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Ground state baryon magnetic moments and nucleon axial vector coupling

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Abstract. Ground-state baryon magnetic moments and nucleon axial vector coupling are calculated using QCD inspired configuration mixing and relativistic corrections. Unlike earlier attempts, we incorporate a natural mass scale for quarks, taken as one third the nucleon mass for up and down quarks, and the strange quark mass suggested by the Lipkin's sum rule. In the parameter-free non-relativistic limit, we find a fairly good fit, which improves upon including relativistic corrections.

Keywords. Baryon magnetic moments; configuration mixing; nucleon axial vector coupling; relativistic effects.

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1. Introduction

Magnetic moments, being second only to masses amongst the observable static properties of 'elementary' particles, are valuable for understanding the dynamical structure of a constituent model. However, at the moment, an overall consistent explanation covering the whole lot of the experimentally well-known cases is still eluding us. Besides there is the problem of a simultaneous fit to the proton magnetic moment (μ_p) and nucleon axial vector coupling (G_A/G_V).

After the qualitative predictions of the simple quark model (SQM), the ground state baryon magnetic moments have attracted a lot of attention in recent years. The renewed interest in the magnetic moments has been caused primarily by the availability of accurate data for the hyperons (Overseth 1980; Cox *et al* 1981) and failure of SQM to make quantitative predictions. Beyond SQM, many attempts have been made by incorporating certain refinements suggested by QCD into the general premises of SQM. The genesis of these so called dynamical models, wherein quark confinement and q-q hyperfine interactions are taken into account, lies in the model of DGG (De Rujula *et al* 1975). They elegantly reproduced, for the first time, the Λ-magnetic moment besides retaining the qualitative results of SU(6). Following DGG, there have been several attempts involving various corrections. Franklin (1979, 1980) has obtained certain sum rules which highlight the importance of symmetry breaking effects. For improvements over SQM, he has stressed the importance of non-static effects, such as exchange currents, orbital exchange effects, and relativistic modifications. Orbital exchange effects have also been included by Lichtenberg (1981) with a slight improvement in the results but at the cost of additional parameters. Isgur and Karl (1980) have calculated magnetic moments in their model, wherein the octet wave-functions are obtained by perturbing a universal (spin flavour independent) harmonic confinement through the QCD inspired one gluon exchange spin-spin forces. They have also incorporated relativistic corrections by invoking the bag model concept of the dependence of the magnetic moment on the radius of the particular baryon. However, the corrections are not dealt with explicitly. Their results show definite improvement over SQM but $\mu(\Sigma)$ and $\mu(\Xi)$ are far from the data. Other authors (Bohm *et al* 1982) have considered the effect of unequal quark masses in addition to colour spin-spin forces for generating the configuration mixing. Their encouraging results once more indicate the importance of configuration mixing. However, besides the configuration mixing and relativistic effects, the latter being important for the case of axial vector coupling as well (Bogoliubov 1968), there is the crucial question of effective quark masses which gives rise to a mass scale for the magnetic moments through the magnetic moment operator. The mass scale problem is very important because the quark masses, if not determined from elsewhere, enter into the calculations as free parameters with their values bearing no simple relationship with baryon masses. This problem has been discussed in detail by Lipkin (1979) and he finds no simple solution to it. Therefore, with the objective of assessing the role played by QCD inspired configuration mixing, we present our calculation of the magnetic moments of the ground state baryons and nucleon axial vector coupling. In doing so, we have taken care of the appropriateness of quark masses and relativistic corrections. To make the paper reasonably self contained, we reproduce in § 2 some essential components used for calculating the magnetic moments. Section 3 gives the calculations and results and § 4 pertains to axial vector coupling. Discussion of the results is given in § 5.

2. Essential components

2.1 *Masses*

In the absence of any dynamical clues as to which effective quark masses should be used, it is worthwhile to consider De Grand's (1980) suggestion. While comparing non-relativistic oscillator (HO) with the bag model, he has noted that in the denominator of the magnetic moment operator it should be the total energy of the quark rather than its "mass". Therefore, guided by this simple criterion, we have assigned a natural quark mass scale for the u and d quark; *viz* $m_u = m_d = 0{\cdot}313$ GeV, which is one third of the nucleon mass. Mass of the strange quark is fixed through Lipkin's (1978) sum rule.

$$m_s - m_u = m_\Lambda - m_p = 0{\cdot}177 \text{ GeV}. \tag{1}$$

This fixes the masses completely and they no longer appear as free parameters.

2.2 *Configuration mixing*

The chromodynamic spin-spin hyperfine interaction mixes the $[56, 0^+]$ HO ground state with $[56', 0^+]$, $[70, 0^+]$ and $[70, 2^+]$ multiplet states at $N = 2$ level. The D-wave admixture is too small (Isgur and Karl 1982) to give rise to any appreciable effects. The non-trivial, extra minimal mixing* piece $[56', 0^+]$, though not affecting the spin-

* The so called minimal non-trivial mixing consisting of $[70, 0^+]$ admixture alone, has been successfully used elsewhere (Gupta and Mitra 1978; Le Yaouanc *et al* 1978).

isospin structure of the ground state, has an appreciable magnitude to give significant contribution. Therefore, the following mixed wave function is used

$$|\Psi_{\text{had}}, 1/2^+\rangle = |a\psi(56, 0^+) + b\psi(56', 0^+) + C\psi(70, 0^+)\rangle. \qquad (2)$$

Details of the wavefunctions specifying the mixing co-efficients (Koniuk and Isgur 1980) a, b, c are given in the appendix. For sake of convenience the small decuplet admixtures have been dropped.

2.3 *Relativistic effects*

Relativistic effects are simulated by considering the spin-wavefunctions in (2) as combinations of free-quark Dirac spinors. This leads to the following magnetic moment operator

$$M = \sum_{i=1}^{3} \frac{1}{2m_i} \hat{e}(i) \left[\left\{ \left(1 - \frac{\rho p^2(i)}{(2m_i)^2}\right) + 2m_i k_q \left(1 - \frac{2p^2(i)}{(2m_i)^2}\right) \right\} \sigma_+(i) + \frac{2}{3} \frac{p_+(i)}{2m_i} \left\{ \frac{p_-(j)}{2m_j} \sigma_+(j) + \frac{p_-(k)}{2m_k} \sigma_+(k) \right\} \right] \qquad (3)$$

for the transverse electromagnetic current (Le Yaouanc *et al* 1977), i, j, k being cyclic. The effect of Dirac spinors is already built into M; so it now acts on the usual Pauli spinors. ρ is a factor which tells us about the extent of relativistic corrections.

3. Calculations and results

After the specification of spin, isospin, and spatial wavefunctions (appendix) the calculation of magnetic moments is straightforward. The contributions proportional to the second term in the square parenthesis in (3) above are quite small compared to the rest of the terms, so we have considerably simplified the algebra by ignoring these terms. In terms of the expectation values the hadron magnetic moment acquires the expression

$$\mu_{\text{had}} = \langle M \rangle = \sum_{i=1}^{3} \langle \hat{e}(i) \rangle \left\{ \frac{1}{2m_i} - \frac{\rho}{4} \times \frac{1}{2m_i^3} \langle p^2(i) \rangle + k_q - \frac{k_q}{2m_i^2} \langle p^2(i) \rangle \right\} \times \langle \sigma_+(i) \rangle. \qquad (4)$$

The first term in the above expression gives, for the non-relativistic, Dirac magnetic moment,

$$\mu_{\text{had}}^{\text{NR}} = \left(\frac{a^2}{2} + \frac{5}{6} b^2 + \frac{5}{12} c^2 - \frac{1}{\sqrt{3}} ab \right) \left[C_3 + \frac{4}{3} D - \frac{1}{3} D_3 \right] - \left(\frac{a^2}{2} + \frac{5}{6} b^2 - \frac{5}{12} c^2 - \frac{1}{\sqrt{3}} ab \right) \frac{4}{\sqrt{3}} Y + \frac{5}{12} c^2 \left(D_3 + \frac{4}{3} C - \frac{1}{3} C_3 - \frac{4}{\sqrt{3}} Y \right) + \left(\frac{2}{3\sqrt{2}} bc - \frac{1}{\sqrt{6}} ac \right) \left[C_3 + \frac{1}{3} D_3 - \frac{4}{3} D \right]. \qquad (5)$$

In writing down the above expression, we have used the symbols $C, D, Y\ldots$ etc. for the sake of convenience. The actual expressions for these symbols are given in table 1. Subsequently, in table 2, we give the results in terms of the relativistic correction factor ρ, the anomalous part parameter k_q, and the HO strength parameter α^2. The results given are for two sets of quark masses *viz.*

(i) $m_1 = m_2 = 3{\cdot}195\ \mathrm{GeV}^{-1}$, $m_3 = 2{\cdot}041\ \mathrm{GeV}^{-1}$, and
(ii) $m_1 = 3{\cdot}226\ \mathrm{GeV}^{-1}$, $m_2 = 3{\cdot}165\ \mathrm{GeV}^{-1}$, and $m_3 = 2.053\ \mathrm{GeV}^{-1}$.

The first set, neglecting small isospin breaking due to u and d quark mass difference, corresponds to the natural quark mass scale of $m_u = m_d = 0{\cdot}313$ GeV, while m_s fixed from Lipkin's sum rule is taken to be 0·490 GeV. In the second set a small $u-d$ quark mass difference of 6 MeV (Isgur 1980) is introduced so that we have $m_u = 0{\cdot}313$ GeV, $m_d = 0{\cdot}316$ GeV, and $m_s = 0{\cdot}487$ GeV. After carefully choosing the parameters, the final numerical estimates are given in table 3.

4. Axial vector coupling

Nucleon axial vector coupling (G_A/G_V) is given by the non-relativistic HO model with unbroken SU(6) to be $\sim 1{\cdot}667$, a result too large compared to the experimental value (Particle Data Group 1973). It is known that the relativistic effects (Bogoliubov 1968) as well as the configuration mixing (Mitra and Sen 1974) bring down its value. That the relativistic effects bring down the result can be seen by noting that the bag model, assuming massless quarks, predicts G_A/G_V equal to 1·09 (De Grand *et al* 1975; Chodos *et al* 1974).

Including relativistic corrections, the nucleon axial vector coupling is given by

$$G_A/G_V = \frac{\left\langle p\uparrow \left| \sum_i \tau_i^+ S_i^z \left(1 - \frac{\rho p^2(i)}{2m_i^2}\right) \right| n\uparrow \right\rangle}{\left\langle p\uparrow \left| \sum_i \tau_i^+ \right| n\uparrow \right\rangle} \tag{6}$$

using the iso-spin and spin overlaps given in the appendix, we obtain for G_A/G_V the expressions given in table 4. Also reproduced therein are the corresponding expressions for μ_p. The final results are presented in table 5.

5. Discussion of results

Before we discuss our results, it is necessary to ascertain the number of effective parameters involved in our calculations. For the non-relativisitc calculations, the results are parameter free since the quark masses and mixing angles have already been fixed. Coming to the relativistic calculations, there are three effective parameters; ρ, k_q and α^2. α, the spring constant gets involved through the momentum dependent part in the magnetic moment operator. While the other two parameters have already been specified the spring constant is related to the characteristic radius in the case of HO wavefunctions.

The α^2 values used in literature show a wide variation, most falling within the range

Table 1. Flavour matrix elements $C_i = \langle \phi' \left| \frac{\hat{e}(i)}{2m_i} \right| \phi' \rangle$, $D_i = \langle \phi'' \left| \frac{\hat{e}(i)}{2m_i} \right| \phi'' \rangle$, and $Y_i = \langle \phi' \left| \frac{\hat{e}(i)}{2m_i} \right| \phi'' \rangle$ with $m_1 = 1/m_u$, $m_2 = 1/m_d$, and $m_3 = 1/m_s^{++}$

Particle	$C_1 = C_2 = C$	C_3	$D_1 = D_2 = D$	D_3	$Y_1 = Y_2 = Y^+$
p	$1/6(m_1 - \frac{1}{2}m_2)$	$1/3(m_1)$	$1/18(5m_1 - \frac{1}{2}m_2)$	$1/9(m_1 - m_2)$	$-1/6\sqrt{3}(m_1 + \frac{1}{2}m_2)$
n	$1/6(m_1 - \frac{1}{2}m_2)$	$-1/6(m_2)$	$1/18(m_1 - 5/2m_2)$	$1/18(4m_1 - m_2)$	$1/6\sqrt{3}(m_1 + \frac{1}{2}m_2)$
Λ	$1/36(5m_1 - 5/2m_2 - m_3)$	$1/18(m_1 - \frac{1}{2}m_2 - 2m_3)$	$1/12(m_1 - \frac{1}{2}m_2 - m_3)$	$1/12(2m_1 - m_2)$	$1/12\sqrt{3}(m_1 + m_3 - \frac{1}{2}m_2)$
Σ^0	$1/12(m_1 - \frac{1}{2}m_2 - m_3)$	$1/3(m_1)$	$1/36(5m_1 - 5/2m_2 - m_3)$	$1/9(m_1 - m_3)$	$1/6\sqrt{12}(m_1 + m_3 - \frac{1}{2}m_2)$
Σ^+	$1/6(m_1 - \frac{1}{2}m_3)$	$1/6(m_1 - \frac{1}{2}m_2)$	$1/18(5m_1 - \frac{1}{2}m_3)$	$1/36(2m_1 - m_2 - 4m_3)$	$-1/6\sqrt{3}(m_1 + \frac{1}{2}m_3)$
Σ^-	$-1/12(m_2 + m_3)$	$-1/6(m_2)$	$-1/36(5m_2 + m_3)$	$-1/18(m_2 + 2m_3)$	$1/12\sqrt{3}(m_2 - m_3)$
Ξ^0	$1/6(m_1 - \frac{1}{2}m_3)$	$-1/6(m_3)$	$1/18(m_1 - 5/2m_3)$	$1/18(4m_1 - m_3)$	$1/6\sqrt{3}(m_1 + \frac{1}{2}m_3)$
Ξ^-	$-1/12(m_2 + m_3)$	$-1/6(m_3)$	$-1/36(m_2 + 5m_3)$	$-1/18(2m_2 + m_3)$	$1/12\sqrt{3}(m_3 - m_2)$

(+) Y_3, overlap being zero for all. (+ +) All m in the numerator.

Table 2. Ground state baryon octet magnetic moment matrix elements in terms of parameters α^2, k_q, and ρ for two sets of quark mass values.

Particle	Set 1: $m_u = m_d = 0{\cdot}313$; $m_s = 0{\cdot}490$	Set 2: $m_u = 0{\cdot}313$; $m_d = 0{\cdot}316$; $m_s = 0{\cdot}487$
p	$1{\cdot}518 - 2{\cdot}585\rho\alpha^2 + 0{\cdot}950k_q - 3{\cdot}236k_q\alpha^2$	$1{\cdot}529 - 2{\cdot}645\rho\alpha^2 + 0{\cdot}950k_q - 3{\cdot}28k_q\alpha^2$
n	$-0{\cdot}984 + 1{\cdot}680\rho\alpha^2 - 0{\cdot}616k_q + 2{\cdot}113k_q\alpha^2$	$-0{\cdot}981 + 1{\cdot}663\rho\alpha^2 - 0{\cdot}616k_q + 2{\cdot}088k_q\alpha^2$
Λ	$-0{\cdot}309 + 0{\cdot}210\rho\alpha^2 - 0{\cdot}310k_q + 0{\cdot}426k_q\alpha^2$	$-0{\cdot}311 + 0{\cdot}209\rho\alpha^2 - 0{\cdot}310k_q + 0{\cdot}430k_q\alpha^2$
Σ^0	$0{\cdot}451 - 0{\cdot}772\rho\alpha^2 + 0{\cdot}320k_q - 1{\cdot}035k_q\alpha^2$	$0{\cdot}462 - 0{\cdot}833\rho\alpha^2 + 0{\cdot}320k_q - 1{\cdot}086k_q\alpha^2$
Σ^+	$1.509 - 2.811\rho\alpha^2 + 0.982k_q - 3{\cdot}587k_q\alpha^2$	$1{\cdot}523 - 2{\cdot}892\rho\alpha^2 + 0{\cdot}982k_q - 3{\cdot}655k_q\alpha^2$
Σ^-	$-0{\cdot}590 + 1{\cdot}266\rho\alpha^2 - 0{\cdot}332k_q + 1{\cdot}517k_q\alpha^2$	$-0{\cdot}583 + 1{\cdot}226\rho\alpha^2 - 0{\cdot}322k_q + 1{\cdot}483k_q\alpha^2$
Ξ^0	$-0{\cdot}778 + 1{\cdot}008\rho\alpha^2 - 0{\cdot}644k_q + 1{\cdot}497k_q\alpha^2$	$-0{\cdot}784 + 1{\cdot}034\rho\alpha^2 - 0{\cdot}644k_q + 1{\cdot}521k_q\alpha^2$
Ξ^-	$-0{\cdot}277 - 0{\cdot}003\rho\alpha^2 - 0{\cdot}330k_q + 0{\cdot}231k_q\alpha^2$	$-0{\cdot}281 + 0{\cdot}012\rho\alpha^2 - 0{\cdot}330k_q + 0{\cdot}247k_q\alpha^2$

Table 3. Numerical estimates of the ground state baryon magnetic moments in nucleon magnetons. Sets I and II identified as in the preceding table.

	Mixed non-relativistic		Mixed relativistic ($k_q = 0{\cdot}075$, $\alpha^2 = 0{\cdot}17\,\mathrm{GeV}^2$; $\rho = 0{\cdot}138$)				
Particle	I	II	I	II	Isgur *et al* (1980)	Bohm *et al* (1982)	Experimental
p	2·848	2·868	2·790	2·807	2·85	2·83	2.793
n	−1·846	−1.840	−1·842	−1·804	−1·85	−1·83	−1.913
Λ	−0·580	−0·583	−0·604	−0·608	−0·61	−0·58	$-0{\cdot}613 \pm 0{\cdot}004$
Σ^+	2·831	2·857	2·760	2·781	2·54	2·72	$2{\cdot}33 \pm 0{\cdot}13$
Σ^-	−1·107	−1·094	−1·062	−1·051	−1·00	−1·15	$-1{\cdot}41 \pm 0{\cdot}25$
Ξ^0	−1·460	−1·471	−1·470	−1·480	−1·20	1·41	$-1{\cdot}250 \pm 0{\cdot}014$
Ξ^-	−0·520	−0·527	−0·561	−0·567	−0·43	−0·63	$-0{\cdot}75 \pm 0{\cdot}07$
Σ^0	0·846	0·867	0·832	0·850	—	—	—

Table 4. μ_p and G_A/G_V corresponding to (a) $m_u = m_d = 0{\cdot}313$ GeV and (b) $m_u = m_d = 0{\cdot}316$ GeV, in terms of ρ and α^2.

Case		$\mu_p/1{\cdot}876$	G_A/G_V
Mixed relativistic	(a)	$1{\cdot}518 - 2{\cdot}585\rho\alpha^2 + 0{\cdot}950k_q - 3{\cdot}236k_q\alpha^2$	$1{\cdot}617 - 15{\cdot}805\rho\alpha^2$
	(b)	$1{\cdot}529 - 2{\cdot}845\rho\alpha^2 + 0{\cdot}950k_q - 3{\cdot}286k_q\alpha^2$	$1{\cdot}617 - 15{\cdot}510\rho\alpha^2$
Mixed non-relativistic	(a)	1·518	1·617
	(b)	1·529	1·617
Unmixed relativistic	(a)	$1{\cdot}597 - 4{\cdot}077\rho\alpha^2 + k_q - 5{\cdot}104k_q\alpha^2$	$1{\cdot}667 - 8{\cdot}508\rho\alpha^2$
	(b)	$1{\cdot}610 - 4{\cdot}171\rho\alpha^2 + k_q - 5{\cdot}182k_q\alpha^2$	$1{\cdot}667 - 8{\cdot}347\rho\alpha^2$
Unmixed non-relativisitc	(a)	1·597	1·667
	(b)	1·610	1·667

Table 5. G_A/G_V for different values of parameters ρ and α^2; ρ having been fixed from the value of proton magnetic moment.

		Corresponding to $k_q = 0.075$			Corresponding to $k_q = 0.10$		
	ρ	0·293	0·138	0·094	0·352	0·161	0·103
	α^2	0·102	0·17	0·21	0·102	0·17	0·21
Mixed	(a)	1·144	1·245	1·305	1·050	1·184	1·275
relativistic	(b)	1·153	1·253	1·311	1·060	1·192	1·282
Unmixed	(a)	1·413	1·467	1·499	1·362	1·434	1·483
relativistic	(b)	1·418	1·471	1·502	1·367	1·439	1·486

0·1 to 0·2, though α^2 greater than 0·2 has also been used. Generally, in the low energy domain, for most of the static properties of hadrons, lower values of α^2 are found to be suitable (Faiman and Hendry 1968, 1969; Metcalf and Walker 1974; Copley *et al* 1969), while for dynamical properties $\alpha^2 \geqslant 0{\cdot}2$ is required (Ono 1976; Foster and Hughes 1981). Therefore, it is somewhat difficult to arrive at a well-defined, unique set of values for the parameters. Nevertheles, using the well determined experimental values of certain quantities, like μ_p, one can make a reasonable estimate of these parameters. After choosing $k_q \approx 0{\cdot}075$ and 0·1 (aiming at a best fit) and considering for α^2 the three possible values 0·102, 0·17 and 0·21, we have obtained the corresponding values for the parameter ρ by fitting to the proton magnetic moment. These sets of values for the parameters ρ, k_q and α^2 are given in table 5. The table also presents the respective values of G_A/G_V. A look at the table indicates that the best fit to G_A/G_V is given by the following set of parameter values:

$$\rho = 0{\cdot}138 \text{ and } \alpha^2 = 0{\cdot}17\,\text{GeV}^2 \text{ with } k_q = 0{\cdot}075.$$

The above choice of α^2 is partly dictated by its success in giving remarkable fit to the resonance photocouplings (Copley *et al* 1969).

The results in the unmixed case are much higher as compared to the case which includes full mixing. The mixed-relativistic case gives an excellent fit. The fit does deteriorate if corresponding to the same α^2 value a different ρ-value (for a different k_q) is chosen (table 5). Therefore, within its limitations, in the model which includes both mixing as well as relativistic corrections, we do obtain a simultaneous fit to both μ_p and G_A/G_V by adopting the set of parameters given above.

Subsequently, using the same set of parameters in expressions for the magnetic moments, we present our results alongwith the experimental values and the results of other recent attempts in table 3. Not much can be discerned from the individual magnetic moments. However, a general survey indicates that the mixed- relativistic case provides a fairly good description of the data. Though the values for the unmixed wavefunctions are not reproduced in the tables, we observed that these values are in general much higher than the data. Typical results in this case are the μ_p values given in the rows 3 and 4 of table 4. One can see that the wavefunction mixing does play an important role in bringing down the values in the right direction. But at the same time, the configuration mixing by itself is inadequate, necessitating thereby the inclusion of relativistic corrections even on phenomenological grounds.

A general survey of the tables 3–5 reveals that the introduction of the isospin violating mass difference does not help in improving the results. In fact they deteriorate, which is in accordance with the earlier findings (Bohm *et al* 1982), wherein a better fit is obtained by taking m_u greater than m_d.

Before we close, a few points must be mentioned. We see that our results, even the non-relativistic ones, are quite comparable to those obtained by other authors (Isgur and Karl 1980; Lichtenberg 1981; Bohm *et al* 1982); the non-relativistic results are more closer to Bohm's. However, our results are parameter free while Bohm *et al* use four explicit parameters apart from the mixing angles which are evaluated using different masses than those used in the magnetic moment calculations. Our relativistic results clearly show further improvement over the NR-case. As compared to Isgur *et al* our investigation has the particular advantage of choosing a natural quark mass scale besides including the relativistic corrections explicitly.

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Appendix

(i) Mixed wavefunctions used are given by the following expressions:

$$N \simeq 0{\cdot}90\, N_8/56, 0^+\rangle - 0{\cdot}34\, N_8/56', 0^+\rangle - 0{\cdot}27\, N/70, 0^+\rangle,$$
$$\Lambda \simeq 0{\cdot}93\, \Lambda_8/56, 0^+\rangle - 0{\cdot}30\, \Lambda_8/56, 0^+\rangle - 0{\cdot}20\, \Lambda_8/70, 0^+\rangle,$$
$$\Sigma \simeq 0{\cdot}97\, \Sigma_8/56, 0^+\rangle - 0{\cdot}18\, \Sigma_8/56, 0^+\rangle - 0{\cdot}16\, \Sigma_8/70, 0^+\rangle,$$
$$\Xi \simeq 0{\cdot}96\, \Xi_8/50, 0^+\rangle - 0{\cdot}25\, \Xi_8/56', 0^+\rangle - 0{\cdot}17\, \Xi_8/70, 0^+\rangle.$$

Where the multiplets are to be projected on to their SU(6) × 0(3) basis states the details of which are given hereunder:

$$\Psi_{(56,0^+)} = \frac{1}{\sqrt{2}}(\chi'\phi' + \chi''\phi'')\psi^s_{00},$$

$$\Psi_{(56',0^+)} = \frac{1}{\sqrt{2}}(\chi'\phi' + \chi''\phi'')\psi^{s'}_{20},$$

$$\Psi_{(70,0^+)} = 1/2\{(\chi'\phi' - \chi''\phi'')\psi''_{20} + (\chi'\phi'' + \chi''\phi')\psi'_{20}\}.$$

The spin and isospin wavefunctions χ's and ϕ's are well-known, while the spacial parts are reproduced below

$$\psi^s_{00} = \alpha^3\pi^{-3/2}\exp[-1/2\alpha^2(\rho^2 + \lambda^2)],$$

$$\psi^{s'}_{20} = \frac{\alpha^2}{\sqrt{3}}(\rho^2 + \lambda^2 - 3/\alpha^2)\psi^s_{00},$$

$$\psi''_{20} = \frac{\alpha^2}{\sqrt{3}}(\rho^2 - \lambda^2)\psi^s_{00}, \quad \text{and} \quad \psi'_{20} = \frac{2\alpha^2}{\sqrt{3}}(\rho\cdot\lambda)\psi^s_{00}.$$

(ii) The various spin, isospin, and space overlaps used in the calculations are given as under:

(a) *Spin overlaps*

$$\langle \chi'_{+1/2}|\sigma_+(1, \text{ or } 2)|\chi'_{-1/2}\rangle = 0$$
$$\langle \chi'_{+1/2}|\sigma_+(3)|\chi'_{-1/2}\rangle = 1$$
$$\langle \chi''_{+1/2}|\sigma_+(1 \text{ or } 2)|\chi''_{-1/2}\rangle = 2/3$$
$$\langle \chi''_{+1/2}|\sigma_+(3)|\chi''_{-1/2}\rangle = -1/3$$
$$\langle \chi'_{+1/2}|\sigma_+(1)|\chi''_{-1/2}\rangle = -\langle \chi'_{+1/2}|\sigma_+(2)|\chi''_{-1/2}\rangle = -1/\sqrt{3}$$
$$\langle \chi'_{+1/2}|\sigma_+(3)|\chi''_{-1/2}\rangle = 0.$$

(b) *Space overlaps*

$$\langle \psi^s|p^2_{1,2,3}|\psi^s\rangle = \alpha^2$$
$$\langle \psi^{s'}|p^2_{1,2,3}|\psi^{s'}\rangle = \langle \psi''|p^2_{1,2,3}|\psi''\rangle = \langle \psi'|p^2_{1,2,3}|\psi'\rangle = \tfrac{5}{3}\alpha^2$$
$$\langle \psi^s|p^2_{1,2,3}|\psi^{s'}\rangle = -2\langle \psi^s|p^2_{1,2}|\psi''\rangle = \langle \psi^s|p^2_3|\psi''\rangle = \frac{1}{\sqrt{3}}\alpha^2$$
$$\langle \psi^{s'}|p^2_3|\psi''\rangle = -2\langle \psi^{s'}|p^2_{1,2}|\psi''\rangle = 2/3\alpha^2$$

$|\psi'\rangle$ gives zero overlaps for the crossed terms.

(c) Spin, isospin overlaps- used in G_A/G_V calculations.

	i value		
Overlap	1	2	3
$\langle \phi''_p|\tau_i^+|\phi''_n\rangle$	2/3	2/3	−1/3
$\langle \phi''_p|\tau_i^+|\phi'_n\rangle$	$-1/\sqrt{3}$	$1/\sqrt{3}$	0
$\langle \phi'_p|\tau_i^+|\phi'_n\rangle$	0	0	1
$\langle \phi'_p|\tau_i^+|\phi''_n\rangle$	$-1/\sqrt{3}$	$1\sqrt{3}$	0

Spin overlaps for the operator S_i^z are same as above.

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