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Angular momentum and isospin properties of chiral fields

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Abstract. In a recently suggested variational quantum field theoretical approach the angular momentum and isospin properties of the pion field surrounding a quark bag are investigated using the Lagrangian of the Cloudy Bag Model.

Keywords. Angular momentum; isospin properties; chiral fields; variational quantum field; mean field approximation.

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1. Introduction

In the last few years various methods have become popular in order to solve chiral bag, chiral solition and Skyrme models for the nucleon and delta system. Most of these procedures (Chodos and Thorn 1975; Brown and Rho 1979; Vento *et al* 1980; Birse and Banerjee 1984; Kälbermann and Eisenberg 1984; Jackson and Rho 1983; Adkins *et al* 1983) seek solutions assuming from the start the so-called hedgehog form (Skyrme 1962) because the formal simplification allows an exact nonperturbative treatment of these models in the mean field approximation. Actually in a recent paper some justification of the hedgehog assumption has been given (Urbano and Goeke 1984c). There it has been shown that the hedgehog is one of a set of shapes of the pion mean field, coupled to the corresponding quark structure, which minimizes the energy of the Cloudy Bag Hamiltonian in the nucleon delta sector. The proof has been obtained in a recently formulated variational quantum field theoretical approach (VQF), whose details can be found in Urbano and Goeke (1984a, b). Unfortunately, the hedgehog approximation for the bag-pion system violates rotational and isospin symmetries. This means that the corresponding state is not an eigenstate of the operators of total angular momentum J and total isospin T. Thus it is important to investigate some of the J- and T-properties of the hedgehog and related solutions of the chiral bag system. This is the objective of this paper and the formalism used is the VQF-theory.

The paper is constructed as follows. Section 2 reviews the VQF-theory with particular emphasis on its mean field aspects, § 3 investigates some angular momentum and isospin properties of the hedgehog solution. The corresponding projection formalism is developed in § 4 with some numerical results for simplified quark-pion structures. A summary and an outlook are given in § 5.

2. The VQF-theory

The detailed formulation of the VQF-approach can be found in Urbano and Goeke (1984a, b). In the present section we review briefly the formalism as far as it is needed for the following. All considerations are presently done using the Lagrangian of the Cloudy Bag Model (Thēberge *et al* 1980). This one is written in the form

$$\hat{H} = \int d^3r\left(-i\sum_{a=1}^{3} q_a^+(\mathbf{r})\alpha\cdot\nabla q_a(\mathbf{r}) + B\right)\theta_v$$
$$+\sum_{j=1}^{3}\int d^3\mathbf{k}\,\omega(k)a_j^+(\mathbf{k})a_j(\mathbf{k})$$
$$+\frac{i}{2f}\int d^3\mathbf{r}\sum_{a=1}^{3}\bar{q}_a(\mathbf{r})\gamma_5\boldsymbol{\tau}^{(a)}\cdot\hat{\boldsymbol{\phi}}(\mathbf{r})q_a(\mathbf{r})\Delta_s \tag{1}$$

Here $\hat{\boldsymbol{\phi}}(\mathbf{r})$ is the quantized pion field operator

$$\hat{\phi}_j(\mathbf{r}) = \frac{1}{(2\pi)^{3/2}}\int\frac{d^3k}{\sqrt{2\omega(k)}}\left[a_j(\mathbf{k})\exp(i\mathbf{k}\cdot\mathbf{r}) + a_j^+(k)\exp(-i\mathbf{k}\cdot\mathbf{r})\right] \tag{2}$$

and the $a_j^+(\mathbf{k})[a_j(\mathbf{k})]$ create (annihilate) a pion with momentum k and cartesian isospin component j. The $q_a(r)$ is the quark field operator, where the index a indicates the colour.

In the nucleon-delta sector the quarks occupy the s-state of the bag with the familiar wave function

$$\psi_a(\mathbf{r}) = \frac{N}{4\pi}\begin{pmatrix} j_0(\Omega r/R) \\ ij_1(\Omega r/R)\boldsymbol{\sigma}^{(a)}\cdot\mathbf{r}\end{pmatrix}|\chi_a\rangle, \tag{3}$$

where $|\chi_a\rangle$ is the spin-flavour state of the quark with colour a. Apparently the most general form of $|\chi_a\rangle$ in the u-d sector is given by

$$|\chi_a(\alpha)\rangle = N(\alpha)\{\alpha_1|u,\uparrow\rangle + \alpha_2|u,\downarrow\rangle + \alpha_3|d,\uparrow\rangle + \alpha_4|d,\downarrow\rangle\}, \tag{4}$$

where u, d indicate up- and down-quarks and the arrows the orientation of the spin component. The $N(\alpha)$ is the normalization constant and the α's are to be treated as variational parameters for the quark structure of the baryon. Due to symmetry the α's are the same for all three colours.

The total baryon is assumed to consist of the quark core given by (3), and the pion cloud described by a coherent state

$$|\xi\rangle = N(\xi)\exp\left[\sum_j\int d^3k\,\xi_j(\mathbf{k})a_j^+(\mathbf{k})\right]|0\rangle. \tag{5}$$

Here the $N(\xi)$ is a normalization factor and the field amplitudes, $\xi_j(\mathbf{k})$, are to be treated as variational parameters for the pion cloud.

Using (3) and (4) the total trial wave function of the baryon reads

$$|\psi(\alpha,\xi)\rangle = \left(\prod_a\psi_a(r)\right)|\xi\rangle. \tag{6}$$

For a fully quantum mechanical treatment the $|\psi(\alpha,\xi)\rangle$ has to be projected on good spin and isospin quantum numbers:

$$|\psi_{JT}(\alpha,\xi)\rangle = \frac{P_JP_T|\psi(\alpha,\xi)\rangle}{\langle\psi(\alpha,\xi)|P_JP_T|\psi(\alpha,\xi)\rangle^{1/2}}. \tag{7}$$

Here P_J and P_T are projection operators introduced into nuclear physics by Peierls and Yoccoz. For a general intrinsic solution (6) with *eg* the spin-flavour structure (4) these projections are complicated since no simplifying symmetry relations can be used. Thus projection techniques are presently only used for special spin-flavour configurations as reported in Urbano and Goeke (1984b) and in §5. Here we first investigate the properties of the intrinsic solution corresponding to the mean field approximation.

3. Mean field approximation

In the mean field approximation the energy is given by the expectation value of H between the states (6), yielding

$$E(\alpha, \xi) = E_{\text{MIT}} + \sum_{j=1}^{3} \int \mathrm{d}^3k\, \omega(k)\xi_j^*(\mathbf{k})\xi_j(\mathbf{k})$$
$$+ g \sum_{j=1}^{3} \int \frac{\mathrm{d}^3k}{\sqrt{2\omega(k)}} \left[i\rho(k)\xi_j(\mathbf{k}) \sum_{a=1}^{3} \langle \chi_a | \sigma^{(a)} \cdot \mathbf{k}\, \tau_j^{(a)} | \chi_a \rangle + \text{c.c.} \right], \tag{8}$$

where $E_{\text{MIT}} = 3\Omega/R + 4\pi B/3R^3 - Z/R$ is the MIT-model energy, $\rho(k) = 3j_1(kR)/kR$ is the Fourier transform of the pion source density, and $g = \Omega/\{6f(\Omega - 1)(2\pi)^{3/2}\}$ is a renormalized coupling constant.

The variation of (8) with regard to $\xi_j^*(\mathbf{k})$ yields $\delta E/\delta\xi_j^*(\mathbf{k}) = 0$ and hence

$$\xi_j(\mathbf{k}) = \frac{ig\rho(k)}{\omega(k)\sqrt{2\omega(k)}} \sum_{a=1}^{3} \langle \chi_a(\alpha) | \sigma^{(a)} \cdot \mathbf{k}\, \tau_j^{(a)} | \chi_a(\alpha) \rangle. \tag{9}$$

Inserting this into (8) the total energy becomes

$$E(\alpha) = E_{\text{MIT}} - 6\pi g^2 A(\alpha) \int_0^\infty \frac{k^4\rho^2(k)}{\omega^2(k)}\, \mathrm{d}k \tag{10}$$

with

$$A(\alpha) = \sum_{i,j=1}^{3} |\langle \chi(\alpha) | \sigma_i \tau_j | \chi(\alpha) \rangle|^2. \tag{11}$$

The $E(\alpha)$ has a minimum whenever $A(\alpha)$ reaches a maximum. As shown in Urbano and Goeke (1984c), evaluating $A(\alpha)$ using (4) yields

$$A(\alpha) = 3 - 2N^4(\alpha)\{(|\alpha_1|^2 + |\alpha_2|^2 - |\alpha_3|^2 - |\alpha_4|^2)^2 + 4|\alpha_1\alpha_3^* + \alpha_2\alpha_4^*|^2\}. \tag{12}$$

One sees immediately that $A(\alpha) \leqslant 3$. The maximum value of $A(\alpha)$ is reached taking, for instance, $\alpha_1 = \alpha_4 = 0$ and $\alpha_2 = -\alpha_3 = 1$. This particular choice corresponds to the quark spin-isospin state

$$|\chi_h\rangle = \frac{1}{\sqrt{2}}\{|u, \downarrow\rangle - |d, \uparrow\rangle\}, \tag{13}$$

which is the well-known hedgehog quark state considered in Chodos and Thorn (1975), Brown and Rho (1979), Vento *et al* (1980), Birse and Banerjee (1984) and Kälbermann and Eisenberg (1984).

Inserting (13) into (9) yields

$$\xi_h(\mathbf{k}) = -i\frac{6g\,\rho(k)}{(2\omega(k))^{3/2}}(k_x\delta_{j1}+k_y\delta_{j2}+k_z\delta_{j3}). \tag{14}$$

For the corresponding pion mean field,

$$\phi_j(\mathbf{r}) = \langle \xi_h|\hat{\phi}_j(\mathbf{r})|\xi_h\rangle, \tag{15}$$

one obtains after an explicit calculation

$$\phi_1(\mathbf{r}) = \frac{x}{r}G(r) \quad \phi_2(\mathbf{r}) = \frac{y}{r}G(r) \quad \phi_3(\mathbf{r}) = \frac{z}{r}G(r), \tag{16}$$

with

$$G(r) = 3g(2/\pi)^{1/2}\int dk\,k^3\frac{\rho(k)}{[\omega(k)]^2}j_1(kr). \tag{17}$$

Both (13) and (16) show the well-known and always assumed hedgehog properties. Apparently they arise from the variational principle formulated above, *ie* VQF.

4. Angular momentum and isospin properties of the hedgehog solution

The hedgehog solution $|\psi_h\rangle$ with χ of (13) and ξ of (14) satisfies for $i = x, y, z$ the equations

$$(J_i^{(q)}+T_i^{(q)})|\psi_h\rangle = 0 \tag{18}$$

and

$$(J_i^{(\pi)}+T_i^{(\pi)})|\psi_h\rangle = 0. \tag{19}$$

The first one (18) is rather trivial to show, if (13) is considered. The second one (19) is more difficult and will be shown explicitly in this section.

The expansion of the coherent state $|\xi_h\rangle$ in terms of zero boson, one-boson, two-boson states, etc., yields

$$|\xi_h\rangle = 0+\sum_j\int d^3k\,\xi_j(\mathbf{k})a_j^+(\mathbf{k})|0\rangle+\frac{1}{2!}\sum_{jj'}\int d^3k\int d^3k'\,\xi_j(\mathbf{k})\xi_{j'}(\mathbf{k}')\,a_j^+(\mathbf{k})a_{j'}^+(\mathbf{k}')|0\rangle+\ldots.$$

Due to the structure of the coherent state $|\xi_h\rangle$ it is sufficient to show that

$$(J_i^{(\pi)}+T_i^{(\pi)})\sum_j\int d^3k\,\xi_j(\mathbf{k})a_j^+(\mathbf{k})|0\rangle = 0, \tag{20}$$

In a cartesian three-dimensional representation one has for the isospin operators the matrices

$$T_1^{(\pi)} = \begin{pmatrix}0&0&0\\0&0&-i\\0&i&0\end{pmatrix} \quad T_2^{(\pi)} = \begin{pmatrix}0&0&i\\0&0&0\\-i&0&0\end{pmatrix} \quad T_3^{(\pi)} = \begin{pmatrix}0&-i&0\\i&0&0\\0&0&0\end{pmatrix}.$$

If one writes $a_j^+(\mathbf{k})|0\rangle = |\mathbf{k},j\rangle$ and

$$|k,1\rangle = \begin{pmatrix}1\\0\\0\end{pmatrix}|\mathbf{k}\rangle \quad |\mathbf{k},2\rangle = \begin{pmatrix}0\\1\\0\end{pmatrix}|\mathbf{k}\rangle \quad |\mathbf{k},3\rangle = \begin{pmatrix}0\\0\\1\end{pmatrix}|\mathbf{k}\rangle,$$

one obtains immediately

$$
\begin{array}{lll}
T_1^{(\pi)}|\mathbf{k},1\rangle = 0 & T_1^{(\pi)}|\mathbf{k},2\rangle = i|\mathbf{k},3\rangle & T_1^{(\pi)}|\mathbf{k},3\rangle = -i(\mathbf{k},2\rangle \\
T_2^{(\pi)}|\mathbf{k},1\rangle = -i|\mathbf{k},3\rangle & T_2^{(\pi)}|\mathbf{k},2\rangle = 0 & T_2^{(\pi)}|\mathbf{k},3\rangle = i|\mathbf{k},1\rangle \\
T_3^{(\pi)}|\mathbf{k},1\rangle = i|\mathbf{k},2\rangle & T_3^{(\pi)}|\mathbf{k},2\rangle = -i|\mathbf{k},1\rangle & T_3^{(\pi)}|\mathbf{k},3\rangle = 0.
\end{array}
$$

Thus one gets *eg* with (14)

$$T_1^\pi \int d^3k\xi_1(\mathbf{k})a_1^+(\mathbf{k})|0\rangle = \int \frac{6g\rho(k)}{[2\omega(k)]^{3/2}}\{-k_y|\mathbf{k},3\rangle + k_z|\mathbf{k},2\rangle\}d^3\mathbf{k}. \quad (21)$$

The operator J_k^π can generally be written as

$$J_k^\pi = \sum_{m,n} \varepsilon_{kmn}\hat{q}_m\hat{p}_n, \quad (22)$$

where $\hat{q}$ and $\hat{p}$ are coordinate and momentum operators. This yields in the momentum representation for example

$$J_1^\pi = i\int d^3k|\mathbf{k}\rangle \frac{\partial}{\partial k_q} k_z\langle\mathbf{k}| - i\int d^3k|\mathbf{k}\rangle\frac{\partial}{\partial k_z}k_y\langle\mathbf{k}|. \quad (23)$$

Hence one obtains

$$
\begin{aligned}
J_1^\pi \int d^3k'\xi_1(\mathbf{k}')a_1^+(\mathbf{k})|0\rangle &= i\,6g\int d^3k'\frac{\rho(k')}{[2\omega(k')]^{3/2}} \\
&\times \int d^3k|\mathbf{k}\rangle\frac{\partial}{\partial k_y}k_z\{-k_x'\langle\mathbf{k}|\mathbf{k}',1\rangle - k_y'\langle\mathbf{k}|\mathbf{k}',2\rangle - k_z'\langle\mathbf{k}|\mathbf{k}',3\rangle\} \\
&- \int d^3k|\mathbf{k}\rangle\frac{\partial}{\partial k_z}k_y\{-k_x'\langle\mathbf{k}|\mathbf{k}',1\rangle - k_y'\langle\mathbf{k}|\mathbf{k}',2\rangle - k_z'\langle\mathbf{k}|\mathbf{k}',3\rangle\},
\end{aligned}
$$

giving

$$
\begin{aligned}
J_1^\pi \int d^3k'\,\xi_1(k')a_1^+(\mathbf{k}')|0\rangle &= -6g\int d^3k\frac{\rho(k)}{[2\omega(k)]^{3/2}} \\
&\times\Big\{|\mathbf{k},1\rangle i\frac{\partial}{\partial k_y}k_z(-k_x) + |k,2\rangle\frac{\partial}{\partial k_y}k_z(-k_y) + |k,3\rangle\frac{\partial}{\partial k_y}k_z(-k_z) \\
&- |\mathbf{k},1\rangle\frac{\partial}{\partial k_z}k_y(-k_x) - |k,2\rangle\frac{\partial}{\partial k_z}k_y(-k_y) - |k,3\rangle\frac{\partial}{\partial k_z}k_y(-k_z)\Big\}.
\end{aligned}
$$

The explicit evaluation yields now

$$J_1^\pi \int d^3k'\xi_1(k')a_1^+(\mathbf{k}')0 = 6g\int d^3k\frac{\rho(k)}{[2\omega(k)]^{3/2}}[-k_z|\mathbf{k},2\rangle + k_y|\mathbf{k},3\rangle]. \quad (24)$$

Comparison of (24) with (21) shows that we have proven the assertion for the considered special case $i = 1$ in (10). The proof for the other cases is similar.

From (8) and (9) there follows the important fact that a decomposition of the total hedgehog state $|\psi_h\rangle$ in terms of states with good spin J and isospin T yields

$$|\psi_h\rangle = \sum_i a_J|J, T = J\rangle. \quad (25)$$

Thus, in order to include quantum correlations, only a projection on one of the quantum numbers, involving only one set of Euler angles, is sufficient for a hedgehog state. However, this simplification does not hold if one first projects on states J, T and then performs a variation. In such a case there is no reason why $|\psi\rangle$ should not have components with $J \neq T$.

5. Projection techniques

A proper way to proceed would be now to perform, first, a projection on good J- and T-quantum numbers according to (7) and second, to vary the corresponding energy into its minimum. This is a rather complicated procedure which has not been investigated yet. A simpler way would consist in performing the projections after the variation, *ie* to perform a projection of a hedgehog state. Even this is complicated since a hedgehog state has nonaxial components in spin and isospin space. In order to explore the projection techniques a simpler case is considered here, where the intrinsic three-quark states are coupled (Urbano and Goeke 1984b; Thēberge *et al* 1980) to the ground state of a proton with spin up, $|N^+_{1/2}\rangle$ mixed with a state $|\Delta^+_{1/2}\rangle$. Thus the bare nucleon state is assumed to be

$$|\mathrm{BN}\rangle = \cos\alpha|N^+_{1/2}\rangle + \sin\alpha|\Delta^+_{1/2}\rangle. \tag{26}$$

For such a state one obtains the energy

$$E_{JT}(\alpha, R) = \frac{\int_0^\pi \mathrm{d}\beta \sin\beta \int_0^\pi \mathrm{d}\tilde\beta \sin\tilde\beta\, \mathrm{d}^J_{1/2\ 1/2}(\beta)\mathrm{d}^T_{1/2\ 1/2}(\tilde\beta) h(\beta,\tilde\beta)}{\int_0^\pi \mathrm{d}\beta \sin\beta \int_0^\pi \mathrm{d}\tilde\beta \sin\tilde\beta\, \mathrm{d}^J_{1/2\ 1/2}(\beta)\mathrm{d}^T_{1/2\ 1/2}(\tilde\beta) n(\beta,\tilde\beta)}, \tag{27}$$

with the overlap kernels

$$\begin{aligned} h(\alpha; \beta, \tilde\beta) &= \langle \mathrm{BN}|\langle\xi|H \exp\{i\beta J_y + i\tilde\beta J_3\}|\mathrm{BN}\rangle|\xi\rangle, \\ n(\alpha; \beta, \tilde\beta) &= \langle \mathrm{BN}|\langle\xi|\exp\{i\beta J_y + i\tilde\beta J_3\}|\mathrm{BN}\rangle|\xi\rangle. \end{aligned} \tag{28}$$

For the evaluation of (28) it is necessary to know the rotated pion field state

$$\exp\{i\beta J_y + i\tilde\beta J_3\}|\xi\rangle = \frac{1}{N}\exp\left\{\sum_j \int \mathrm{d}^3k\, \xi_j(\mathbf{k}_j; \beta\tilde\beta) a_j^+(\mathbf{k})\right\}|0\rangle,$$

with

$$\xi_j(\mathbf{k}, \beta\tilde\beta) = \sum_{j'} [R_3(\tilde\beta)]_{jj'}\, \xi_{j'}(R_y^{-1}(\beta)\mathbf{k}),$$

where N is a normalization constant. Following the formalism of Urbano and Goeke (1984a) some results of the calculations are shown in figure 1. There the MIT-energy, the intrinsic energy and the projected energy are contrasted. One realizes that the effect of the projection is biggest at small bag radii where the pion field is strongest. Actually for very small R-values one would obtain a collapse of the system if this is not prevented by an additional mechanism. In the present investigation this is done by assuming the $q\bar q$-component of the pion to have a finite extension (Urbano and Goeke 1984b) characterized by its radius η_π. For a physical pion the η_π is related to the pion decay

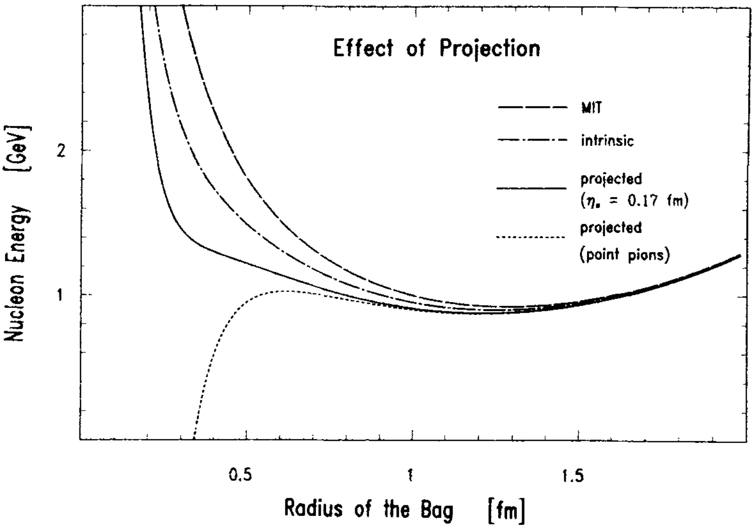

Figure 1. The total nucleon energy is given for various bag radii R. Considered are the MIT-model, the VQF-formalism in the mean field approximation (intrinsic) and with angular momentum and isospin projections included (projected). For the last case $\eta_\pi = 0{\cdot}17$ fm and $\eta_\pi = 0$ fm are considered to demonstrate the instability for small radii for vanishing η_π. The parameters of the calculation are $B^{1/4} = 0{\cdot}125$ GeV, $g = 14\,\mathrm{GeV}^{-1}$, $Z = 1{\cdot}75$.

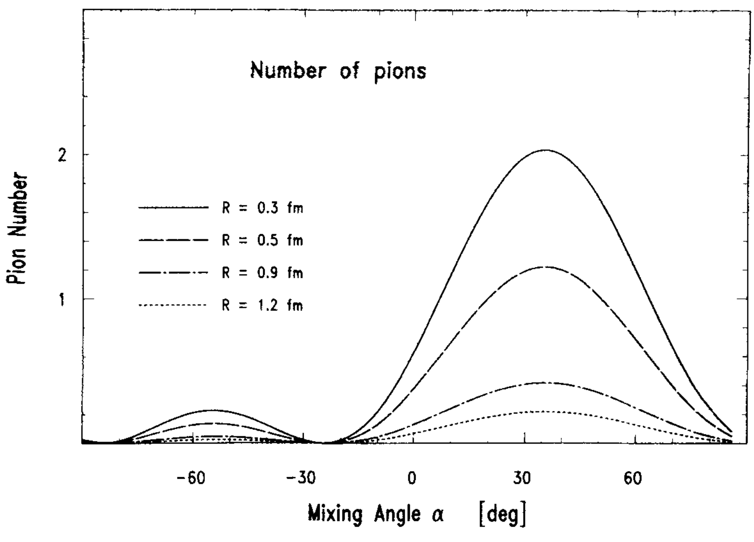

Figure 2. The average number of pions in the intrinsic nucleon state is given in dependence on the mixing angle α for various bag radii R. The parameters of the calculation are those of figure 1.

constant and comes out to be $\eta_\pi = 0{\cdot}17$ fm. This value is used in the calculations.

An interesting quantity is the average number of pions in the cloud. This is given by

$$N_\pi = \langle \xi | \sum_{j=1}^{3} \int \mathrm{d}^3 k a_j^+(\mathbf{k}) a_j(\mathbf{k}) | \xi \rangle,$$

and for the projected states analogously. Some results are found in figures 2 and 3. Figure 2 shows for various R-values the intrinsic number of pions in dependence on the mixing angle α. One sees a clear variation showing maximal values at α around 40°. For bag radii of about 1·2 fm only small values of N_π are obtained which are at most $N_\pi \approx 0{\cdot}2$. The projected results are different. They are displayed in figure 3 for various η_π-values in dependence of R. A comparison between both figures shows that the projected values are noticeably larger than the nonprojected ones. This holds even for

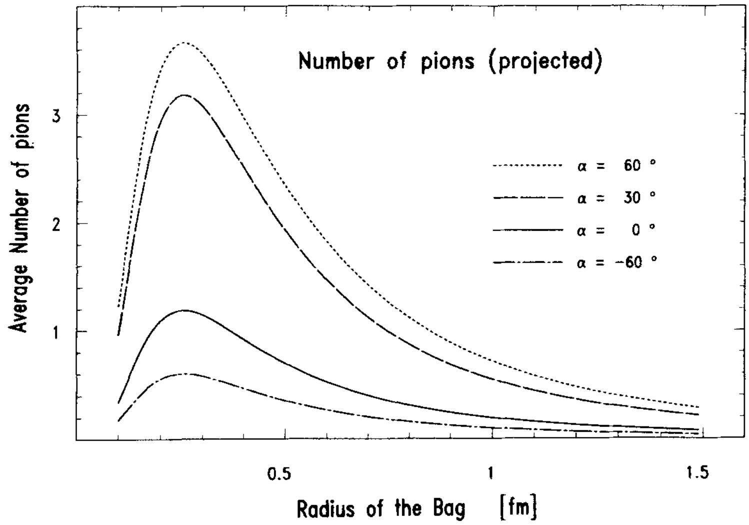

Figure 3. The average number of pions in the projected nucleon state is given in dependence on the bag radius R for various mixing angles α. The parameters of the calculation are those of figure 1.

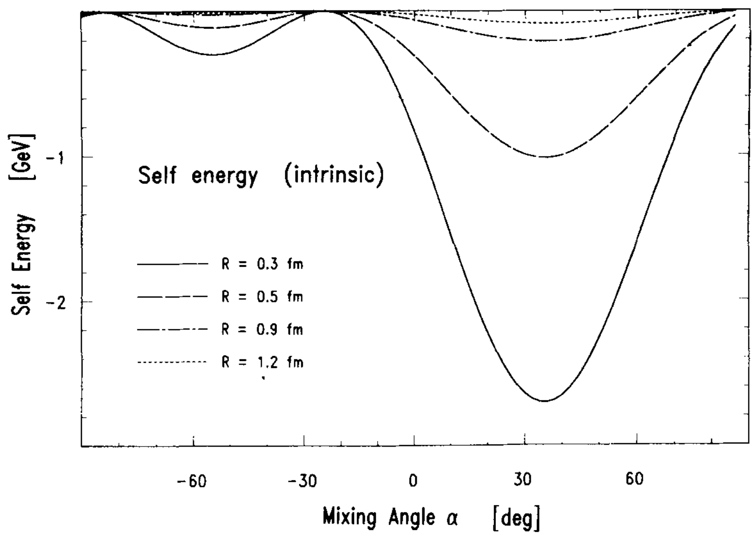

Figure 4. The self-energy the bag acquires due to the coupling with the poin field. Considered is the intrinsic solution with the parameters of figure 1 for various α- and R-values.

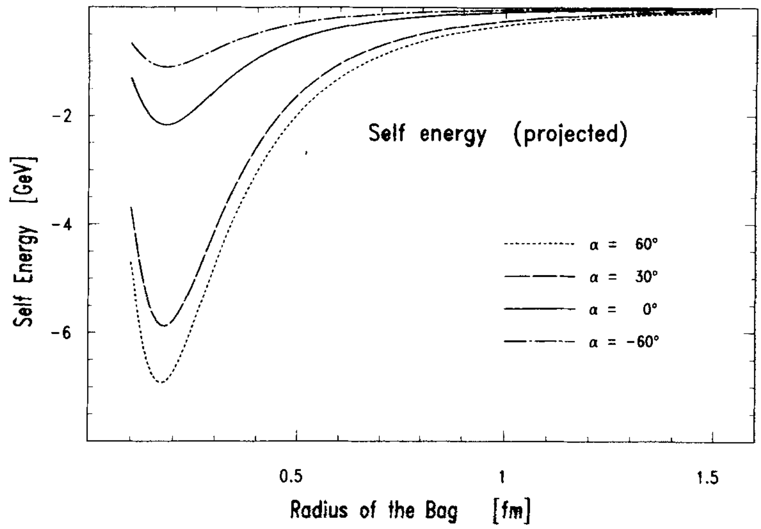

Figure 5. The self-energy of the projected state for various α- and R-values. The parameters are taken from figure 1.

$\alpha = 0°$ and $R \simeq 1{\cdot}2$ fm where the effect of the projections on the energy is rather small.

The self-energy of the system shows a behaviour as it is displayed in figures 4 and 5. It is noticeably larger in magnitude for the projected case. For all the above data it should be noted that due to the finite pion size the effective coupling constant goes to zero for $R \to 0$.

6. Summary

The objective of the present investigation was to illuminate angular momentum and isospin properties of chiral bag models. To this end the variational quantum field theoretical approach (VQF) of Urbano and Goeke was considered. It was first demonstrated that the well known hedgehog structure appears by variational techniques if a Cloudy Bag Hamiltonian is used. It was then shown that this hedgehog allows for simplifications of the projection techniques to be used. Those were then studied in a simplified model showing that particularly the average number of pions in the field is a rather sensitive number. Stability of the system can be guaranteed if a finite size of the pions in the cloud is assumed.

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