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On the peripheral nature of the target fragmentation phenomena

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Abstract. The mechanism of target fragmentation phenomena is explored in a statistical model. It is shown that peripheral interaction arising out of large impact parameter can describe the mass yield distribution of the products from the fragmentation of ^{63}Cu by the bombardment of p, ^{12}C and ^{40}Ar with energies of 28, 25 and 80 GeV respectively. Important insights into the dynamics is obtained from these reactions as the target remaining the same, the projectile mass varies by forty units and the incident energy per nucleon by fourteen units. Surface properties of the target and projectile are shown to play an important role. Other features like limiting fragmentation and projectile dependence are also borne out in this study.

Keywords. Target fragmentation; statistical model; peripheral nature of phenomena; limiting fragmentation.

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1. Introduction

Studies of the interaction of complex nuclei with relativistic energies have revealed a variety of phenomena (Goldhaber and Heckman 1978; Gross *et al* 1982) like target fragmentation, projectile fragmentation and formation of highly excited compound system composed of matter from both projectile and target which undergo explosion into multiple small fragments etc. Understanding of the mechanism of these reactions which gives rise to such a wide variety of phenomena is quite a challenging problem. One can intuitively feel that the nature of the interaction would range from very gentle to extremely violent depending upon whether the two nuclei touch peripherally or undergo head-on collision. It is expected that the occurrence of different phenomena should have a direct bearing on the impact parameter involved in the reaction. Thus the scale of energy transfer and the resulting temperature should span a very long range. At the upper end of this scale the collisions are more central and we have the phenomena of the formation of a composite system comprising of matter both from the projectile and the target. Here the temperature involved is more than 100 MeV. Numerous calculations (Das Gupta and Makjian 1981) performed in the fireball model based on statistical mechanics have met with considerable success. Comparatively the phenomena of target fragmentation are much less understood and models less developed. These phenomena are expected to be on the other end of the scale with a much lower temperature (below 10 MeV) and a large impact parameter characteristic of peripheral interaction. A model based on this picture which can describe the experimental data like mass yield distribution and other important features like limiting fragmentation (Kaufman and Steinberg 1980) *i.e.* insensitiveness of the mass yield distribution to the incident energy of the projectile when the latter reaches a limiting value would definitely be a step forward. Some models have been proposed (Bowman *et al* 1973; Cumming 1980; Morrissey *et al* 1978; Randrup and Koonin 1981) in the past to account for the different features of these phenomena with varying degrees of success. However, our understanding of the mechanism is far from satisfactory. In this paper, we report our attempt in this regard. We have chosen the three reactions $p+\mathrm{Cu}$, $^{12}\mathrm{C}+\mathrm{Cu}$ and $^{40}\mathrm{Ar}+\mathrm{Cu}$ with 28 GeV, 25 GeV and 80 GeV respectively, which have been studied extensively. These reactions provide a unique opportunity to study the dynamics and gain insight into the mechanism as the target remains the same and the projectile mass varies from 1–40 units and the energy per nucleon of the projectile also varies widely from 2–28 GeV.

Recently we have proposed (Gross *et al* 1982) a statistical model for target fragmentation to describe the mass yield distribution of various reactions. It assumes that the projectile on contact with the target transfers energy which excites the target. After the collision, the excited target decays statistically into fragments of various kinds. In the framework of grand canonical ensemble, treating temperature as a parameter we have described (Gross *et al* 1982) the mass yield distribution observed in a large number of proton-induced reactions. This model had two very important drawbacks: (i) the incident energy and the projectile mass were not input in the model. In reality, the mass yield distribution should be a function of these quantities. By treating temperature as a parameter, the explicit dependence of these two quantities on the energy transfer had been avoided. (ii) The dependence of the mass yield distribution on the impact parameter was not included. To unearth the mechanism of the phenomena from the experimental mass yield distribution, we have tried here to remedy these two defects in our model. We have now succeeded in showing that the peripheral interaction with a large impact parameter (greater than the sum of the half density radii) plays the most crucial role in fragmentation phenomena.

2. Theoretical framework and calculations

In the collision of two complex nuclei, the calculation of energy transfer to the target from first principle is not feasible at the present stage. One has to devise a phenomenological model to determine it. In order to understand qualitatively the feature of limiting fragmentation, we have used a phenomenological expression for energy transfer ΔE in our recent study (equation 20, Gross *et al* 1982). However, it was not sufficiently realistic as the nuclei were assumed to be spheres of uniform density. The target fragmentation phenomena are likely to originate from peripheral collision, so, the surface property of the nuclei is expected to play an important role. We improve the above expression for ΔE by taking into account this element additionally in calculating the energy transfer, which is subsequently used as input in our statistical model to calculate the mass yield distribution. Then the formula for ΔE has the following factors:

(i) It is proportional to the kinetic energy per unit volume of the projectile

$$E_{\mathrm{inc}}^{\mathrm{kin}}/(\tfrac{4}{3}\pi r_0^3 A_p). \tag{1}$$

So $\quad \Delta E \propto E_{\mathrm{inc}}^{\mathrm{kin}}/A_p.$

(ii) It is proportional to the contact area. As shown in Appendix 1

$$\Delta E \propto A_T^{1/3} A_P^{1/3} (A_T^{1/3} + A_P^{1/3})^{-1} (1-\beta^2)^{1/2}. \tag{2}$$

(iii) It is proportional to the contact time between the target and projectile. In Appendix 2 it is shown that

$$\Delta E \propto A_T^{1/6} \beta^{-1}. \tag{3}$$

In addition to the above three features which we have considered earlier (Gross *et al* 1981), the more important one which carries the clear signature of the peripheral nature of the reaction is the surface properties of the nuclei. In the above, we have taken the target and projectile to have constant density sharp surface whereas, in reality, the nuclei have fuzzed out density surfaces. This aspect should have a great bearing on the reaction mechanism of the fragmentation phenomena. We consider this feature as follows:

(iv) ΔE should be proportional to the combined density profile of the target and projectile at the distance of the closest approach during collision. Taking the standard Wood-Saxon form for the density distribution we can write

$$\Delta E \propto \frac{\rho_0}{\{1+\exp(r_P - R_P)/a\}} \cdot \frac{\rho_0}{\{1+\exp(r_T - R_T)/a\}} \tag{4}$$

where r_P and r_T are the coordinates of the points referred to the centres of the projectile and target as origins respectively. R_P and R_T are half density radii and a the diffuseness parameter. For the values of r_P and r_T relevant to peripheral collision we replace r_P and r_T by their average values

$$\langle r_P \rangle = (1+\alpha) R_P \quad \text{and} \quad \langle r_T \rangle = (1+\alpha) R_T,$$

where, α is related to the impact parameter through the relation

$$b = \langle r_P \rangle + \langle r_T \rangle = R_P + R_T + \alpha (R_P + R_T). \tag{5}$$

Thus, $\alpha = -1$ corresponds to the central collision and the positive values of α correspond to impact parameter larger than the sum of the half density radii of projectile and target. Then averaging the impact parameter amounts to finding out an average value of α through the relation

$$\langle \alpha \rangle = \left[\int_{\alpha_{in}}^{\alpha_f} \frac{\alpha \, d\alpha}{\{1+\exp(\alpha R_P/a)\}\{1+\exp(\alpha R_T/a)\}} \right] \times \left[\int_{\alpha_{in}}^{\alpha_f} \frac{d\alpha}{\{1+\exp(\alpha R_P/a)\}\{1+\exp(\alpha R_T/a)\}} \right]^{-1}. \tag{6}$$

Now collecting all factors, we have

$$\Delta E = \lambda A_T^{1/2} A_P^{-2/3} (A_T^{1/3} + A_P^{1/3})^{-1} (1-\beta^2)^{1/2} \beta^{-1} \times \{1+\exp(\langle \alpha \rangle R_P/a)\}^{-1} \{1+\exp(\langle \alpha \rangle R_T/a)\}^{-1} E_{inc}^{kin}. \tag{7}$$

We use the standard value 0·54 fm for the diffuseness parameter a. Thus to calculate the energy transfer ΔE through (7) for any reaction, we have to know the value of the parameter λ and also an average value $\langle\alpha\rangle$ which we determine by the use of experimental mass yield distribution as described below.

We use our statistical model proposed earlier (Gross *et al* 1982) to get the mass yield distribution. In this model we worked in the grand canonical ensemble and determined the chemical potentials μ_p and μ_n for proton and neutron respectively by taking into account the charge and baryon number conservation. Since we had no way of knowing the energy of the excited target, we had treated temperature as a parameter whose value was determined by fitting with the experimental mass yield distribution (equations (7) and (8) of Gross *et al* 1982). We had followed this procedure in our previous publication (Gross *et al* 1982). However, once the temperature T of the decaying target is thus determined, we can calculate the energy transfer from the standard relation

$$\Delta E = -\frac{\partial}{\partial\beta}\ln\xi(\mu_p, \mu_n, \beta), \tag{8}$$

where, $\beta = 1/kT$ and ξ is the grand partition function. The excitation energy ΔE so determined is used in (7) to get the value of λ and also to estimate $\langle\alpha\rangle$.

The result of a detailed study of averaging over α which is also equivalent to the averaging over the impact parameter b is presented in table 1 for the reaction $^{12}C + ^{63}Cu$. α_{in} and α_f are the initial and final values of α used in the integration (equation (6)) and b_{in} and b_f are the corresponding impact parameters. ρ^c_{in} and ρ^c_f are the corresponding initial and final densities of the combined density profile in the intersection plane when the two nuclei are at the closest approach during collision. These quantities are presented in percentage of the central nuclear density ρ_0. The first column gives the sum of the half density radii of the projectile and target. In the first three rows we have presented the results where the initial combined density ρ^c_{in} is kept constant at 5% and the final density varies from 84 to 92%, $\langle\alpha\rangle$ and b change somewhat significantly. In the last three rows we have presented the results where the final density is kept fixed at 86% and the initial density varies from 5 to 15%. We find that $\langle\alpha\rangle$ and $\langle b\rangle$ have hardly changed. These two sets show that averaging over α which is equivalent to the averaging over b is sensitive to α_f (more central collision). The values of the two parameters $\langle\alpha\rangle$ and λ (equation (7)) were determined consensually to be 0·14 and 1·032 respectively from the experimental mass yield distribution of the above three reactions. The sensitivity of $\langle\alpha\rangle$ (correspondingly of b) to the mass yield distribution in a representative case like $^{12}C + Cu$ is discussed later.

In our statistical model we calculate the multiplicity $\langle n_i\rangle$ of fragment i (equation (7) of Gross *et al* 1982) which is subsequently used to determine the cross-section σ_i

Table 1. Impact parameter averaging in the reaction $^{12}C + ^{63}Cu$.

$R_c + R_{c_u}$	α_{in}	α_f	ρ^c_{in} (%)	ρ^c_f (%)	b_{in} (fm)	b_f (fm)	$\langle\alpha\rangle$	$\langle b\rangle$ (fm)	$(\langle\alpha^2\rangle - \langle\alpha\rangle^2)^{1/2}$
6·89	0·05	0·62	5	84	7·26	11·15	0·151	7·93	0·088
	0·04	0·62	5	88	7·16	11·15	0·140	7·85	0·089
	0·03	0·62	5	92	7·08	11·15	0·130	7·79	0·089
	0·05	0·62	5	86	7·21	11·15	0·145	7·89	0·089
	0·05	0·50	10	86	7·21	10·32	0·143	7·88	0·084
	0·05	0·42	15	86	7·21	9·82	0·140	7·86	0·079

through the relation

$$\sigma_i = \langle n_i \rangle \sigma_F^{A_P, A_T}$$

where, $\sigma_F^{A_P, A_T}$ is the total fragment cross-section. Cumming *et al* (1978) estimated that the total fragmentation cross-section is about 70% of the total inelastic cross-section. Heckman *et al* (1978) used the following empirical parametrization for total inelastic cross-section

$$\sigma_{\text{in}}^{A_P, A_T} = \pi r_0^2 [(A_P^{1/3} + A_T^{1/3}) - b(A_P^{-1/3} + A_T^{-1/3})^2], \tag{9}$$

with $r_0 = 1{\cdot}36$ and $b = 0{\cdot}75$. For $^{12}C + Cu$ the total inelastic cross-section has been experimentally measured (Heckman *et al* 1978) to be 1730 ± 36 mb. This is comparable with the empirical value of 1922 mb of Heckman *et al* (1978) computed through (9). For total fragmentation cross-section we have used the same parametrization as (9) with $r_0 = 1{\cdot}1$ and $b = 1{\cdot}11$. With these values we obtain $\sigma_F^{A_P, A_T}$ for $^{12}C + ^{63}Cu$ to be 1150 mb which is about 67% of the total inelastic cross-section. Thus our estimation of σ_F is reasonable.

3. Results and discussion

The energy transfers ΔE and the temperatures T obtained in our study for the three reactions are presented in table 2. The mass yield distributions for the three reactions are compared with the experimental data in figure 1. The agreement is very satisfactory. We find from table 2 that the energy transfers and the temperatures in all the three reactions are quite similar. This gives rise to similar values of multiplicity $\langle n_i \rangle$ in our statistical model calculation. Thus the values of fragmentation cross-section obtained using (9) differ from one another through $\sigma_F^{A_P, A_T}$. Thus the larger cross-section for ^{40}Ar ions over that of proton is solely due to increased total reaction cross-section of the former compared to the latter. Cumming *et al* (1978) from their analysis of experimental data had anticipated this very interesting feature of the reaction mechanism. The present calculation clearly bears this out.

In figure 2, we have presented the density profile of the Ar + Cu system at closest approach which emerged from our study. As pointed out earlier, we have taken $\langle \alpha \rangle$ equal to 0·14 for all the three reactions. This gives rise to an impact parameter of 9·27 fm. This is larger than the half density radii of 8·14 fm. Cumming *et al* (1978) studied the

Table 2. Summary of the statistical model calculation in the three reactions. ΔE and T are the energy transfer to the target and its temperature. μ_p and μ_n are the proton and neutron chemical potentials.

Reaction	Incident energy (GeV)	ΔE (MeV)	T (MeV)	μ_p	μ_n
$p + ^{63}Cu$	28	156·45	4·80	−12·366	−8·533
$^{12}C + ^{63}Cu$	25	169·90	4·99	−12·583	−8·508
$^{40}Ar + ^{63}Cu$	80	170·30	5·00	−12·594	−8·507

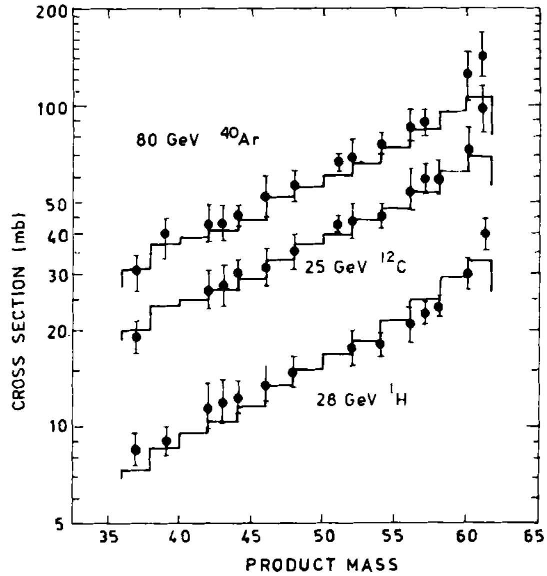

Figure 1. Calculated limiting mass yield distribution for fragmentation of Cu target. The histograms are for 28 GeV p, 25 GeV ^{12}C and 80 GeV ^{40}Ar projectiles. Data (solid dot) taken from Cumming *et al* (1978).

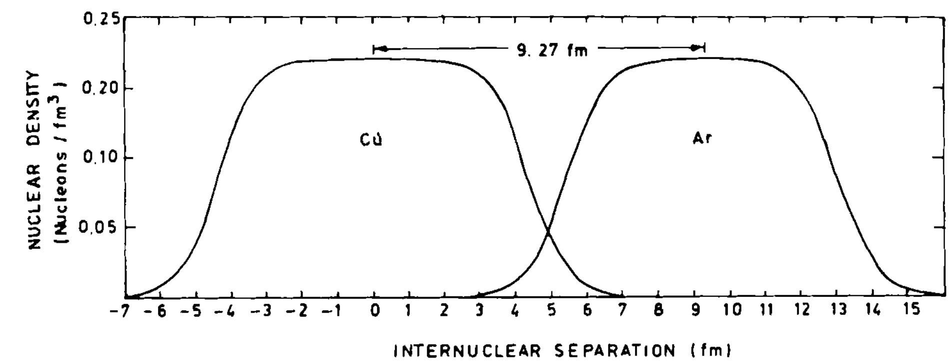

Figure 2. Nuclear density distribution at closest approach for $^{40}Ar + ^{63}Cu$ reaction showing the peripheral nature of the interaction.

impact parameter dependence of this reaction following a phenomenological model of Barshay *et al* (1975). They found that an impact parameter equal to 8·2 fm would make the largest contribution to σ_R. On this basis, they expect that the most effective impact parameters which would contribute to the fragmentation process are larger than the half density radii. The present calculation clearly confirms their expectation. In fact, we have tried to fit the mass yield distribution of these reactions with a lower value of $\langle \alpha \rangle$ *i.e.* by including impact parameter lower than 9·27 fm and we find that we cannot succeed. In figure 3 we have demonstrated the dependence of mass yield distribution on the impact parameter for the reaction $^{12}C + Cu$. The solid line is the result obtained

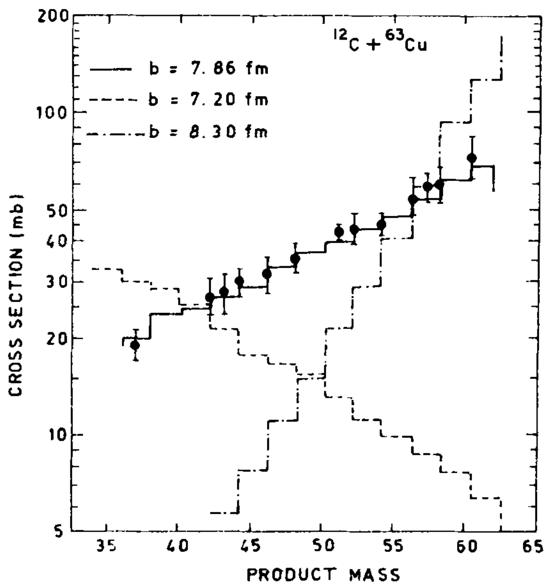

Figure 3. Impact parameter dependence of the mass yield distribution of $^{12}C + ^{63}Cu$ reaction.

with $\langle \alpha \rangle = 0{\cdot}14$ which corresponds to $b = 7{\cdot}86$ fm. This agrees well with experiment. The dotted and dash-dotted histograms are obtained with smaller and larger impact parameters of 7·2 and 8·3 fm respectively. It can be seen that in the case of the former which corresponds to a more central collision, the mass yield distribution is widely different from the experiment. A measure of the range of impact parameter which contributes towards the process is given by $(\langle \alpha^2 \rangle - \langle \alpha \rangle^2)^{1/2}$. Their values are presented in table 1 which corresponds to a spread of about 0·5 fm.

4. Conclusion

We have attempted to develop a semi-phenomenological model, in which given the projectile, target and the incident energy, the mass yield distribution arising out of target fragmentation can be calculated. Considering the complexity of the problem, the present success shows that our attempt is a step forward and provides encouragement for further study. It is shown that target fragmentation mainly originates from peripheral interaction with an impact parameter larger than the sum of the half density radii. Further it is shown that the projectile dependence of the cross-section solely comes from the total reaction cross-section. Clearly more extensive studies are needed to establish these features of the dynamics.

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Appendix 1

Dependence of energy transfer on contact area

The energy transfer is proportional to the contact area between the target and the projectile. In zeroth order approximation we consider the projectile and target nucleus as spheres of radii $R_P = r_0 A_P^{1/3}$ and $R_T = r_0 A_T^{1/3}$ respectively. The geometry is shown in figure 4 in which the distances AQ, QP and PR are called x, y and D respectively. We assume the depth of overlap (from surface to surface along the line connecting the two centres perpendicular to the beam axis) D roughly to be a constant with $D \leqslant \min(R_P, R_T)$. Thus the collision is supposed to be somewhat peripheral in nature. The area of contact is taken to be πx^2. From the figure it can be seen that

$$R_T^2 - (R_T - y)^2 = x^2, \tag{A1}$$

$$R_P^2 - \{R_P - (D - y)\}^2 = x^2. \tag{A2}$$

From (A1) and (A2) y is obtained as

$$y = \frac{D(2R_P - D)}{2(R_T + R_P - D)}. \tag{A3}$$

Equation (A1) can be written as

$$x^2 = y(2RT - y).$$

Using (A3) for y and neglecting the term in D^2 we obtain

$$x^2 = \frac{2R_P R_T D}{(R_T + R_P)}.$$

As seen from the rest frame of the target, the projectile will suffer Lorentz contraction in the direction of motion. This relativistic effect is taken into account by multiplying by the usual factor $(1 - \beta^2)^{1/2}$. Hence the energy loss ΔE can be written as

$$\Delta E \propto A_T^{1/3} A_P^{1/3} (A_T^{1/3} + A_P^{1/3})^{-1} (1 - \beta^2)^{1/2}.$$

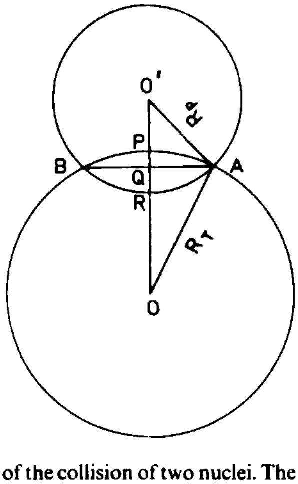

Figure 4. Geometry of the collision of two nuclei. The circles with radii R_T and R_P represent the target and projectile nuclei respectively.

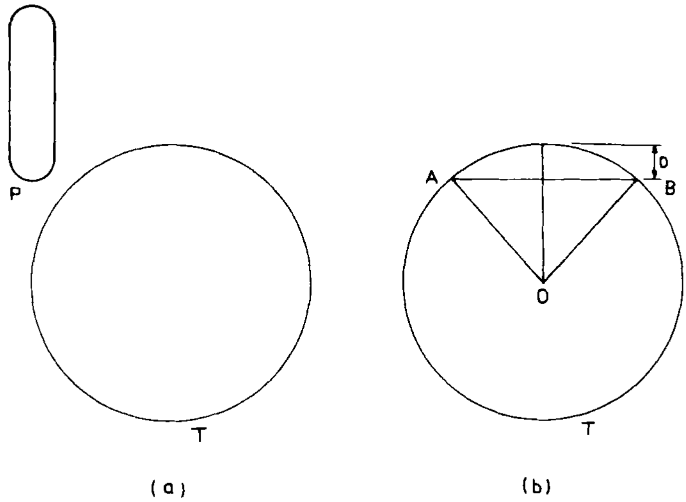

Figure 5. **a.** Target nucleus T at rest and projectile nucleus P Lorentz contracted due to motion. **b.** Contact between target and projectile. T represents the target nucleus and AB the contact path followed by projectile.

Appendix 2

Dependence of energy transfer on contact time

The contact time is the duration during which the projectile and the target remain in contact. Because of Lorentz contraction, the projectile will appear like a disc in the rest frame of the target as shown in figure (5a). The dimension of the target will predominantly determine the contact time. This is exactly true in the case of limiting fragmentation. As shown in figure (5b), the contact time can be taken to be equal to AB/v where v is the velocity of the projectile. From figure (5b).

$$AB = 2[R_T^2 - (R_T - D)^2]^{1/2},$$

$$= 2[2R_T D]^{1/2}.$$

So $$\Delta E \propto [2R_T D]^{1/2} \beta^{-1}.$$

Taking the collision to be predominantly peripheral in nature and assuming D to be roughly a constant as before one obtains

$$\Delta E \propto A_T^{1/6} \beta^{-1}.$$

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