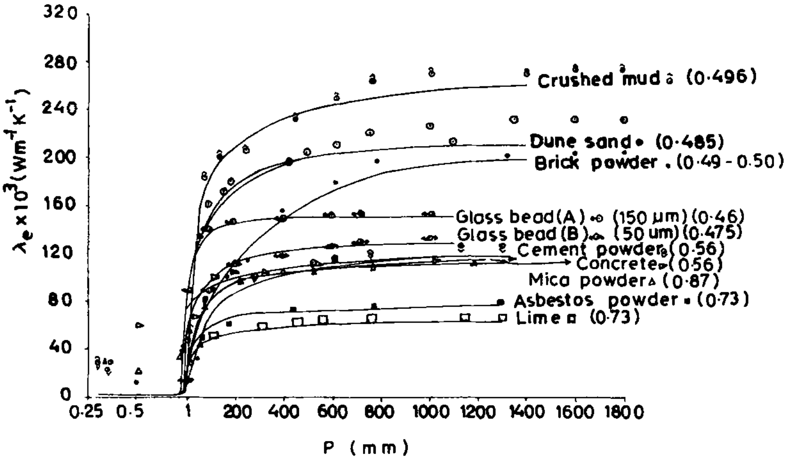

Figure 1. Effective thermal conductivity of loose building-materials and glass beads at varying interstitial air pressures. The continuous lines represent the calculated values of λ_e through (17) while the symbols represent the experimental values (Pande *et al* 1984; Wechsler and Glaser 1964).

3. Comparison with experimental results and discussions

Using (17) λ_e values of building materials like crushed mud, dune sand, brick powder, cement powder, concrete, mica powder, asbestos powder and lime (Pande *et al* 1984) and glass beads (Wechsler and Glaser 1964) have been calculated. The calculated values of λ_e along with the experimental values of these materials are plotted in figure 1. An agreement is seen for nearly all the considered materials in the pressure region 1 to 600 mm of Hg. Below 1 mm of Hg the reported values of λ_e are much larger as compared to the calculated values. The difference is due to the radiative heat transfer (λ_r) through the pore. At about atmospheric pressure the reported values are slightly higher as compared to the calculated values indicating a very small heat transfer through convection.

A plot of λ_e (in normalized units of λ_e/λ_{e0}) vs d of (17) at different interstitial air pressures is shown in figure 2 which investigates the dependence of λ_c upon pore and particle size. The measured values of λ_e of glass beads (Wechsler and Glaser 1964) at very low interstitial air pressure (0·01 mm of Hg) closely resembles the curve plotted at this pressure (figure 2). λ_{e0} is taken as 0·2 $\mathrm{Wm^{-1} K^{-1}}$ and d is determined through (21). We find from figure 2 that λ_c increases very sharply as the pore size or particle size increases in the pressure region 1 to 600 mm of Hg, while below the air pressure of 1 mm of Hg (0·1–0) λ_c is nearly independent of pore or particle diameter. The value of λ_c of loose materials whose pore diameter $d < 100\ \mu$m is only 6 % of its value at normal pressure when interstitial air pressure $P < 0{\cdot}1$ mm of Hg. This indicates that a microfined powder at very low pressure should work as a better insulator as compared to hazardous fibrous insulation material.

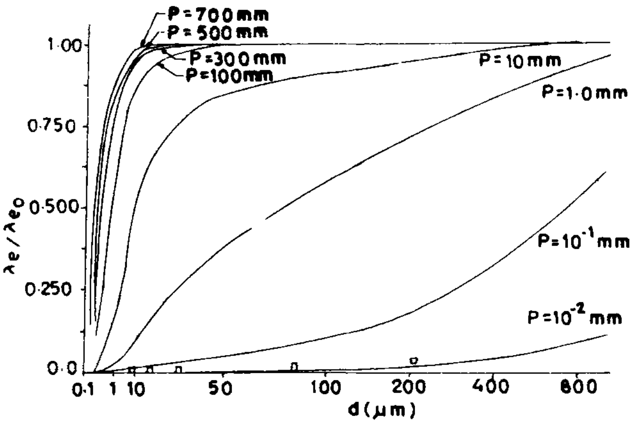

Figure 2. Variation of λ_e with pore diameter d through (17) at different interstitial air pressure. The symbol (□) represents the experimental value of λ_e of glass beads of different sizes at the interstitial air pressure of 10^{-2} mm of Hg. (Wechsler *et al* 1972).

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