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Is reggeon field theory rendered obsolete by QCD?

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Abstract. We affirm the power of the eikonal approximation in reggeon field theory (RFT) and point out its merit as contrasted with renormalization group method for calculating reggeon self-energy correction due to pomeron exchanges. Relative merits of RFT and QCD in appropriate momentum regions of hadronic interactions are also examined to suggest a negative answer to the title.

Keywords. Reggeon field theory; quantum chromodynamics; eikonal approximation.

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1. Introduction

Although its genesis can be traced to the days of geometrical optics, the eikonal approximation has over the years acquitted itself as a tool of extensive utility in particle physics. In fact, the point of departure of the present paper is the employment of the eikonal approximation in RFT. It is befitting, therefore, to test the scope of the eikonal method in RFT by means of one of the ever open problems of field theory *viz*, self-energy. To be precise, we study reggeon self-energy correction due to pomeron exchange, envisaging a triple point interaction; and our field theory is RFT, naturally. A bare calculation of the above correction has recently been reported (Sidhanta 1984). Here we clarify its methodology and also establish the complementarity, rather than contrariety, between RFT and QCD as theories of strong interaction.

While vindication of RFT in the specific energy-momentum region of strong interaction will be taken up in the next section, it will be pertinent at this point to recall that the eikonal approximation as such admits of a systematic development (Orzalesi 1974) by the sophisticated method of Schwinger functionals within the Lagrangian field theoretic formalism. And RFT, as the name indicates, is, avowedly, a variant of the field theory. In this sense, then, the present paper which applies eikonal technique to RFT, is an exercise in field theory rather than an *ad hoc* phenomenology. As a matter of fact the calculation in the present paper starts from the Lagrangian

$$\mathscr{L}(x,t) = \frac{i}{2}\psi^{+}\frac{\partial}{\partial t}\psi - \alpha_0'\nabla\psi^{+}\nabla\psi - \Delta_0\psi^{+}\psi + \{g\psi^{+}\psi\phi + \text{h.c}\}, \tag{1}$$

where ψ and ϕ represent the reggeon and pomeron fields and α_0' is the slope of the reggeon trajectory while $\Delta = 1-\alpha_0'$. The above Lagrangian results in the field equation

$$\left[i\frac{\partial}{\partial t} - \alpha_0'\nabla^2 - \Delta_0 + g\phi(x,t)\right]\psi(x,t) = 0. \tag{2}$$

The functional eikonal formalism applied here to the RFT can presumably be employed in QED for self-energy correction. This seems to be a point of vantage. Another propitious point is the immunity from certain undesirable features that accompany the renormalization group (RG) treatment of the present self-energy problem (see remarks following (14)). Regarding the method of RG it is to be emphasized that for certain limiting values of external momenta alone the mass scale loses physical import. Therefore its use is always contingent on how nearly those limiting values have been fulfilled in a given case. In QCD itself despite the fact that in the deep inelastic region the condition is reasonably met, an alternative to RG for calculating structure functions was undertaken which was considerably successful (Altarelli and Parisi 1977). Here we supplant the RG by addressing eikonal Green's function to RFT formalism where we envisage a triple reggeon interaction. By our method the asymptotic form of the amplitude due to poles in the t-channel can be reproduced since P', P, A_2-exchanges can be identified with reggeons. To exemplify the power of eikonal in RFT in this context we reproduce the final result from Sidhanta (1984). It is felt however that a rapid survey of what eikonal consists of, would be in order also, and this is given in §3.

2. On the compatibility of RFT and QCD

It is expedient at the outset to refute certain views which consider Regge model sterile at the moment when QCD has held out new promise for hadron physics. The aesthetic value of QCD as a nonabelian gauge theory is admittedly very high. But it passes one's understanding how that 'high aesthetic value' may argue to the end of use-value of practical calculations offered by the Regge model in its own specific region of hadron physics. It is to be borne in mind that although QCD is spectacular in the deep inelastic region, its asymptotic freedom which makes perturbative calculation unblemished over there, does not subsist when small p_T-region is considered where the Regge theory still reigns supreme. A very recent paper by Collins and Kearney (1984) has reiterated the merit of Regge approach for exclusive processes, in the said region. Till a nonperturbative QCD calculation evolves, things are not going to change.

Moreover, the Regge model with its later innovations (reggeon calculus/RFT) is in no way in conflict with the quark model. It is well known that the Regge model's classification based on external quantum numbers shows interesting correspondence with SU_3 assignment based on internal quantum numbers. Also, the Reggeized notion of 'orbital excitation' has inspired speculation that increasingly high orbital excitation of quarks in a potential well may explain higher masses for the states composed of quarks. To confuse RFT with the original 1959 Regge pole notion would be no less naive than confining QCD to the 1964 model of quarks. Both disciplines have made progress—unequally albeit. We discover the same functional integration technique adopted in RFT for correct propagator formulation as we would in any field theory of the day. The ladder diagrams were first popularized by Reggeists but, in QCD itself one re-discovers them for proving leading mass singularities whenever the gluon is collinear with the incoming quark. Both RFT and QCD lean heavily on Lagrangian field theory and both make copious use of graph technique and RG method. In a sense, QCD is a field theory of confined particles called quarks with three-gluon vertex admissible and RFT is a field theory of quasi-particles called reggeons with nonspatial degrees of freedom in-admissible. Of course, the spin $> \frac{3}{2}$ riddle is present but its resolution requires further development of supersymmetry theory. In any case, it would be stretching a point to declare RFT obsolete even in the wake of gauge theories with their pageant and promise.

3. Eikonal in RFT for reggeon self energy correction

In terms of the graph technique the eikonal approximation consists in discarding the higher momentum powers in the propagator. Now, the reggeon propagator followed by the exchange of a poweron of energy E_R and momentum ω is:

$$D(\omega, p) = \frac{1}{\omega - E_R - \tilde{\alpha}'_0 (p-k)^2 - \Delta_0} = \frac{1}{\omega - \tilde{\alpha}'_0 p^2 + \beta(k) - \Delta_0}, \tag{3}$$

where the refined eikonal approximation (RE) is defined by

$$\beta_{\rm RE} = -E_k - \tilde{\alpha}'_0 (k^2 - p\cdot k), \tag{4}$$

in constrast to the ordinary eikonal (OE) which is

$$\beta_{0E} = -E_k + 2\tilde{\alpha}'_0 p\cdot k. \tag{5}$$

In what follows we explain how to develop the functional method to calculate radiation correction for the reggeon propagator arising from all possible pomeron exchange where, of course, no ψ loop is included.

The Green's function in the presence of an external field is cast into the following eikonal form:

$$\begin{aligned} G(x, y|g\phi) &= (x|G\phi|y), \\ &= i\int_0^v V(v, x, y|g\phi)\exp[iv(\omega - \tilde{\alpha}'_0 p^2 + i\varepsilon)], \end{aligned} \tag{6}$$

where

$$V(v, x, y|g\phi) = \int \exp[iv(\omega - \tilde{\alpha}'_0 p^2 + i\varepsilon) - ip(x-y) + i\chi(v, x, p|g\phi)]. \tag{7}$$

The eikonal function admits of the perturbative expansion

$$\chi = \sum g^n \chi_n, \tag{8}$$

and satisfies the differential equation

$$\partial\chi_{\rm RE}/\partial v = i\beta(k) + g\phi(x, t), \tag{9}$$

with

$$\beta(k) = -E_k - \tilde{\alpha}'_0 (k^2 - 2p\cdot k). \tag{10}$$

To second order of perturbative expansion G reads:

$$G_2(x, y|g\phi) = -i\int dv \exp[iv(\omega - \tilde{\alpha}'_0 p^2 - \Delta_0 + i\varepsilon) - ip(x-y) + ig\chi_1 + ig^2\chi_2]. \tag{11}$$

The self-energy correction to reggeon propagator according to figure 1 (where

Figure 1. Self energy correction to the reggeon propagator due to pomeron exchange.

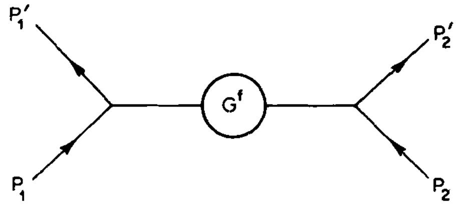

Figure 2. Exchange of pole in the t-channel.

pomeron exchange without poweron loop occurs) is given by:

$$\mathscr{G}_2(\omega, p) = -i\int \exp[i\nu(\omega - \bar{\alpha}_0 p^2 - \Delta_0 + i\varepsilon) + \chi_2(\nu)], \tag{12}$$

where the function $\chi_2(\nu)$ corresponding to the second order of the expansion (8) is the self-energy correction to the physical reggeon propagator (invested with pomeron clothing) $\chi_2(\nu)$ is found by operating the second order Green's function (§§2.4–2.6) with

$$K\left(\frac{\partial}{\partial\phi}\right) = \exp\left[\frac{1}{2}\iint \mathrm{d}Z\,\mathrm{d}\omega \frac{\delta}{\delta\phi(Z)} D_F(Z-\omega)\frac{\delta}{\delta\phi(\omega)}\right]$$

and going to the limit $\phi \to 0$

where $$\chi_2(\nu) = \frac{i}{2}\int (E^2 - k^2)D(k)\int_0^\nu \exp\{i\tau_1\beta(k)\}\,\mathrm{d}\tau_1$$
$$\times \int_0^\nu \exp\{i\tau_2\beta(-k)\}\,\mathrm{d}\tau_1\,\bar{\mathrm{d}}k\,\mathrm{d}E_k \tag{13}$$

(Recall at this point the final remark in §1 concerning the subsumption of meson-exchanges (P, ρ, A_2) under reggeons. See figure 2 for visualizing how the physical reggeon is incorporated in the scheme by considering the correction due to its pomeron dressing). Without any further ado about eikonal formalism (see Orzalesi 1974) we now turn to the actual calculation of the above self-energy correction, which is the pivotal point of this paper. To avoid repetition we only state the results already obtained (Sidhanta 1984)

$$A(S, t) = S^{\alpha}(\log S)^{\frac{1}{4}}. \tag{14}$$

Recall that Abarbanel and Bronzan (1974) had given:

$$\sigma_T \sim (\log S)^{\frac{1}{6}}$$

RG approach suffers from an expansion in powers of $\varepsilon = 4 - D$ whose convergence is not indubitable. Besides, the case of more than one IR stable point is apt to befog the dynamics of the process itself. Our eikonal approach is immune from these lapses of the RG method.

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