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Quark-pion coupling constant in a chiral quark model

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Abstract. Incorporating chiral-symmetry to the potential model of quarks with confining potential $U(r) = \frac{1}{2}(1+\gamma^0)ar^2$ with $m_q = 10$ MeV and $a = 2{\cdot}273\,\mathrm{fm}^{-3}$ that gives a reasonable quark-core contribution to μ_p, $\langle r^2 \rangle_p^{1/2}$ and g_A, the quark-pion coupling constant for quarks in a nucleon is estimated. $G_{qq\pi}^2/4\pi$ obtained between 0·4 and 0·5 is consistent with those extracted from experimental vector meson decay-width ratios by Suzuki and Bhaduri. The nucleon-pion coupling constant $G_{NN\pi}^2/4\pi$ comes out to be of the order of 13·1 in reasonable agreement with the experimental value.

Keywords. Quark; confinement; vertex function; quark-pion coupling constant; chiral symmetry.

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1. Introduction

In a phenomenological model of baryons, if one considers the quarks as point Dirac particles moving independently in an effective potential taken as an equal admixture of scalar and vector parts, the static electromagnetic properties of low lying baryons can be explained reasonably well (Ferreira 1977; Ferreira *et al* 1980; Barik and Das 1983a, b; Barik *et al* 1985). However unlike the electromagnetic and isospin currents, the axial vector current carried by the quarks is not conserved in this model. Such a situation is inherent with all the potential models confining quarks including the bag model. But in view of the experimental success of PCAC and hence the fact that chiral SU(2) × SU(2) is one of the best symmetries of strong interaction, it is desirable to conserve the total axial vector current in any of these models describing hádrons. This is usually done at a phenomenological level (Chodos and Thorn 1975; Brown *et al* 1979a, b; Vento *et al* 1980; Theberge *et al* 1980, 1981; Thomas *et al* 1981; Thomas 1983) by introducing elementary pion field that also carries an axial current such that the four divergence of the total axial vector current satisfies the PCAC condition. In spite of many successful applications of chiral bag models, it is not totally free from certain objections particularly for its insistence on excluding pions from the interior of the static, spherical bag. Therefore we attempt a simpler alternative approach to formulate a chiral potential model with equally mixed scalar and vector harmonic potential used (Barik *et al* 1985) for studying the static properties of baryons. Our main objective here is to determine the quark-pion coupling constant in this model to examine its consistency with the estimates made earlier by other workers (Suzuki and Bhaduri 1983; Faimen and Hendry 1983; Hendry 1982).

2. Independent quark model with chiral symmetry

We consider that quarks in a hadronic core move independently in an effective central potential

$$U(r) = \tfrac{1}{2}(1+\gamma^{\circ})\,V(r), \tag{1}$$

obeying the Dirac equation and implying thereby a Lagrangian density

$$\mathscr{L}_q = \tfrac{i}{2}\bar{q}(x)\gamma^{\mu}\overleftrightarrow{\partial}_{\mu}q(x) - m_q\bar{q}(x)q(x) - \bar{q}(x)U(r)\bar{q}(x). \tag{2}$$

Then under a global infinitesimal chiral transformation

$$q(x) \rightarrow q(x) - i\gamma^5\left(\frac{\boldsymbol{\tau}.\boldsymbol{\varepsilon}}{2}\right)q(x), \tag{3}$$

the axial vector current of the quarks

$$\mathbf{A}_q^{\mu}(x) = \bar{q}(x)\gamma^{\mu}\gamma^5\frac{\boldsymbol{\tau}}{2}q(x), \tag{4}$$

associated with such a transformation, is not conserved since its four divergence is

$$\partial_{\mu}\mathbf{A}^{\mu}(x) = iG(r)\bar{q}(x)\gamma^5\boldsymbol{\tau}q(x), \tag{5}$$

where $G(r) = (V(r)/2 + m_q)$. This is due to the fact that just like the surface term $-\frac{1}{2}\bar{q}q\,\Delta_s$ in the bag model Lagrangian density, the term $G(r)\bar{q}q$ in $\mathscr{L}_q$ corresponding to the quark mass m_q and the scalar potential $\frac{1}{2}V(r)$, is chirally odd. The vector part of the potential poses no problem in this respect. Now to restore chiral symmetry in the usual manner, we can introduce a zero mass pion field with the interaction Lagrangian density,

$$\mathscr{L}_I = \frac{-i}{f_{\pi}}G(r)\bar{q}(x)\gamma^5(\boldsymbol{\tau}\cdot\boldsymbol{\phi})q(r), \tag{6}$$

when $f_{\pi} = 93$ MeV is the pion decay constant. Then the total axial vector current due to quark and pion together, *i.e.* $\mathbf{A}^{\mu}(x) = [\bar{q}\gamma^{\mu}\gamma^5\boldsymbol{\tau}/2\,q + f_{\pi}\partial^{\mu}\boldsymbol{\phi}]$ gets conserved with $\partial_{\mu}\mathbf{A}^{\mu}(x) = 0$. However if we give the pion field a mass m_{π}, then

$$\partial_{\mu}\mathbf{A}^{\mu}(x) = -f_{\pi}m_{\pi}^2\boldsymbol{\phi}, \tag{7}$$

yielding the usual PCAC relation in the current quark level.

First of all, neglecting the pion coupling with the quarks, one can study the bare hadrons in terms of its individual quarks obeying the Dirac equation

$$[i\gamma^{\mu}\partial_{\mu} - m_q - U(r)]\,q(x) = 0. \tag{8}$$

Taking $U(r)$ in (1) with $V(r) = ar^2$, $(a > 0)$ and m_q as the current quark mass, the spatial orbits of all the individual quarks in the low lying baryon ground states can be written in their 1S $\frac{1}{2}$ configuration as,

$$q(\mathbf{r}) = \frac{1}{(4\pi)^{1/2}}\begin{pmatrix} ig(r)/r \\ \boldsymbol{\sigma}\cdot\hat{r}f(r)/r \end{pmatrix}, \tag{9}$$

when, with $\lambda_q = (E_q + m_q)$ and $r_0 = (a\lambda_q)^{-1/4}$, the reduced radial parts of the upper and lower components can be written as,

$$g(r) = N_q(r/r_0)\exp(-r^2/2r_0^2),$$

$$f(r) = -\frac{N_q}{\lambda_q r_0}(r/r_0)^2 \exp(-r^2/2r_0^2). \tag{10}$$

Here E_q is the ground state ($1S\frac{1}{2}$) individual quark binding energy obtainable from the energy eigen value condition

$$(\lambda_q/a)^{1/2}(E_q - m_q) = 3, \tag{11}$$

and N_q is the overall normalisation factor satisfying the relation,

$$N_q^2 \sqrt{\pi} r_0/8\lambda_q = 1/(3E_q + m_q). \tag{12}$$

These solutions resulting from (8) can be utilized to describe the bare nucleons represented by the quark-core alone. In fact, in an independent quark model approach, where the quarks in a nucleon are assumed to satisfy the Dirac equation as given in (8) with $V(r) = ar^2$, we obtained (Barik *et al* 1985) a fairly reasonable description of the bare nucleons with its static properties in terms of magnetic moment μ_p, charge radius $\langle r^2 \rangle_p^{1/2}$ and the axial constant g_A for neutron β-decay being estimated after centre of mass correction as

$$(\mu_p, \langle r^2 \rangle_p^{1/2}, g_A) \equiv (2{\cdot}6\,\mu_N, 0{\cdot}72 fm, 1{\cdot}02). \tag{13}$$

Here, the potential parameter $a = 2{\cdot}273\ \text{fm}^{-3}$, the quark masses $m_u = m_d = 10$ MeV and as a consequence of (11), the quark binding energy in the $1S\frac{1}{2}$ configuration $E_u = E_d = 540$ MeV, have been used. Therefore in the present work, where our main objective is to build such a potential model for nucleons incorporating the chiral symmetry to study the pion coupling to quarks, we would adopt the same set of parameters that describes the bare nucleon properties in a reasonable manner.

3. Pion-quark coupling constant

We intend to study mainly the coupling of quarks in a nucleon to pions, in a chiral symmetric potential model. Therefore, in view of the fact that chiral SU(2) × SU(2) is experimentally found to be an excellent symmetry of strong interaction having its physical realization in pion with its small mass as the corresponding Goldstone boson, we concentrate our discussion mainly in the (u, d) flavour sector only. Then as a first step in this direction, let us assume that the interaction Lagrangian density in (6) can be written effectively as,

$$\mathscr{L}_I = -iG_{qq\pi}\bar{q}(x)\gamma^5(\boldsymbol{\tau}\cdot\boldsymbol{\phi})q(x), \tag{14}$$

with $G_{qq\pi}$ as the effective quark-pion coupling strength. Then in a classical field approximation, taking the emitted pion field ϕ_j in the process $q \to q + \pi$ as a plane wave with momentum $\mathbf{k}$, we can write the interaction Hamiltonian as,

$$H_{\text{int}} \simeq iG_{qq\pi}\int d^3\mathbf{r}\bar{q}(\mathbf{r})\gamma^5 q(\mathbf{r})\exp(i\mathbf{k}\cdot\mathbf{r})\tau_j. \tag{15}$$

Now from (6) we can also similarly obtain

$$H_{\text{int}} \simeq \frac{i}{f_\pi}\int d^3\mathbf{r}\bar{q}(\mathbf{r})\gamma^5 q(\mathbf{r})G(r)\exp(i\mathbf{k}\cdot\mathbf{r})\tau_j. \tag{16}$$

Then comparing (15) and (16), we can obtain a much simpler estimate of $G_{qq\pi}$ as,

$$G_{qq\pi} = \frac{1}{f_\pi} \frac{\int d^3\mathbf{r} G(r) \bar{q}(\mathbf{r}) \gamma^5 q(\mathbf{r}) \exp(i\mathbf{k}\cdot\mathbf{r})}{\int d^3\mathbf{r} \bar{q}(\mathbf{r}) \gamma^5 q(\mathbf{r}) \exp(i\mathbf{k}\cdot\mathbf{r})}. \tag{17}$$

Now taking the $1S\frac{1}{2}$ spatial wave functions of the quarks as given in (9) and (10), we obtain,

$$G_{qq\pi} = \frac{1}{f_\pi} \frac{\int_0^\infty dr\, r^{5/2} G(r) J_{3/2}(kr) \exp(-r^2/r_0^2)}{\int_0^\infty dr\, r^{5/2} J_{3/2}(kr) \exp(-r^2/r_0^2)}. \tag{18}$$

Using the standard integral result,

$$\int_0^\infty dX\, X^\mu \exp(-\alpha X^2) J_\nu(\beta X) = \frac{\beta^\nu}{2^{\nu+1}} \alpha^{-\frac{1}{2}(\mu+\nu+1)}$$

$$\frac{\Gamma\left(\frac{\mu+\nu+1}{2}\right)}{\Gamma(\nu+1)} \exp(-\beta^2/4\alpha)\; F\left(\frac{\nu-\mu+1}{2}, \nu+1, \beta^2/4\alpha\right), \tag{19}$$

expression (18) can be simplified to give,

$$G_{qq\pi} = \frac{1}{f_\pi}\left[\frac{5}{4} a r_0^2 \left(1 - \frac{1}{10} k^2 r_0^2\right) + m_q\right]. \tag{20}$$

Then with a soft pion approximation we can approximate

$$G_{qq\pi} \simeq \frac{1}{f_\pi}\left[\frac{5 a r_0^2}{4} + m_q\right] = \frac{(5E_q + 7m_q)}{12 f_\pi} \tag{21}$$

so that, the quark-pion coupling constant comes out as,

$$\frac{G^2_{qq\pi}}{4\pi} = 0{\cdot}49. \tag{22}$$

This is in good agreement with the estimate obtained by Suzuki and Bhaduri (1983) from the ratio $\Gamma(\bar{\rho} \to \pi\bar{\gamma})/\Gamma(\rho^\circ \to e^+ e^-)$.

A better estimate of the quark-pion coupling constant can be made in a more reasonable way by looking at the NN_π-vertex. For the interaction Lagrangian density (6), the NN_π-vertex function, in a point pion approximation, can be written as,

$$V_j^{N'N}(\mathbf{k}) = -\frac{i}{f_\pi}(2\omega_k)^{-\frac{1}{2}} \int d^3\mathbf{r}\, G(r) \exp(i\mathbf{k}\cdot\mathbf{r}) \langle N'/\sum_q \bar{q}(\mathbf{r})\gamma^5 q(\mathbf{r})\tau_j|N\rangle. \tag{23}$$

Here j is the isospin index and $\omega_k = (k^2 + m_\pi^2)^{1/2}$ is the pion energy. Since for the NN_π-vertex, the spatial orbits of all the quarks in the initial and final nucleon state are the same $1S\frac{1}{2}$, using (10) and (11) in (23), we can obtain,

$$V_j^{N'N}(\mathbf{k}) = \frac{i}{\sqrt{2} f_\pi}(2\omega_k)^{-\frac{1}{2}} \frac{N_q^2 \sqrt{\pi}\, k^{-3/2}}{\lambda_q r_0^4} I(k) \langle N'|\sum_q (\sigma_q\cdot\mathbf{k})\tau_j|N\rangle.$$

where,

$$I(k) = 2\int_0^\infty \mathrm{d}r\, r^{5/2}\, G(r)\, J_{3/2}(kr) \exp(-r^2/r_0^2). \tag{24}$$

Using (19) for $I(k)$ we can write

$$\begin{aligned} V_j^{N'N}(\mathbf{k}) &= \frac{i}{2f_\pi}(2\omega_k)^{-\frac{1}{2}}\left[\frac{3}{5}g_A u(k)\right]\langle N'|\sum_q(\sigma_q\cdot\mathbf{k})\tau_j|N\rangle \\ &= \langle N'|\Sigma v_j^{qq}(\mathbf{k})|N\rangle. \end{aligned} \tag{25}$$

Hence we obtain the quark-pion vertex operator function

$$v_j^{qq}(\mathbf{k}) = \frac{i}{2f_\pi}(2\omega_k)^{-\frac{1}{2}}\left[\frac{3}{5}g_A u(k)\right](\sigma_q\cdot\mathbf{k})\tau_j, \tag{26}$$

where, g_A is the axial vector coupling constant which can be obtained in this model as (Barik *et al* 1985)

$$g_A = \frac{5}{9}\left(\frac{5E_q + 7m_q}{3E_q + m_q}\right)$$

and $u(k)$ is the form factor given by,

$$u(k) = \left[1 - \frac{(E_q - m_q)k^2 r_0^2}{2(5E_q + 7m_q)}\right]\exp(-k^2 r_0^2/4), \tag{27}$$

which for $k \to 0$ reduces to one. Now comparing (26) with the corresponding expression in Chew-Low type model (Chew 1954; Chew and Low 1955; Wick 1955), which is written in terms of the pseudo-vector $qq\pi$-coupling $f_{qq\pi}$ as,

$$v_j^{qq}(\mathbf{k}) = i(2\omega_k)^{-\frac{1}{2}}\sqrt{4\pi}\,(f_{qq\pi}/m_\pi)\,u(k)\,(\sigma_q\cdot\mathbf{k})\tau_j, \tag{28}$$

we have,

$$\sqrt{4\pi}\,(f_{qq\pi}/m_\pi) = \frac{1}{2f_\pi}(3g_A/5). \tag{29}$$

This is the equivalent Goldberger-Treiman relation, which with the familiar equivalence of pseudo-scalar and pseudo-vector coupling constants yields,

$$(G_{qq\pi}/2M_q) = \sqrt{4\pi}\,(f_{qq\pi}/m_\pi) = \frac{1}{2f_\pi}(3g_A/5), \tag{30}$$

where, M_q is the effective constituent quark mass taken as one-third of the $N \to \Delta$ spin $\to$ isospin average mass *i.e.* 390 MeV.

Then we have,

$$\begin{aligned} \frac{G_{qq\pi}^2}{4\pi} &= \frac{1}{4\pi}\left(\frac{M_q}{f_\pi}\right)^2\left[\frac{3}{5}g_A\right]^2, \\ &\simeq 0{\cdot}449. \end{aligned} \tag{31}$$

However if we consider the CM correction for g_A, then using the corrected g_A value from (13), we get,

$$G_{qq\pi}^2/4\pi \simeq 0{\cdot}524. \tag{32}$$

The pion coupling to the quarks has been considered so far to be a point particle. But one can introduce the finite size of the pion according to the prescriptions of (de Kam and Pirner 1982; Phatak 1983) by visualizing the pion absorption as a process in which a quark of the bare nucleon is replaced by a quark of the pion after it is annihilated by the antiquark of the pion. Then the NN_π-vertex function can be written as,

$$V_j^{N'N}(\mathbf{k}) = -\frac{i}{f_\pi}(2\omega_k)^{-\frac{1}{2}}\int d^3 r d^3\rho G(r)\exp(i\mathbf{k}\cdot\mathbf{r})P(\rho)$$

$$\langle N'|\sum_q \bar{q}(\mathbf{r}+\rho/2)\gamma^5\tau_j q(\mathbf{r}-\rho/2)|N\rangle. \tag{33}$$

Here 'ρ' is the $q\bar{q}$-separation distance and $P(\rho)$ is the probability function for finding such a $q\bar{q}$-pair in the pion. Introducing a size parameter R_π for the pion, one can choose,

$$P(\rho) = \frac{3}{4\pi}\frac{1}{R_\pi^3}\theta(R_\pi - \rho). \tag{34}$$

However with a reasonable approximation to replace $\boldsymbol{\sigma}\cdot(\mathbf{r}\pm\rho/2)/|\mathbf{r}\pm\rho/2|$ by $\boldsymbol{\sigma}\cdot\hat{r}$ and $(|\mathbf{r}+\rho/2|+|\mathbf{r}-\rho/2|)\simeq 2r$, (33) can be simplified to give the quark-pion vertex operator function

$$v_j^{qq}(\mathbf{k}) \simeq \frac{i}{2f_\pi}(2\omega_k)^{-1/2}\frac{3g_A}{5}u(k)F(R_\pi)(\sigma_q\cdot\mathbf{k})\tau_j,$$

where,

$$\begin{aligned} F(R_\pi) &= 4\pi\int_0^\infty d\rho\,\rho^2 P(\rho)\exp(-\rho^2/4r_0^2), \\ &= \frac{12r_0^3}{R_\pi^3}\gamma(3/2, R_\pi^2/4r_0^2), \\ &\simeq \left[1+\frac{1}{10}(R_\pi/r_0)^2\right]\exp(-R_\pi^2/4r_0^2). \end{aligned} \tag{35}$$

Then proceeding as before and taking the CM correction into account for g_A in (35), we can obtain,

$$\frac{G_{qq\pi}^2}{4\pi} = \left(\frac{G_{qq\pi}^2}{4\pi}\right)_0 F^2(R_\pi), \tag{36}$$

when $(G_{qq\pi}^2/4\pi)_0$ is the coupling constant obtained with point pion approximation (equation (32)). It is obvious that the effect of the finite size is to reduce the coupling constant depending upon the size-parameter $R_\pi\cdot R_\pi$ is expected not to be the pion charge radius, but rather the radius of the $q\bar{q}$-pair distribution within the pion, which is observed to be considerably smaller (Oset *et al* 1984) than the charge radius of the pion. According to the estimate of Brodsky and Lepage (Brodsky and Lepage 1981; Brodsky 1982) $R_\pi \simeq 0.4$ fm or smaller. Similar values are also obtained in a microscopic chiral model of the pion (Bernard *et al* 1984). Therefore taking a range of values for R_π as 0·4, 0·3 and 0·2 fm respectively we obtain,

$$G_{qq\pi}^2/4\pi = [0.463, 0.489, 0.508]. \tag{37}$$

4. Conclusion

The coupling strength $G_{qq\pi}^2/4\pi$ determined by Suzuki and Bhaduri (1983) from the vector meson decay ratios with a static approximation can be cited here for a comparison. They obtain it as about (i) 0·4 from $\Gamma(\bar{\rho} \to \pi\bar{\gamma})/\Gamma(\bar{\rho} \to e^+e^-)$ (ii) 0·5 from $\Gamma(\omega \to \pi^\circ\gamma)/\Gamma(\omega \to e^+e^-)$ and (iii) 0·88 from $\Gamma(\rho \to \pi^-\pi^\circ)/\Gamma(\rho \to \pi^-\gamma)$. We find that except for case (iii), the values of the quark-pion coupling constant extracted from the experimental vector meson decay widths are quite comparable with our theoretical estimates in this model given in (22), (31), (32) and (37). However from the observations of Hendry (1982) examining the decay of excited N and Δ, one obtains $G_{qq\pi}^2/4\pi \simeq 1·1$, which is much larger than our estimate.

The nucleon-pion coupling constant $(G_{NN_\pi}^2/4\pi)_0$ in this model comes out to be of the order of 13·1 which compares well with the experimental value 14·4. The finite size of the pion, however, reduces the values of $(G_{NN_\pi}^2/4\pi)$ to 11·59, 12·23 and 12·71 for R_π equals 0·4, 0·3 and 0·2 fm respectively.

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