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Pion-proton elastic scattering at 200 GeV/c

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Abstract. Recent measurements of differential and elastic cross-sections, slope parameters and ratios of the real and imaginary parts of the forward scattering amplitudes for pion-proton elastic scattering at 200 GeV/c have been fitted by using a simple Regge pole model with phenomenological residue functions. The computed results for total cross-sections have also been compared with the experimental data.

Keywords. Pion-proton elastic scattering; Regge pole model; differential cross-sections; elastic cross-sections.

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Recently, Schiz *et al* (1981) made a high-statistics study of $\pi^+ p$ and $\pi^- p$ elastic scatterings at 200 GeV/c incident momentum. The $\pi^+ p$ and $\pi^- p$ distributions contain $2{\cdot}22 \times 10^5$, and $4{\cdot}28 \times 10^5$ events, respectively. The t ranges from $-0{\cdot}021$ to $-0{\cdot}665$ $(\mathrm{GeV/c})^2$ (scattering angles from approximately 0·7–4 mrad). Thus in a single experiment this group measured $d\sigma/dt$ over the small-to-intermediate-t region. The resulting $d\sigma/dt$ distributions for $\pi^+ p$ and $\pi^- p$ elastic scatterings are shown in figure 1. The errors shown are statistical only; there is an uncertainty in the overall normalization of 4%. This uncertainty is due mainly to the statistical error involved in the method of counting the number of incident-beam particles. The data have not been corrected so as to extrapolate to the optical point. The displayed $d\sigma/dt$ distributions have been corrected for inelastic contamination, for radiative effects and for plural nuclear scattering. Also the contribution due to Coulomb scattering (including the Coulomb-nuclear interference contribution) has been removed. The Coulomb correction is negligible above $-t \approx 0{\cdot}35$ $(\mathrm{GeV/c})^2$ and only slightly significant below. The data by Schiz *et al* (1981) display high statistical accuracy and bridge at a range not covered by any other experiment.

The data also allow an investigation of the t-dependence of the logarithmic slope parameter $B(t)$ defined by

$$B(t) \equiv \frac{d}{dt}\left(\ln \frac{d\sigma}{dt}\right).$$

Figure 2 shows measurements of $B(t)$ for $\pi^+ p$ and $\pi^- p$ by several experiments (Schiz *et al* 1981; Ayres *et al* 1976, 1977; Akerlof *et al* 1976; Barbiellini *et al* 1972; Bartener *et al* 1973; Schamberger *et al* 1978). The forward slopes measured by Schiz *et al* (1981) are derived from fits over smaller ranges of t than used by the other authors. This is

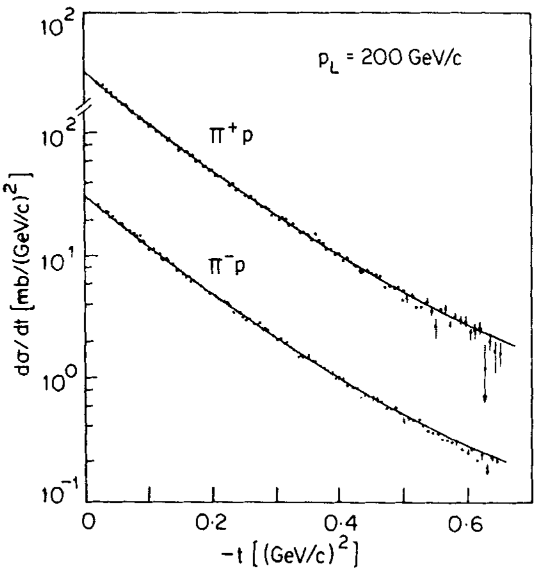

Figure 1. The differential cross-sections $d\sigma/dt$ for pion-proton elastic scattering at $P_L = 200$ GeV/c plotted against $-t$. The curves represent the predictions of the model described in the text.

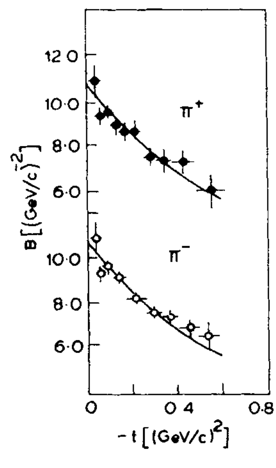

Figure 2. The slope parameters B plotted against $-t$. The curves represent the predictions of the model described in the text.

especially true for those measured by Ayres *et al* (1976, 1977) and Akerlof *et al* (1976) where fits to a quadratic form were performed over their full t range ($-0{\cdot}02$ to $-0{\cdot}40$ for Ayres *et al* 1976, 1977; $-0{\cdot}07$ to $-0{\cdot}08$ for Akerlof *et al* 1976). In the region $0{\cdot}10 < -t < 0{\cdot}60$ (GeV/c)^2, the local slope decreases with increasing t. From $0{\cdot}03 < -t$

< 0.10 $(\mathrm{GeV/c})^2$, the local slope is relatively flat; finally there is a sharp increase in the value in the region $0.02 < -t < 0.03$ $(\mathrm{GeV/c})^2$.

By integrating the $d\sigma/dt$ measurements over t, the total elastic cross-sections were computed. In fact, to calculate the contributions of those t-regions for which direct measurement were not made, Schiz *et al* (1981) used the results of the fits from which the local slopes were obtained. They found that when the $d\sigma/dt$ distributions were extrapolated to $t = 0$, these were consistent within experimental errors with the optical point. Table 1 presents the total elastic cross-sections and the ratios of the total elastic cross-section to the total cross-section at 200 GeV/c. The errors include, in addition to the statistical uncertainties, the systematic uncertainties due to extrapolation of the measured $d\sigma/dt$ distributions over unmeasured t regions. The results of Schiz *et al* (1981) agree well with those of Akerlof *et al* (1976) but are in poor agreement with the statistically more precise data of Ayres *et al* (1977).

It has been shown (Schiz *et al* 1981) that in the interval $0.025 < -t < 0.620$ $(\mathrm{GeV/c})^2$ the differential cross-section measurements can be explained by using the Chou-Yang (1968). We will show in this paper that these high statistical measurements of differential and integrated cross-sections, the local slope parameter values (Schiz *et al* 1981; Ayres *et al* 1976, 1977; Akerlof *et al* 1976) and the results for total cross-sections obtained earlier (Carroll *et al* 1974) can be fitted by using a simple Regge pole model with phenomenological residue functions.

The number of independent helicity amplitudes in the reactions $\pi^{\pm}p \to \pi^{\pm}p$ is 2. The forward peaks indicate that the non-flip amplitudes dominate in the forward direction. Thus for small $-t$ the behaviour of the scattering process is governed by the single non-flip amplitude. Moreover within experimental errors, the $d\sigma/dt$ data for $\pi^{\pm}p \to \pi^{\pm}p$ are equal at 200 GeV/c (Schiz *et al* 1981). This means that at this momentum, only the pomeron and the f trajectory can dominate the behaviour of these scattering processes, as contributions due to these trajectories do not change sign in going from π^+p to π^-p elastic scattering. If, at this high momentum, only the high lying pomeron with intercept 1·045 dominates the behaviour of the scattering process, then we may write the scattering amplitude for any one of these two processes as

$$T = \gamma(t)\xi(t)s^{\alpha(t)}\sqrt{mb}\ (\mathrm{GeV}),$$

where the scaling factor s_0 has been taken as 1 $(\mathrm{GeV})^2$. The differential and total cross-sections are then given by

$$d\sigma/dt = \frac{1}{sp^2}|T|^2 = \gamma^2(t)\frac{s^{2\alpha(t)-1}}{p^2}\ \mathrm{mb}/(\mathrm{GeV/c})^2$$

$$\sigma_T = \frac{4\sqrt{\pi}(0.3895)^{1/2}}{p\sqrt{s}}\ \mathrm{Im}\,T(s, t = 0)\ \mathrm{mb},$$

Table 1. Total elastic cross-sections at 200 GeV/c.

σ_{el} (mb)		σ_{el}/σ_T	
Expt.	Theoret.	Expt.	Theoret.
π^-p 3·32 ± 0·15	3·28	0·136 ± 0·006	0·133
π^+p 3·17 ± 0·06	3·28	0·133 ± 0·003	0·133

where the sine factor in the denominator of ξ has been absorbed in the residue function. The ratio of the real and imaginary parts of the forward elastic nuclear amplitude is given by

$$\rho = -\cot\frac{\pi\alpha}{2}.$$

The slope $B(t)$ at any t value is given by

$$B(t) = \frac{\mathrm{d}}{\mathrm{d}t}\left(\ln\frac{\mathrm{d}\sigma}{\mathrm{d}t}\right).$$

The integrated elastic cross-section is given by

$$\sigma_{\mathrm{int}} = \int\frac{\mathrm{d}\sigma}{\mathrm{d}t}\,\mathrm{d}t.$$

A very good fit with differential cross-section data for both the reactions is obtained by the following phenomenological choice of the residue function (Saleem and Aleem 1983a, b)

$$\gamma(t) = (1{\cdot}35e^{6{\cdot}5t} + 0{\cdot}79e^{1{\cdot}75t}).$$

The equation of the pomeron trajectory has been chosen as

$$\alpha(t) = 1{\cdot}045 + 0{\cdot}1\,t.$$

Figure 1 shows the differential cross-sections for $\pi^{\pm}p \to \pi^{\pm}p$ as measured by Schiz *et al* (1981). The solid curves represent the predictions of our simple Regge pole model.

Figure 2 shows the slope parameter $B(t)$ as a function of t as calculated by using the equation

$$B(t) = \frac{\mathrm{d}}{\mathrm{d}t}\left(\ln\frac{\mathrm{d}\sigma}{\mathrm{d}t}\right)$$

and the experimental values as obtained by various authors. Both for π^+p and π^-p elastic scatterings, the agreement is very good.

Table 2 shows the total cross-sections for $\pi^{\pm}p \to \pi^{\pm}p$ at 200 GeV/c. The theoretical values for π^+p as well as π^-p differ from the experimental results by about 2 and 4% respectively. This difference indicates the extent to which the Regge trajectories contribute.

The σ_{el} for $\pi^{\pm}p$ had been calculated in the range 0 to $-0{\cdot}65(\mathrm{GeV/c})^2$. The theoretical and experimental results are shown in table 1. The agreement is again good. The calculated ratio of σ_{el} to σ_T is also given in the same table and is consistent with experiment. The ratio of the real and imaginary parts of T at $t = 0$ is given in table 2. The predicted values agree with the experimental values. It may be pointed out that the

Table 2. Total cross-sections and ratio of the real and imaginary parts of the scattering amplitude at $t = 0$.

	σ_T(mb)		ρ	
	Expt.	Theoret.	Expt.	Theoret.
π^-p	23·84 ± 0·06*	24·75	0·053 ± 0.017*	0·07
π^+p	24·33 ± 0·04*	24·75	0·064 ± 0.02 *	0·07

*The data are from Carroll *et al* (1974) and Fajardo *et al* (1981).

exact contribution of the f trajectory at 200 GeV/c or high momenta to pion-proton scattering can be obtained only after precise measurements at momenta other than 200 GeV/c have been made. The choice of the intercept $\alpha(0) = 1{\cdot}045$ for the pomeron trajectory is imperative because only a pomeron trajectory with intercept greater than unity can lead to a rise in the total cross-section with momentum. This choice violates the Froissart bound. However this occurs only at energies which will not be available in the near future.

The energy dependence of σ_{el}/σ_T plays a very significant role to distinguish between different theoretical models for high energy pion-proton elastic scattering. We have calculated σ_{el} and σ_T for pion-proton elastic scattering at 200 GeV and higher energies. It is found that both σ_T and σ_{el} increase with energy. However σ_T increases slowly as compared with σ_{el} so that the ratio of σ_{el} to σ_T increases with energy. Regge field theory with critical pomeron predicts that σ_{el}/σ_T should slowly decrease asymptotically, whereas the geometrical scaling model projects a constant σ_{el}/σ_T at high energies. The experimental measurements at energies higher than 200 GeV have not yet been made. If, in agreement with the predictions of our model, σ_{el}/σ_T increases with energy, then we say that the pion expands with energy becoming also more absorbing in the central region of low impact parameter.

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