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R-matrix plus potential analysis of unbound states of ^{33}S

RAMA DAS and S N MUKHERJEE*
SERC, Daresbury Laboratory, Daresbury, Warrington WA4 4AD, England
*Department of Physics, Banaras Hindu University, Varanasi 221 005, India

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Abstract. Single channel single level *R*-matrix plus potential analysis of the resonances observed in the $^{32}S(n, n)$ reaction for the neutron energy lying from 20–1100 keV has been carried out to determine the properties of these resonances. This analysis is further supported by a single channel multilevel *R*-matrix analysis of the data. Spectroscopic factors for the resonances have also been calculated by our *R*-matrix method. Its comparison with those obtained by DWBA analysis of the (d, p) data is discussed.

Keywords. $^{32}S(n, n)$ reaction; *R*-matrix analysis; DWBA analysis.

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1. Introduction

Recently, Halperin *et al* (1980) presented a complete study of the resonance structure in the $n + {}^{32}S$ system, in which 64 resonances have been observed within the neutron energy region 20–1100 keV. These authors assigned spins and parities to 28 of these resonances by a single-level *R*-matrix analysis which were reanalysed by Johnson and Winters (1980) by multilevel single-channel *R*-matrix analysis. Some of the resonances studied have also been observed by Liljestrand *et al* (1975) and Bommer *et al* (1976) by means of the (d, p) reaction into the continuum on a ^{32}S target at deuteron incident energy of 12 MeV. The spin and parity assignments for some of the resonances made by these authors did not agree with those of Halperin *et al* (1980). We have performed a detailed single channel, single level and a multilevel *R*-matrix analysis of the resonances for the $n + {}^{32}S$ system reported by Halperin *et al* (1980) to clarify the situation regarding the spin and parity assignments for these resonances and to compare the spectroscopic factors calculated by *R*-matrix method with those calculated by DWBA.

Westin and Adams (1971) described the following procedure for determining the spectroscopic factors for the neutron resonances by *R*-matrix method. The total elastic scattering cross-section of the neutron from the target nucleus is calculated with some appropriate potential (usually of Woods-Saxon form). This is then parametrized in terms of the usual single-channel *R*-matrix parameters (*i.e.* background term, *R*-matrix radius, potential resonance, reduced widths, energy and boundary condition at the given energy). Once the potential scattering is parametrized, the experimental data can be fitted using different values for the reduced widths and energies of the resonances. By comparing potential scattering parameters with the experimental fitting parameters, the spectroscopic factor for a resonance can be defined. In Westin and Adams (1971)

method, however, the calculated spectroscopic factors depend strongly on R-matrix parameters. To overcome this drawback, some corrections to this method have been suggested. This is discussed in detail in the following sections.

For the $n + {}^{32}S$ reaction the elastic scattering is the dominant reaction mode for the neutron energy within the region 20–1100 keV. The (n, α) channel contributes very little to the cross-section in this region. So only the (n, n) channel is considered in the present work.

2. Potential scattering using R-function

In this section it will be shown briefly how the potential scattering can be described by an R-matrix formalism. If the neutron elastic scattering is considered as a single problem of neutron scattering from a potential, its solution can be easily found by solving the Schrödinger equation with a Woods-Saxon potential (V_{ws}). By varying the parameters of the potential a reasonable fit to the elastic scattering data can be obtained. This fit, however, could be further improved by including the compound resonances in the framework of the R-matrix theory (Westin and Adams 1971; Takenchi and Moldauer 1970; Johnson 1973).

The Schrödinger equation in the interior region ($r_c < a_c$) can be written as

$$(T + V_{ws})u_{plj}(r_c) = E_{plj}u_{plj}(r_c), \tag{1}$$

where T is the kinetic energy operator and V_{ws} is the Woods-Saxon optical model potential, p labels a particular lj resonance in the elastic channel c, a_c is the R-matrix channel radius and E_{plj} is the R-matrix energy parameter connected with the neutron centre-of-mass energy at which the resonance is seen experimentally. The radial wavefunction u_{plj} obeys the boundary condition

$$\frac{a_c}{u_{plj}(a_c)}\left[u'_{plj}(r_c)\right]_{a_c} = b_{plj}, \tag{2}$$

where prime denotes differentiation with respect to r_c.

When only the neutron elastic scattering channel is allowed, the R-matrix reduces to a R-function and for the single particle resonance labelled plj the R-function is taken to be

$$R_{lj} = R^0_{lj} + \gamma^2_{plj}/(E_{plj} - E), \tag{3}$$

where the parameters R^0_{lj} describes the background effect and γ^2_{plj} denotes the reduced width. When the optical model radial wavefunction is normalized to unity inside the R-matrix radius a_c, the form of the reduced width is (Cooper *et al* 1974)

$$\gamma^2_{plj} = \left(\frac{\hbar^2}{2\mu_c a_c}\right)u^2_{plj}(a_c)\Big/\int_0^{a_c} u_{plj}(r_c)\mathrm{d}r_c, \tag{4}$$

where μ_c is the reduced mass of the system.

The relation between the elastic scattering U-matrix (collision matrix) element and the R-function is

$$U_{lj} = \exp(2i\delta_{lj})[1 + i\Gamma^{s,p}_{plj}/\{(E_{plj} - E) + (\Delta - \tfrac{1}{2}i\Gamma^{s,p}_{plj})\}], \tag{5}$$

where

$$\delta_{lj} = -\phi_{plj} + \tan^{-1}[(R^0_{lj}p_c/\{1 + R^0_{lj}(B_c - S_c)\}] \tag{6}$$

and $2\phi_{plj}$ is the change in the optical model phaseshift ($= \pi/2$) at the resonance. The single particle width $\Gamma^{s.p.}_{plj}$ is related to the reduced widths γ^2_{plj} and the penetration factor p_c by the following expression:

$$\Gamma^{s.p.}_{plj} = 2p_c\gamma^2_{plj}/D. \tag{7}$$

The expression for the level shift Δ is given by

$$\Delta = \frac{\gamma^2_{plj}}{D}\left[(B_c - S_c) + R^0_{lj}(B_c - S_c)^2 + R^0_{lj}p_c^2\right], \tag{8}$$

where

$$D = \left[1 + R^0_{lj}(B_c - S_c)\right]^2 + (R^0_{lj}p_c)^2, \tag{9}$$

where B_c and S_c are the natural boundary condition parameters (NBCP) and the shift function respectively. B_c (NBCP) is a free, real and neutron energy independent parameter and can be chosen according to the best R-matrix parametrization of potential resonance scattering. By using the reduced width calculated by the optical model wavefunctions and using the optical model phase-shifts instead of the hard sphere phase-shifts in the above equation the R-matrix parametrization of the potential scattering and the extraction of single particle width is made easier (Mahaux and Weidenmuller 1967; Cugnon 1975; Namjoshi *et al* 1976).

Thus neutron scattering from a potential can be solved by one of the two ways. One way is to match the logarithmic derivatives of the internal and external wavefunctions at the boundary of potential at each energy and find the phase-shift as a function of energy. The second is to use the R-matrix procedure for the partial wave of interest.

In the present analysis of ^{32}S(n, n) data, a potential is chosen which has a real central part of the Woods-Saxon form and a spin-orbit term of the Thomas form. We have fixed the radius and diffuseness parameters of both the real and the spin-orbit terms to 1·22 fm and 0·7 fm respectively. The depth of the spin-orbit potential is taken to be 3 MeV. The depth of the real potential has been determined for a given resonance energy to produce a resonance. These values for $l = 0, 1, 2$ resonances lie in the range 26–32 MeV, 39–50 MeV and 34–36 MeV respectively.

To convert the potential scattering calculation into the R-matrix form, we need to choose an energy E_{plj} and an R-matrix radius a_c. Then using the wavefunction generated by the potential at E_{plj} and a_c, γ^2_{plj} (4) and boundary condition b_{plj} (2) can be computed.

In the R-matrix optical model analysis a_c must be chosen carefully. Westin and Adams (1971) suggest that a_c should be outside the point where the potential contributes to greater than about 0·1 %. Takeuchi and Moldauer (1970) also placed a_c at either 7 fm or 10 fm for ^{17}O, which are outside the potential. On the other hand, Johnson (1973) put it inside the potential range. We have chosen a_c equal to the distance where the nuclear potential drops to 0·02 % of its maximum value which is equal to 10 fm in the present case. This choice of a_c provides a best fit to the data.

In one-level approximation, NBCP (B_c) is chosen to be equal to b_{plj} for each resonance plj. For numerical estimation of B_c most of the authors (Bond and Firk 1977; Hickey *et al* 1974) advocate the value:

$$B_c = b_{plj} = \left[S_c(E_{plj})\right]_{r_c = a_c}, \tag{10}$$

where S_c represents the shift function evaluated at $E = E_{plj}$. By this choice of B_c and proper selection of background R-matrix R^0_{lj}, the level shift parameter (equations (8)

and (9)) may be made negligible giving rise to minimum shift of the energy E_{plj} from the experimental resonance energy (Westin and Adams 1971). Furthermore, such a choice of B_c along with the choice of R-matrix radius a_c equal to the nuclear potential radius enables us to join smoothly the interior and exterior wavefunctions at the boundary. The boundary condition parameters used by Cooper *et al* (1975) are different from those used by us in the sense that they have used an R-matrix radius greater than the nuclear potential radius.

In the present calculation a simultaneous search over E_{plj} and R^0_{lj} is made to provide a best fit to each peak of the experimental total cross-section of Halperin *et al* (1980). We use the χ^2 minimization procedure. As remarked earlier, Halperin *et al* (1980) observed 64 resonances for $n + {}^{32}S$ systems in their work. Of these, 28 were spin and parity assigned. In the present analysis those 14 resonances of their work are selected which were also observed by Bommer *et al* (1976) in their (d, p) measurements on ^{32}S target, so that the R-matrix and DWBA results could be compared.

In table 1, the results of our R-matrix optical model analysis for $l > 0$ resonances are presented. Level shifts in each resonance are negligible and hence not listed. As discussed earlier, for each chosen energy E_{plj} we have searched over the depth of the potential which produces a resonance. Corresponding to each well depth at energy E_{plj}, b_{plj} and γ^2_{plj} are calculated. Since we have the freedom of changing E_{plj} and hence the corresponding depth of the potential well, we could reproduce all the single particle resonances corresponding to a particular partial wave lj. Possibly this explains why energy-independent optical model potential chosen by Westin and Adams (1971) could not reproduce the double $d_{3/2}$ resonances in carbon and ended up in giving total spectroscopic strengths greater than one. Further spectroscopic factors for each $d_{3/2}$

Table 1. The results of single level R-function optical model analysis for p and d-wave resonances.*

E_χ (MeV) (n, n) Halperin *et al* (1980)	$E_{c.m.}$ (MeV) (n, n) Halperin *et al* (1980)		lj^π	E_{plj} (MeV)	b_{plj}	γ^2_{plj} (MeV) (n, n) Present work	R^0_{lj}	$\sigma_{tot.}$ peak (barn)	χ^2
8·8387	0·1957	1	$1/2^-$	0·1870	−0·5264	0·1987	0·0	8·31	0·165
8·9060	0·2630	1	$1/2^-$	0·2580	−0·4482	0·2055	0·0	5·47	0·252
8·9218	0·2788	1	$3/2^-$	0·2690	−0·4389	0·2060	0·0	11·21	0·193
9·3449	0·7019	1	$1/2^-$	0·6890	−0·2343	0·1658	0·5	2·30	0·291
9·3605	0·7175	1	$3/2^-$	0·7030	−0·2305	0·1629	0·0	4·22	0·223
9·5349	0·8919	1	$3/2^-$	0·8690	−0·1936	0·1250	0·0	3·64	0·230
9·6587	1·0157	1	$3/2^-$	1·0050	−0·1729	0·0948	0·3	2·81	0·218
9·2112	0·5682	2	$5/2^+$	0·5610	−1·0820	0·0518	0·0	10·38	0·041
9·4021	0·7591	2	$5/2^+$	0·7510	−0·8881	0·0602	0·0	6·93	0·018
9·4360	0·7930	2	$5/2^+$	0·7860	−0·8582	0·0616	0·0	6·51	0·035
9·5606	0·9176	2	$5/2^+$	0·9120	−0·7603	0·0661	0·0	5·03	0·138

* Channel radius (a_c) and the last particle binding energy (B_n) used in the analysis are 10 fm and 8·643 MeV respectively.

resonance in their case strongly depends on the R-matrix parameters. Similar observations have been made by Cooper *et al* (1975).

We have also been able to reproduce S-wave resonances at $E_x = 98{\cdot}8$, $364{\cdot}3$ and $674{\cdot}1$ keV through R-function potential analysis. The ability of optical model R-function to reproduce S-wave resonances in the absence of a barrier was tested by Takeuchi and Moldauer (1970). Since in the absence of a barrier, width parameters for $l = 0$ resonances are not meaningful, these are not reported here.

The spectroscopic factor in (d, p) stripping to unbound state is generally defined as the ratio of experimental cross-section and that calculated by DWBA. The neutron width $\Gamma_n^{(d,p)}$ calculated from (d, p) reaction is then written as

$$\Gamma_n^{(d,p)} = S\Gamma_n^{\text{s.p.}}, \tag{11}$$

where S is the ratio of experimental and theoretical (d, p) cross-section and $\Gamma_n^{\text{s.p.}}$ is the single particle neutron width.

The spin and parity assignments for eleven $l > 0$ resonances have also been re-examined in the single level R-function optical model analysis. Our lj^π assignments for nine $l \neq 0$ resonances agree with those made by single level single channel R-matrix analysis of the data by Halperin *et al* (1980). It may be noted that these authors have used hard sphere phase shift instead of optical model phase shifts used in present case and the reduced widths obtained by fitting experimental data instead of the reduced width estimated through optical model wavefunction in the present analysis. However in our analysis, p-wave resonance at $E_x = 9658{\cdot}7$ keV is assigned $j^\pi = 3/2^-$ instead of $1/2^-$ assigned by them. Furthermore the resonance at $E_x = 9402{\cdot}1$ keV is confirmed to be $d_{5/2}$ resonance instead of an ambiguous $l > 1$ reported by Halperin *et al* (1980) and $d_{3/2}$ assigned by Liljestrand *et al* (1975). Bommer *et al* (1976) assigned only orbital angular momentum (l) to the eleven $l \neq 0$ resonances being discussed here and our assignments agree in all cases with their assignments. Thus the present analysis confirms that the resonance at $E_x = 9344{\cdot}9$ keV corresponds to $l = 1$ as has been assigned by Bommer *et al* instead of $l = 3$ as assigned by Liljestrand *et al*. Final confirmation of all common resonances through a single-channel multilevel (SCML) R-matrix fit to total elastic scattering cross-section and the extraction of spectroscopic factors are discussed in § 3.

3. Multilevel single channel R-matrix fit to the total elastic scattering cross-section data and the spectroscopic factors

For the actual scattering to be parametrized the potential scattering R-function (3) must be replaced by one representing the true nuclear wavefunction at the channel surface. The SCML R-function is given as

$$R_{lj} = \sum_{\lambda=1}^{\infty} \gamma_{\lambda lj}^2/(E_{\lambda lj} - E). \tag{12}$$

As the experiments are performed over limited energy range, it is difficult to include all the states in (12) and an approximation must, therefore, be made. Usually the R-function is split into two segments, one part containing explicitly all the levels within the energy region of interest and the other including distant levels outside the region so that

(12) takes the form

$$R_{lj} = \sum_{\lambda} \gamma_{\lambda lj}^2/(E_{\lambda lj} - E) + R_{lj}^0, \tag{13}$$

where $E_{\lambda lj}$ and $\gamma_{\lambda lj}$ are respectively the eigen energies and the neutron reduced widths parameters for actual levels λlj. The quantity R_{lj}^0 includes the effect of other levels not included in the analysis. The form of the actual reduced width is given by

$$\gamma_{\lambda lj} = (\hbar^2/2\mu_c a_c)^{1/2} \int \Phi_c \xi_{\lambda lj} \, ds, \tag{14}$$

where Φ_c is the channel surface wavefunction and $\xi_{\lambda lj}$ is the actual complete channel wavefunction for λlj state. The relation between collision matrix and R-function is

$$U_{lj} = \exp(2i\phi_l)\left[1 + \frac{2iR_{lj}p_c}{1 + R_{lj}(B_{lj} - S_c - ip_c)}\right], \tag{15}$$

where ϕ_l is hard sphere phase shift. The partial width $\Gamma_{\lambda lj}$ is expressed as $\Gamma_{\lambda lj} = 2p_c\gamma_{\lambda lj}^2$. The total scattering cross-section has the usual form

$$\sigma_{\text{tot}}^{(n,n)} = \frac{\pi}{k^2} \sum_{lj} \frac{(2j+1)}{(2I_1+1)(2I_2+1)} \left|1 - U_{lj}\right|^2. \tag{16}$$

The natural boundary condition parameter B_{lj} appearing in (15) is different from that defined by (10) which corresponds to one-level approximation of the R-matrix theory. In the present SCML R-matrix fitting process, B_{lj} is considered as real energy independent free parameters for each lj and its values are chosen to obtain the best fit to the shape of the experimental total cross-section. On the other hand, $b_{\lambda lj}$ is the boundary condition parameter for each lj state and is given by the logarithmic derivative of the actual radial wavefunction $u_{\lambda lj}$ at $r_c = a_c$. In the present work we assume $B_{lj} \neq b_{\lambda lj} \neq b_{\lambda' lj}$ for $\lambda \neq \lambda'$ for each lj.

For single-channel case, the spectroscopic factor for a particular resonance state λlj is defined as (Cooper *et al* 1974)

$$S_{\lambda lj} = \gamma_{\lambda lj}^2/\gamma_{plj}^2. \tag{17}$$

Thus $S_{\lambda lj}$ may be interpreted as the square of the fraction of optical model wavefunction (u_{plj}) in the actual internal wavefunction (u_{plj}) at the R-matrix surface, $r_c = a_c$. The normally tacit assumption, that a single internal R-matrix eigenfunction is directly related to the actual internal wavefunction for the nuclear state, allows S to be considered equivalent to the spectroscopic factor defined for the stripping reaction as long as the surface radius is taken outside the range of nuclear force. For the spectroscopic factor $S_{\lambda lj}$ to be least dependent on the R-matrix parameters, both $\gamma_{\lambda lj}^2$ and γ_{plj}^2 must be estimated using the same value of channel radius a_c and true nuclear radial wavefunction ($u_{plj}(r_c)$) and the single particle wavefunction ($u_{plj}(r_c)$) must satisfy identical boundary condition at channel surface $r_c = a_c$, *i.e.* $b_{\lambda lj} = b_{plj}$.

In the present multilevel single channel R-matrix analysis of $^{32}S + n$ data of Halperin *et al* (1980) we have included only 28 resonances; 5 for $l = 0, j = 1/2$ ($\lambda = 1$–5); 7 for $l = 1, j = 1/2$ ($\lambda = 1$–7); 7 for $l = 1, j = 3/2$ ($\lambda = 1$–7) and 9 for $l = 2, j = 5/2$ ($\lambda = 1$–9). A multilevel fit to the total cross-section data is shown in figures (1a) and (1b) and the corresponding resonance parameters $E_{\lambda lj}$ (energy), $\gamma_{\lambda lj}^2$ (reduced width) and B_{lj} (NBCP)

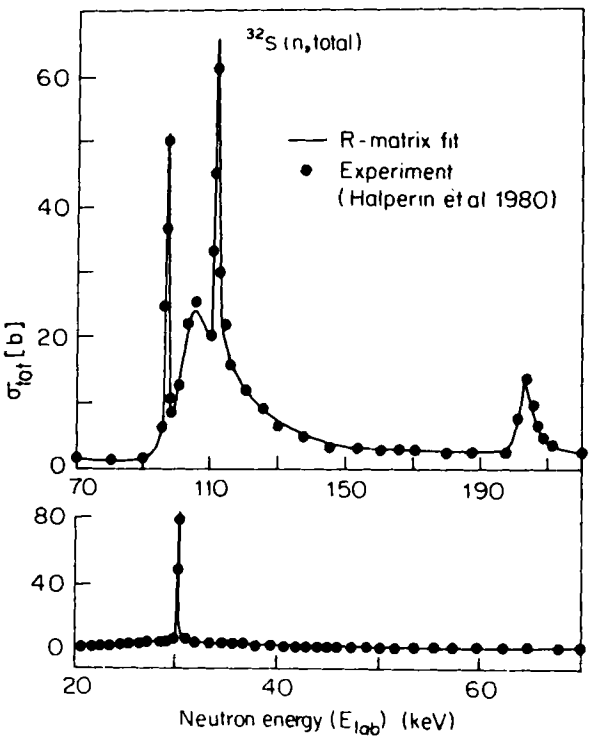

Figure 1a. SCML *R*-matrix fit to the total neutron elastic scattering cross-section data of Halperin *et al* (1980) from 20–220 keV (shape fitting).

up to 1100 keV neutron energy (E_{lab}) are listed in table 2. Partial widths $\Gamma^n_{\lambda j}$ obtained from the present analysis (listed in the last column) compare nicely with those of Halperin *et al* (listed in the 5th column).

The fit to the $^{32}S + n$ cross-section data was performed by an interactive *R*-matrix programme which included a χ^2 minimization routine. The programme varies B_{lj} (NBCP), $E_{\lambda j}$ (resonance energy), $\gamma_{\lambda lj}$ (reduced width) and R^0_{lj} in order to obtain the best fit to the data. The process of fitting, however, involves first guessing a set of parameters for the resonances. In SCML analysis B_{lj} is treated as a free real and energy independent parameter and it is searched for each partial wave (lj) to get the best fit to the data. Similarly the background parameter R^0_{lj} is also treated as a free parameter and is searched for each partial wave. In figure 2 the best fit values of R^0_{lj} for each partial wave are shown with respect to the neutron energy. In order to get a meaningful comparison between the present SCML and the optical model SCSL analysis of the data (presented in last section), the value of the channel radius is chosen to be the same in both analyses. It is worthwhile to note that the multilevel fit of Johnson and Winters (1980) for the same data differs from ours in the sense that these authors do not treat B_{lj} and R^0_{lj} as free parameters. In their analysis B_{lj} is considered as a real energy dependent parameter and is put equal to the shift function at each neutron c.m. energy and R^0_{lj} was estimated from the non-resonant phase-shifts (see Johnson and Winters 1980 for details). The imposition of these constraints results in their getting too high theoretical peak cross-

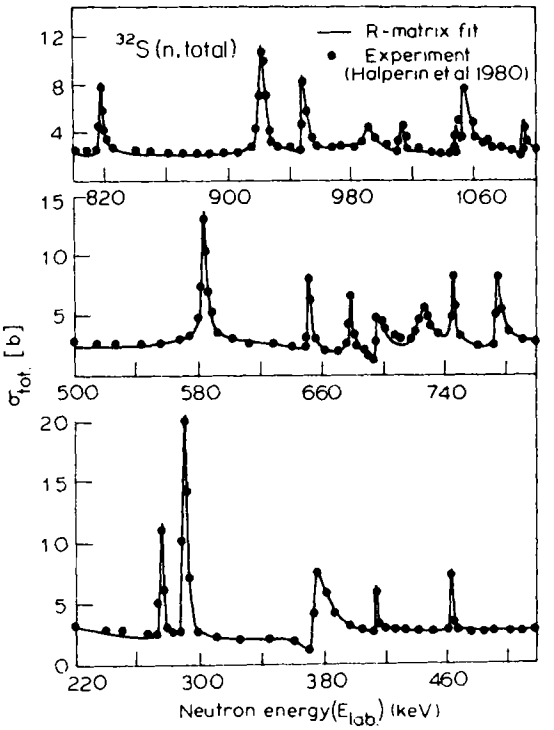

Figure 1b. SCML *R*-matrix fit to the total neutron elastic scattering cross-section data of Halperin *et al* (1980) from 220–1100 keV.

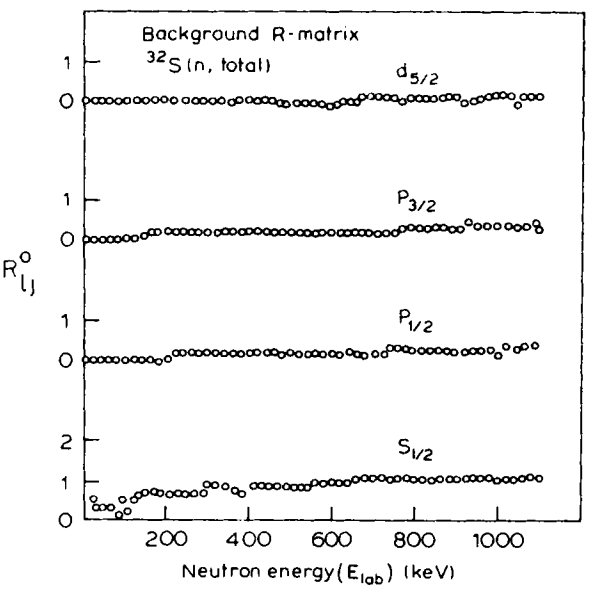

Figure 2. The background *R*-matrix R^0_{lj} used in the fits shown in figures 1a and 1b.

Table 2. *R*-matrix parameters from single channel multilevel *R*-matrix shown in figures 1 and 2.

lj^π Present work		lj^π	E_χ (keV) Halperin *et al* (1980)	$E_{c.m.}$ (keV) (n, n)	Γ_n (keV)	$E_{\lambda lj}$ (keV)	B_{lj} (n, n)	$\gamma^2_{\lambda lj}$ (keV) Present work	$\Gamma^n_{\lambda lj}$ (keV)
	0	$1/2^+$	8741·8	98·8	15·000	98·0		11·0	14·960
	0	$1/2^+$	9007·3	364·3	7·850	358·8		3·00	7·836
0 $1/2^+$	0	$1/2^+$	9317·1	674·1	11·810	669·1	0·0	3·00	10·716
	0	$1/2^+$	9656·7	1013·7	2·100	1008·5		0·50	2·190
	0	$1/2^+$	9674·6	1031·6	0·310	1033·0		0·07	0·309
	1	$1/2^-$	8671·7	28·7	0·064	29·6		0·60	0·056
	1	$1/2^-$	8838·7	195·7	3·050	198·8		3·60	3·406
1 $1/2^-$	1	$1/2^-$	8906·0	263·0	1·660	267·6		1·50	1·890
	1	$1/2^-$	9344·9	701·9	4·500	702·1	−1·351	1·40	3·917
	1	$1/2^-$	9597·1	954·1	8·300	958·0		1·60	5·571
	1	$1/2^-$	9619·4	976·4	1·550	979·4		0·70	2·464
	1	$1/2^-$	9658·7	1015·7	2·500	1015·3		0·70	2·541
	1	$3/2^-$	8736·7	93·7	0·234	94·1		0·50	0·208
	1	$3/2^-$	8751·0	108·0	1·116	109·6		2·00	0·988
	1	$3/2^-$	8921·8	278·8	2·240	281·2		1·60	2·099
1 $3/2^-$	1	$3/2^-$	9042·0	399·0	0·145	400·5	−1·501	0·08	0·144
	1	$3/2^-$	9360·5	717·5	2·240	720·5		0·90	2·570
	1	$3/2^-$	9534·9	891·9	5·550	892·1		1·40	4·648
	1	$3/2^-$	9700·4	1057·4	0·820	1092·0		0·20	0·744
	2	$5/2^+$	9090·9	447·9	0·244	449·0		0·30	0·199
	2	$5/2^+$	9211·2	568·2	3·860	572·1		3·80	3·730
	2	$5/2^+$	9271·7	628·7	1·420	633·2		1·20	1·375
	2	$5/2^+$	9297·0	645·0	0·790	657·5		0·60	0·726
2 $5/2^+$		(1)	9402·1	759·1	0·250	754·4	−3·701	1·70	2·526
	2	$5/2^+$	9436·0	793·0	1·690	795·2		0·90	1·435
	2	$5/2^+$	9534·0	891·0	2·415	894·1		1·30	2·421
	2	$5/2^+$	9560·6	917·6	3·090	921·2		1·50	2·895
	2	$5/2^+$	9665·4	1022·4	3·900	1023·0		1·70	3·723

Channel radius $a_c = 10$ fm.

sections. However, in the present case by appropriate selection of free parameters B_{lj} and R^0_{lj}, an excellent fit to the experimental peak cross-sections has been obtained. The value of χ^2 for 153 data points is 2·342, giving χ^2 per datum equal to 0·015.

The lj assignments performed in the previous section by the optical model SCSL analysis have been re-examined here in our SCML analysis. We confirm lj assignments of 13 out of the 14 common resonances considered in the previous section. However the p-wave resonance at $E_x = 9658{\cdot}7$ keV is assigned $j^\pi = 1/2^-$ in SCML analysis instead of $j^\pi = 3/2^-$ as assigned in SCSL optical model analysis in the previous section. For some of the resonances our lj assignments differ from those of Liljestrand *et al* (1975) who use DWBA analysis of the (d, p) reactions to unbound states for the study of these assignments. This is shown in table 3. In the $^{32}S(n, n)$ data of Halperin *et al* (1980) there

Table 3. Differences in lj^{π} assignments.

E_X (keV) (*n*, *n*)	lj^{π} (*n*, *n*)	E_X (keV) (*d*, *p*) and (*n*, *n*)		E_X (keV)		E_X (keV)	
Halperin *et al* (1980)	Present work	Bommer *et al* (1976)		Liljestrand *et al* (1975)		Peterson *et al* (1950)	
9211·2	$2\ 5/2^+$	$2\ 5/2^+$	9211·0	$2\ 5/2^+$	9209·0	$2\ 3/2^+$	9211·0 (1, 2)
9344·9	$1\ 1/2^-$	$1\ 1/2^-$	9348·0	1	9350·0	$2\ 7/2^-$	
9402·1	($l > 1$)	$2\ 5/2^+$	9400·0	2	9400·0	$2\ 3/2^+$	
9658·7	$1\ 1/2^-$	$1(3/2^-$, $1/2^-)$	9666·0	1			

Table 4. Comparison of spectroscopic factors *via* (*n*, *n*) and (*d*, *p*) studies.

lj^{π}	E_X (keV) *R*-matrix Present work	S (*n*, *n*)	lj^{π}	E_X (keV) DWBA	$S_{(d,p)}$
				Liljestrand *et al* (1975)	
$1\ 1/2^-$	8838·7	0·018	$1\ 1/2^-$	8838	0·005
$1\ 1/2^-$	8906·0	0·007			
$1\ 3/2^-$	8921·8	0·008			
$2\ 5/2^+$	9211·2	0·073	$2\ 3/2^+$	9209	0·011
$1\ 1/2^-$	9344·9	0·008	$3\ 7/2^-$	9350	0·004
$1\ 3/2^-$	9360·5	0·006			
$2\ 5/2^+$	9402·1	0·028	$2\ 3/2^+$	9400	0·021
$2\ 5/2^+$	9436·0	0·015			
$1\ 3/2^-$	9534·9	0·011			
$2\ 5/2^+$	9560·6	0·023			
1	9658·7	0·007			

are two very closely-spaced resonances: one at $E_x = 9397{\cdot}2$ keV assigned $j^{\pi} = 3/2^-$ and another at $E_x = 9402{\cdot}1$ keV ambiguously assigned as $l > 1$. In our SCML *R*-matrix analysis we could reproduce only one resonance at $E_x = 9402{\cdot}1$ keV which we assign as $j^{\pi} = 5/2^+$. This agrees with the same *lj* assigned to this resonance in our potential plus single level *R*-matrix analysis (SCSL) of the data presented in the last section. Also the resonances at $E_x = 9211{\cdot}2$ keV and $E_x = 9344{\cdot}9$ keV have $j^{\pi} = 5/2^+$ and $1/2^-$ respectively in our work contrary to $j^{\pi} = 3/2^+$ and $7/2^-$ respectively as assigned to them by Liljestrand *et al* (1975). In the work of Liljestrand *et al* the angular distributions for the (*d*, *p*) reactions to the resonant states of ^{33}S has been analysed by DWBA method. However, the analysis of angular distributions alone may not be sufficient for performing *lj* assignments for the resonances. Another quantity, the so-called "stripping enhancement factor", provides a more clear-cut signature for the orbital angular momentum transfer in stripping reactions to the resonant states as has

been shown by Mukherjee *et al* (1977). It is also evident from the fact that the assignments of Bommer *et al* (1976) performed by analysing the stripping enhancement factor instead of the angular distributions agree perfectly with our assignments. Another good signature of l transfer in these reactions is the vector analysing powers. It is desirable that more work on these lines be performed to remove the apparent limitations of the work of Liljestrand *et al* (1975).

The spectroscopic factors calculated by using 17 for the eleven $l > 0$ common resonances are listed in table 4. In order to make spectroscopic factors least dependent on the R-matrix parameters we have chosen the same value of channel radius a_c in the evaluation of $\gamma^2_{\lambda lj}$ and $\gamma^2_{\rho lj}$. It was also ensured that the actual nuclear wavefunction and single particle wavefunction satisfy identical boundary condition. Further in both analysis (SCSL and SCML) the background term R^0_{lj} has been considered as a free parameter, and chosen according to the best fit to the data. In this table spectroscopic factor for some of the resonances calculated by Liljestrand *et al* (1975) by their DWBA analysis of (d, p) data is also listed. We see that our spectroscopic factors are larger by a factor 1·3 to 6·6 as compared to those of Liljestrand *et al*. The DWBA calculation of Liljestrand *et al*, performed by using the Cocker's (1973) technique, should be repeated by employing the improved technique of Vincent and Fortune (1970, 1973).

4. Summary and conclusion

Our investigations suggest considerable agreement between (d, p) and (n, n) analyses of neutron resonances unlike the results of Liljestrand *et al* (1975). This points to the fact that a careful R-matrix plus potential analysis of the (n, n) data is quite desirable to get a good comparison between the same resonances observed in (d, p) and (n, n) experiments.

The lj assignments of the resonances seen in the reaction $^{32}S(n, n)$ performed by Halperin *et al* (1980) have been re-examined by us by potential plus SCSL R-matrix analysis as well as by SCML R-matrix fit to the data.

We have confirmed the lj assignments of some of the resonances which were ambiguously lj assigned in the work of Halperin *et al* (1980). Further, we have compared our lj^π assignments for the common resonances with those performed by DWBA analysis of the (d, p) data by Liljestrand *et al* (1975) and by Bommer *et al* (1976). While our assignments generally agree with those of Bommer *et al*, they do not agree with those of Liljestrand *et al*.

The spectroscopic factors calculated by us by the R-matrix analysis also do not agree with those calculated by Liljestrand *et al*. The reanalysis of the (d, p) data of Liljestrand *et al* by employing the improved technique of Vincent and Fortune (1970) for calculating the stripping to unbound states may provide a better agreement between (n, n) and (d, p) results.

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