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Are weak vector bosons composite?

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Abstract. Recent CERN $p\bar{p}$ collider data on anomalous Z° events suggest, among other possibilities, a composite structure for the weak intermediate vector bosons. We present a short review of these developments and examine how far the scenario for weak interactions with such composite models of the weak vector bosons presents a viable alternative to the standard electroweak theory. In particular, we show how the scale of the dynamics underlying the composite structure is set by the magnitude of the weak mixing angle $\sin^2\theta_w$ and point out the possibility of accommodating the anomalous $Z^\circ - l\bar{l}\gamma$ decay events presently observed within this picture.

Keywords. Vector bosons; composite models; Fritzsch-Mandelbaum model; weak interactions; lepton interaction.

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1. Introduction

It is now widely believed that the basic forces involved in the interactions of elementary particles—the weak, electromagnetic and strong—arise from a single dynamical principle, namely that of local gauge symmetry. At present, the most widely accepted scenario is that in which the strong interactions arise from the unbroken colour gauge group $SU(3)_c$ whereas the electroweak forces are based on the flavour gauge group $SU(2)_L \times U(1)$. However, in electroweak forces, the gauge symmetry is broken down to $U(1)_{e.m.}$ which alone remains an unbroken local symmetry. If this picture is correct, there is a degree of asymmetry in the manner in which the basic dynamical principle operates in the two cases; this renders the description somewhat lacking in aesthetic appeal. To this one may add another aspect of dissimilarity between the strong and electroweak forces. This relates to the manner in which relevant mass scales are assigned to strong interactions on the one hand and to electroweak forces on the other, connected with the so called "gauge-hierarchy" problem. In the standard model of elementary particle interactions based on $SU(3)_c \times SU(2)_L \times U(1)$, the mass scales of the various interactions are set in very different ways. As is well known, the running gauge coupling parameter α_s corresponding to a particular gauge group like $SU(3)_c$ depends on an arbitrary constant with the dimensions of mass, denoted in this case, by Λ_3: $\alpha_s(q^2) \equiv \alpha_s(q^2/\Lambda_3{}^2)$. The scale of strong interactions is determined by the Λ_3 value at which the running gauge coupling parameter α_s becomes large. This happens at $\Lambda_3 \simeq 0{\cdot}3$ GeV. However, the fundamental mass scale of elementary particle physics at which a grand unified gauge symmetry like, say, SU(5) breaks down to the standard $SU(3)_c \times SU(2) \times U(1)$ is believed to be $\sim 10^{15}$ GeV. At this point, the strong gauge coupling parameter is only $\sim 1/50$. The strong interaction mass scale is thus much smaller than this fundamental scale, which is explained by the fact that α_s only changes logarithmically with energy: it therefore takes a large energy scale to become significant in magnitude. One finds this way of setting the mass scale of a theory and of understanding its relation to the fundamental mass scale of the unbroken gauge symmetry quite satisfactory. In contrast, the mass scale of weak interactions is set by the magnitude of the vacuum expectation value of a scalar field which plays no other role in the theory.

In view of this, several models have recently been proposed (Peskin 1981; Fritzsch and Mandelbaum 1981a, b, c; Abbott and Farhi 1981a, b; Greenberg and Sucher 1981; Barbieri *et al* 1981; Buchmüller *et al* 1983; Schrempp and Schrempp 1983) in which the observed intermediate weak vector bosons appear as bound states of more fundamental constituents. In such models, the results of the standard $SU(2)_L \times U(1)$ electroweak theory, as confirmed by low energy weak interaction phenomena, are regarded as phenomenological manifestations of some deeper confinement dynamics called quantum haplodynamics (QHD) in much the same way as the strong interaction phenomena of the earlier decades are viewed as the phenomenological manifestations of the confining QCD with vector bosons like the ρ appearing as bound states of quarks and antiquarks. A degree of symmetry is thus restored in the relationship between the fundamental exact gauge symmetry and the strong, weak and electromagnetic interactions.

It also appears that there is experimental evidence which suggests a need for a change in our understanding of the structure of weak interactions. The CERN $p\bar{p}$ collider experiments which confirmed the existence of the $W^\pm$ and the Z° in accord with the predictions of the standard $SU(2)_L \times U(1)$ gauge model also provided evidence which may possibly be interpreted as a signal of the composite structure of the weak vector bosons (Arnison *et al* UA1 Collab. 1983a, b, c; Bagnaia *et al* UA2 Collab. 1983; Banner *et al* UA2 Collab. 1983). The evidence consists of the "anomalous" Z° decay events in which Z° decays into an e^+e^- (or $\mu^+\mu^-$) pair which is accompanied by a hard photon of energy $\sim$ 30 GeV. The invariant mass of the lepton pair shows a peak at $\sim$ 50 GeV. The number of such γ emission events now observed seems to indicate a branching ratio of $\sim 20\,\%$ for such events in relation to all lepton pair decay events. The experiments also indicate that the photons cannot be regarded as arising out of external bremsstrahlung since the angle between the direction of photon emission and one of the lepton pair is large ($\sim 15 \pm 5^\circ$). One of the mechanisms that may account for the emission of such hard photons is inner bremsstrahlung from the charged constituents of the intermediate vector bosons. An alternative picture would be one in which there exists a neutral scalar boson U° which can decay into a lepton pair; one then has $Z^\circ \to U^\circ + \gamma \to l\bar{l} + \gamma$. This decay sequence can also be accommodated in a bound state model of weak vector bosons.

We next focus our attention on the specific bound state model of weak vector bosons proposed by Fritzsch and Mandelbaum (1981a). We shall later discuss alternative suggestions due to Barbieri *et al* (1981) as also due to Abbott and Farhi (1981a, b).

2. The Fritzsch-Mandelbaum model of a composite weak boson

In this model (Fritzsch and Mandelbaum 1981a) the W boson triplet ($W^\pm$, W°) are considered to be composites of pairs of spin 1/2 haplons α and β (together with their

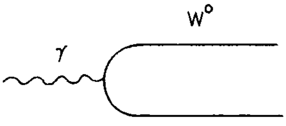

Figure 1. γ-W° mixing.

antiparticles) which form an isodoublet with charges $\pm 1/2$. The haplons and their antiparticles are held together by the basic forces of QHD mediated by hypergluons. The correspondence between the weak bosons and the haplons is as follows:

$$W^+ = \bar{\alpha}\beta, \quad W^- = \bar{\alpha}\beta, \quad W_0 = \frac{1}{\sqrt{2}}(\bar{\alpha}\alpha - \bar{\beta}\beta).$$

With respect to colour, the haplons may be either singlets or triplets; with respect to hypercolour, they may be N-plets. In this model, the photon is elementary. The situation with respect to the W's is similar to that obtained for the ρ's in strong interaction dynamics where ρ is visualized as a composite of a (u, d) quark doublet and the corresponding antiparticle doublet. The neutral current phenomena are distinguished from the charged current ones on account of γ–W° mixing
This phenomenon of γ–W° mixing gives rise to Z° which is more massive than $W^\pm$. The Weinberg mixing angle $\sin^2\theta_w$ is then directly related to the parameters characterizing this mixing phenomenon. At first sight it may appear puzzling that any model in which the global SU(2) symmetry (with respect to which) $W^\pm, W^\circ$ appear as a triplet) is broken by γ–W° mixing should admit a value of $\sin^2\theta_w$ greater than the order of the fine structure constant α. The resolution of this puzzle and a precise determination of the value of this parameter would thus be a central problem in any composite model. We shall see how a comparatively large value of $\sin^2\theta_w \sim 0{\cdot}23$ puts dynamical constraints on composite models of weak vector bosons with masses comparable to the scale of the binding forces of QHD.

That $\sin^2\theta_w$ may be related to the γ–W° mixing strength was noted by Bjorken (Bjorken 1979; deGroot and Schildknecht 1981) and independently by Hung and Sakurai (Hung and Sakurai 1978; Chen and Sakurai 1982) before suggestions that the weak vector bosons may be composite. Hung and Sakurai assume a phenomenological Lagrangian which incorporates γ-W° mixing together with the canonical Lagrangians for the vector bosons, photons and fermions. With such a phenomenological Lagrangian, the calculation of the exact consequences of mixing using Feynman diagrams is extremely inconvenient: one has to sum all the Feynman diagrams for a given process. There exists an alternative procedure based on the propagator-mixing mechanism which does not require such a sum. Hung and Sakurai employ this method to obtain the γ–W° mixing parameter. One writes the relevant part of the Lagrangian in a matrix form:

$$L_{\gamma\text{-}W^\circ} = \tfrac{1}{2}\phi^T_\mu[\Box^2 K - M^2)\delta_{\mu\nu} - \partial_\mu\partial_\nu K]\phi_\nu + \phi^T_\mu J_\mu + \text{total derivatives} \quad (1)$$

Here $\phi_\mu \equiv \begin{pmatrix}\tilde{A}_\mu \\ J_\mu\end{pmatrix}$ $\qquad J_\mu \equiv \begin{pmatrix}\tilde{e} & J^{\text{e.m.}}_\mu \\ g & J^3_\mu\end{pmatrix}$

$$K \equiv \begin{pmatrix}1 & \lambda_{\gamma\text{-}W} \\ \lambda_{\gamma\text{-}W} & 1\end{pmatrix} \qquad M^2 \equiv \begin{pmatrix}0 & 0 \\ 0 & m^2_W\end{pmatrix}.$$

T denotes the transpose, $\sim$ denotes the fields (or parameters) before mixing and g denotes the charged weak coupling. In writing (1), we have assumed the γ–W° Lagrangian to be of the form

$$L_{\gamma\text{-}W^\circ} = -\tfrac{1}{4}\lambda_{\gamma\text{-}W}[\tilde{F}_{\mu\nu}W^\circ_{\mu\nu} + W^\circ_{\mu\nu}\tilde{F}_{\mu\nu}].$$

From the Lagrangian given by (1), one extracts the inverse of the mixed propagator in momentum space:

$$\Delta^{-1}_{\mu\nu}(q^2) = (q^2K + M^2)\delta_{\mu\nu} - q_\mu q_\nu K. \tag{2}$$

This matrix propagator is now required to have poles at the physical masses of the particles involved which, in the present case, are the photon and the Z° boson. To find the poles, it is sufficient to consider the transverse part of the inverse propagator matrix only. Denoting this by $\Delta T^{-1}(q^2)$, we have

$$\Delta T^{-1}(q^2) = (q^2K + M^2). \tag{3}$$

The physical masses are determined from the requirement

$$\det[\Delta T^{-1}(q^2)]\Big|_{q^2=-m_i^2} = 0, \tag{4}$$

where $m_i^2 = 0, i = 1$ and $m_i^2 = m_z^2, i = 2$ corresponding to the photon and the Z° meson respectively. The physical photon mass turns out to be zero, whereas the mass of the Z° is given by

$$m_z^2 = \frac{m_{W^2}}{1-\lambda^2_{\gamma\text{-}W}}. \tag{5}$$

The corresponding effective Lagrangian is then written as

$$L_{\text{eff}} = \tfrac{1}{2}J^+_\mu \Delta T^{-1}(q^2)J_\mu \tag{6}$$

Substituting for ΔT^{-1} and J_μ one easily finds the following structure for the neutral current interaction

$$L^{\text{NC}}_{\text{eff}} = \frac{4G_F}{\sqrt{2}}\left(J^3_\lambda - \frac{e}{g}\lambda_{\gamma\text{-}W}J^{\text{e.m.}}_\lambda\right)\left(J^3_\lambda - \frac{e}{g}\lambda_{\gamma\text{-}W}J^{\text{e.m.}}_\lambda\right), \tag{7}$$

where $g^2/m_W^2 = 4\sqrt{2}G_F$. Comparing (7) with the effective Lagrangian for neutral current processes in the SU(2) × U(1) Weinberg-Salam theory, *viz*

$$L^{\text{WS}}_{\text{eff}} = \frac{4G_F}{\sqrt{2}}(J^3_\mu - \sin^2\theta_W J^{\text{e.m.}}_\mu)(J^3_\mu - \sin^2\theta_W J^{\text{e.m.}}_\mu)$$

we find $\sin^2\theta_W = \frac{e}{g}\lambda_{\gamma\text{-}W}$. (8)

3. Determination of $\sin^2\theta_W$ in a composite model

In the composite model scenario, the W bosons are composite, whereas the photon is elementary. Hence, there is no obvious way of electroweak unification; $\sin^2\theta_W$ is no longer a parameter as in the standard electroweak model and should be determined in terms of the parameters of the composite model. This can be seen as follows. If we define the vector meson-photon transition matrix element as

$$\langle 0|J_\mu^{\text{electroweak}}(0)|W^\circ\rangle = (2\pi)^{-3/2} M_W^2 e\varepsilon_\mu \lambda_{\gamma\text{-}W} \tag{9}$$

we can relate this matrix element to the value of the wavefunction of the composite W-meson at the origin. In a relativistic bound state model, the wavefunction of a composite state, constructed with the haplon field is given in momentum space by

$$\chi(q,p) = (2\pi)^{-3/2} \int d^4 x \exp(iq\cdot x) \langle 0|T(\psi(x/2)\bar{\psi}(-x/2))|W\rangle, \tag{10}$$

where ψ is the operator for the haplon fields, p the total 4-momentum of the bound state and q the relative 4-momentum. If we write the electromagnetic current in terms of the haplon fields

$$J_\mu^{\text{e.m.}} = :\bar{\psi}\gamma_\mu Q_H \psi: \tag{11}$$

where Q_H stands for the haplon charge factors then it is easily shown, using (9), (10) and (11) that

$$e\varepsilon_\mu M_W^2 \lambda_{\gamma\text{-}W} = \text{Trace}\, \gamma_\mu Q_H \chi_{(x=0)}. \tag{12}$$

Since $\chi(x)$ is the wavefunction of a vector meson, we put

$$\chi(x) = \chi^V(x),$$

and obtain from (12)

$$eM_W^2 \lambda_{\gamma\text{-}W} = 4\langle Q\rangle \chi^V(0), \tag{13}$$

which results in the following expression for $\sin^2\theta_W$:

$$\sin^2\theta_W = 4\frac{e^2}{g}\langle Q\rangle \frac{\chi^V(0)}{M_W^2}. \tag{14}$$

In the Bethe-Salpeter formalism, $\chi^V(x)$ satisfies (Böhm and Kramer 1977)

$$[\Box - m^2 + \tfrac{1}{4}M^2 - K_0(R)]X^V(x) = 0, \tag{15}$$

where m is the haplon mass, $p = (0, iM)$ and $K_0(R) \equiv K_0(x_\mu x_\mu)^{1/2}$ stands for the confining haplon-hypergluon forces. If we assume that $K_0(R) = \alpha_0 + \beta_0 R^2$ with $\alpha_0 = -M^2$ and $\beta_0 = \Lambda_H^4$ where Λ_H is the scale of the confining haplon-hypergluon dynamics, we obtain from the normalized Bethe-Salpeter wavefunction the value of $X^V(0)$ as

$$|X^V(0)|^2 = \frac{1}{16\pi^2} M_W^2 \Lambda_H^2. \tag{16}$$

From (14) and (16) we obtain the $\sin^2\theta_W$ value as:

$$\sin^2\theta_W \simeq \frac{4\alpha}{g}\langle Q\rangle \left(\frac{\Lambda_H}{M_W}\right). \tag{17}$$

The Q value in any specific dynamical model may be written down; for instance, in the Fritzsch-Mandelbaum version, one has

$$\langle Q\rangle^2 = e_H^2 n_H n_c, \tag{18}$$

where $e_H^2 = \frac{1}{2}$ and n_H and n_c stand for the number of hypercolour and colour degrees of freedom respectively. We see that the $\sin^2\theta_W$ value puts a constraint on the mass scale of the confining dynamics Λ_H. With $e_H^2 = \frac{1}{2}$, $N_c = N_H = 3$, the experimental value of $\sin^2\theta_W$ puts the constraint (Fritzsch and Mandelbaum 1982)

$$\Lambda_H \gtrsim G_F^{-1/2}. \tag{19}$$

Equation (8) was derived on the assumption that only a single triplet of iso-vector W bosons dominates. Kuroda and Schildknecht (1983) showed that the relation also holds if a spectrum of vector bosons $W, W^1, \ldots$ exists, provided the level spacing is sufficiently large. The justification for such an assumption is as follows. It is known that successful estimates of the γ–ρ° transition in QCD are based on the application of asymptotic freedom sum rules (Shifman *et al* 1979) which may be regarded as better founded versions of the local duality relationship which is generally invoked for the γ–ρ° junction in strong interaction physics. Asymptotic freedom sum rules relate the low energy behaviour of various current-current correlation functions to their high energy behaviour arising from asymptotically free constituents. Assuming similar sum rules to be valid even for confining haplons, Gounaris *et al* (1983) showed that a consistent picture emerges within the framework of asymptotic freedom sum rules in QHD as well. Combining a hypercolour scale $\Lambda_H \simeq G_F^{-1/2}$ with the assumption that excited vector boson masses are $\sim 0{\cdot}5$ TeV, a large dynamical enhancement of $\lambda_{\gamma\text{-}W}$ can occur, in agreement with the measured value of $\sin^2\theta_W$. Barbieri and Mohapatra (1983) made the same conclusion when using asymptotic freedom sum rules in models in which the sum over the squares of the electric charges of the haplons (preons) exceeds the same for the quarks and leptons. To adapt the sum rules to the present case, one writes

$$\begin{aligned}\pi_{\mu\nu}(q^2) &= i\int \mathrm{d}^4 x \exp(iq\cdot x)\langle 0|T(j_\mu(x)j_\nu(0)|0\rangle,\\ &= \pi(q^2)(q_\mu q_\nu - q^2 g_{\mu\nu}) + \pi_1(q^2)q_\mu q_\nu.\end{aligned} \tag{20}$$

If we define

$$R(s) \equiv \frac{24\pi}{e^2}\,\mathrm{Im}\,\pi(s),$$

the asymptotic freedom sum rule reads

$$\int \mathrm{d}s e^{-s/M^2} R(s) = M^2 \langle T_3\rangle_\rho^2\left[1 + 0\left(\frac{\alpha_H(M^2)}{\pi}, \frac{\Lambda_H^2}{M^2}\right)\right], \tag{21}$$

where $\langle T_3\rangle_\rho$ denotes the average isovector haplonic charges within a weak isovector state. The current j_μ is, in analogy with its structure in QCD, given by the haplon fields, *viz*

$$j_\mu = -\frac{1}{2}\sum_{c,H}(\bar{\alpha}_L\gamma_\mu\alpha_L - \bar{\beta}_L\gamma_\mu\beta_L), \tag{22}$$

with the sum running over the number of colour and hypercolour degrees of freedom of the haplons; the mass scale M^2 is assumed $\gtrsim \Lambda_H^2$. The expression for $R(s)$ may be derived from the effective γ–W mixing Lagrangrian as considered earlier; alternatively, the sum rule may be saturated by insertion of poles such as the Z°. A consistent value of $\sin^2\theta_W$ now emerges if one assumes the excited neutral bosons to have masses much greater than the Z°.

The analogy between QCD and QHD has been considered seriously by Fritzsch *et al* (1982) beyond the implications described above. They postulate a local current algebra for the weak isovector currents of the subconstituents of leptons, quarks and intermediate vector bosons and discuss the universality of the weak interactions. We turn to a brief discussion of their ideas.

4. Universality of weak interactions in the composite model

In QCD the universality of the coupling of vector mesons to hadrons is a consequence of the underlying quark structure and pole dominance. For example, consider the coupling of a hadron A consisting of u or d quarks and (say) one or more heavy quarks c or s or b. The dependence of the matrix element of the third component of the isovector quark current on the momentum transfer variable t is given by

$$F^V(t) = (m_\rho^2/f_\rho) f_{\rho AA}/(m_\rho^2 - t) \tag{23}$$

where $m_\rho^2/f_\rho = \langle 0|\tfrac{1}{2}(\bar{u}\gamma_\mu d)|\rho^\circ\rangle$

and $f_{\rho AA}$ is the ρ coupling to the hadron A.

If $\langle A|j_\mu^3(0)|\rangle = \bar{u}\gamma_\mu u F^V(t) T_3$ is normalized to unity at $t = 0$ in accordance with the algebra satisfied by the isospin charges, we obtain

$$f_\rho = f_{\rho AA}. \tag{24}$$

Consequently, $f_{\rho K^+K^-} = f_{\rho D^+ D^-} = \ldots$; the vector meson couples universally to all hadrons of the type $q\bar{s}, q\bar{c}\ldots$ where q is a u or a d quark. This example shows that vector meson dominance implies highly nontrivial constraints. Now, one observes in nature that left handed quarks and leptons form doublets under the weak isospin group $SU(2)_L$ and the weak charges obey the weak isospin algebra. As in QCD, we assume that the charge densities at equal times obey the local current algebra and further that these currents are bilinear in the constituent fields α and β which form W's in much the same way in which the u's and d's form the vector meson ρ. Thus if we assume that the W's, q's and leptons all have common constituents α and β, then we may demand universal normalization at $t = 0$ of the electron, neutrino and quark form factors: $F_e(0) = F_{\nu e}(0) = F_u(0) = \ldots = 1$. From W-dominance we will now have

$$F_f(t) = \frac{m_W^2}{f_W}\frac{f_{Wff}}{(m_W^2 - t)}. \tag{24a}$$

This, in turn, will predict universal coupling of W with the electron, neutrino etc. *i.e.* we

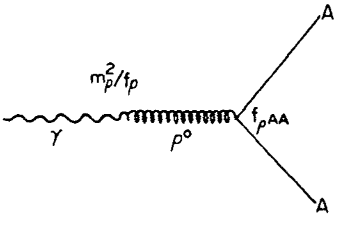

Figure 2. ρ-meson universality.

have $f_W = f_{Wf\bar{f}} = g$, the weak W decay constant. We therefore see that W dominance and a composite structure for W's, quarks and leptons leads to the observed universality of the weak coupling of the W to quarks and leptons.

It therefore appears that the basic features of the standard electroweak theory are quite consistent with an underlying confining dynamics similar to the role played by QCD in strong interaction physics. It is, therefore, natural to inquire if there are any experiments suggestive of a composite structure for the W's. In the next section, we turn our attention to the anomalous Z° events recently observed in the CERN proton-antiproton collider experiments.

5. Anomalous Z° events and the composite model hypothesis

Recently, the UA1 and UA2 groups at CERN (Arnison *et al* 1983 a, b, c; Banner *et al* 1983; Bagnaia *et al* 1983) established beyond doubt the existence of $W^\pm$ and Z° at the mass values predicted by the $SU(2)_L \times U(1)$ gauge theory. Despite this sccess, it appears that there are some anomalous decay events which indicate a possible departure from our present belief that the Z°'s are elementary as in the standard electroweak model. In these anomalous events, one finds that along with the emission of a lepton pair from the Z°, there emerges a hard photon of energy ~ 30 GeV. The photon emitted is not collinear with either lepton which excludes external bremsstrahlung as a source of the photon. Indeed, as mentioned in the introduction, one observes that the photon momentum makes an angle $\sim (15 \pm 5^\circ)$ with respect to the momentum of one of the leptons. Further study shows that the invariant mass distribution for the $l^+ l^-$ pair peaks near 50 GeV. The present data shows that 20% of the $Z^\circ + l\bar{l}$ decay events are accompanied by hard photons.

We first note that in the standard model, the $Z^\circ + l\bar{l}\gamma$ process takes place through higher order processes if the hard photons do not arise from internal bremsstrahlung. Since an appreciable fraction of Z° decays into lepton pairs is accompanied by a photon, it is unlikely that higher order processes in the standard gauge model could explain the phenomena except for other mechanisms based on excited leptons and quarks which may be considered as elementary exotics. Within the composite model picture, however, these anomalous events may be easily accommodated. One may, for instance, envision the existence of a scalar meson U° with a mass ~ 50 GeV into which the Z° decays accompanied by a photon, followed by the U° decaying into a lepton pair (Baur *et al* 1984; Peccei 1984).

In Fritzsch and Mandelbaum's version of the composite model, we have noted in §2 that the $W^\pm$, W° appear as particular combinations of the constituent haplons; in addition, one has a neutral singlet vector meson V° appearing as the combination $[1/(2)^{1/2}](\alpha\bar{\alpha} + \beta\bar{\beta})$. It is assumed that the haplon constituents are in the relative s-state. As in QCD, they are expected to exist with their scalar partners $X^\pm$, X° and U°: for the scalars, the haplon-antihaplon pair occur with their spins antiparallel. The hypergluon mechanism responsible for the binding is assumed to be such that the V° particle is extremely heavy and does not participate in weak processes. The postulated similarity between QHD and QCD suggests the possible existence of a chiral symmetry at the QHD level. Consequently, the X's may be thought of as Goldstone bosons, not unlike the π-mesons in QCD. The couplings of the ordinary fermions to the X's are probably highly suppressed ($\sim m_f/m_W$) as a result of chiral symmetry and the corresponding PCAC relation (analogous to the Goldberger-Trieman relation). This explains why they are not observed in weak decays such as $\pi^+ \to l^+ \nu_l$. The situation for the V° particle is, however, quite different. ν-hardron scattering can admit a 10% contribution from scalar currents; conceivably, the U° particle contributes to such scalar currents. One thus expect that if the U° particle has a mass $\sim$ 40–50 GeV, its coupling to fermions must not exceed about 1/6 that for charged W bosons to fermions. Assuming universality as for the coupling of W bosons, one has

$$g_{Ue\bar{e}} = g_{U\mu\bar{\mu}} = \ldots = g_{Uuz\bar{u}} = g_{Ud\bar{d}} = \ldots \tag{25}$$

These couplings being flavour diagonal do not contribute to the decay $K^\circ_L \to \mu^+\mu^-$. The U_0 particle could, however, decay into two photons and the decay rate, obtained in the same way as its QCD analog, is given by

$$\Gamma_{U\to 2\gamma} = 8\pi\alpha^2 Q^4 F_U^2/M_U, \tag{26}$$

where Q is the haplon charge, F_U the U° decay constant and M_U its mass. With $F_U \simeq 100$ GeV, $M_U = 50$ GeV, one finds $\gamma_{U\to 2\gamma} \simeq 17$ MeV. Using universal lepton and quark couplings, the branching ratio for $U^\circ \to l\bar{l}$ is easily calculated; Fritzsch and Mandelbaum find

$$B(U^\circ \to l\bar{l}) = 4\text{–}5\%, \tag{27}$$

with $20\,\text{GeV} < M_U < 80\,\text{GeV}$ depending on whether one assumes $M_U \gg 2m_t$ or $M_U \ll 2m_t$ where m_t stands for the mass of the top quark, yet to be observed. From the well-known result for the decay rate of a vector meson into a scalar particle and a photon in QCD, we can write (Jackson 1976)

$$\Gamma(Z + U\gamma) = \tfrac{16}{3}\alpha k^3 \mu^2, \tag{28}$$

where k is the photon momentum and μ the magnetic moment of the haplon *i.e.* $\mu_i = Q_i/2m_i$ where Q_i is the charge of the ith type of haplon and m_i the corresponding mass. Assuming $m_\alpha = m_\beta = M_v/2$ and using $M_v = 50$ GeV, one finds that for a hard photon of energy 30 GeV the branching ratio

$$\frac{B(Z^\circ \to e^+e^-\gamma)}{B(Z^\circ \to e^+e^-)} \simeq 20\%, \tag{29}$$

a value consistent with the present experimental data. We note, however, that the exchange of the U° particle in deep inelastic ν-hadron scattering may cause a deviation in the y-distribution from that which is observed and expected in the standard electroweak theory. Consequently, we discuss an alternative mechanism for the decay $Z^\circ + l^+l^-\gamma$ in bound state models through a four fermi interaction between haplons and leptons with strength $\sim g$, the weak coupling constant, the strength of the coupling being determined by the requirement that the observed four Fermi coupling constant be $\sim G_F$, the Fermi coupling constant.

6. Four fermi haplon lepton interactions and anomalous Z° decays

We consider an alternative mechanism for anomalous Z° decays in bound state models through a direct four fermi haplon lepton coupling without the presence of a scalar U° (Biswas *et al* 1984). We assume an interaction Hamiltonian of the form

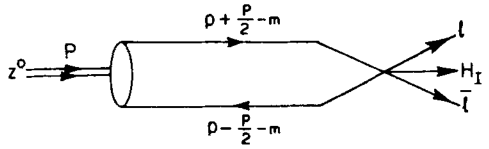

Figure 3. $Z^\circ \to l\bar{l}$ in the composite vector meson model.

$$H_I = f\bar{h}\gamma_\nu(a' - \gamma_5)h\bar{l}\gamma_\nu(a - \gamma_5)l, \tag{30}$$

with f an unknown weak parameter and $a = 1 - 4\sin^2\theta_W$ as in the standard Weinberg-Salam model. The parameter a' denoting the relative strength of the current is also treated as an unknown constant. Consider first the decay $Z^\circ \to l\bar{l}$ in the composite model hypothesis using the Hamiltonian given by (3). Assuming, for simplicity, the leptons to be fundamental, we have the following Feynman diagram
The choice of V, A interactions in (30) is dictated by the observation that the contribution of scalar and pseudoscalar currents to neutral current phenomena is small, particularly in deep inelastic ν-hadron scattering. The amplitude for process may now be written as

$$\begin{aligned} A(Z^\circ \to l\bar{l}) = \frac{i}{(2\pi)^4}\int d^4p\,\mathrm{Tr}\Big\{\Gamma(P; p + \frac{P}{2} - m, p - \frac{P}{2} - m) \\ \cdot\left(p + \frac{P}{2} - m\right)^{-1}\gamma_\nu(a^1 - \gamma_5)\left(p - \frac{P}{2} - m\right)^{-1}\Big\} \\ \cdot\bar{v}_l\gamma_\nu(a - \gamma_5)v_l. \end{aligned} \tag{31}$$

If one uses the relation between the vertex function Γ and the Bethe-Salpeter wave function for the composite Z°, *viz*

$$\begin{aligned} \psi(p, P) = \left(p + \frac{P}{2} - m\right)^{-1}\Gamma\left(P; p + \frac{P}{2} - m, p - \frac{P}{2} - m\right) \\ \cdot\left(p - \frac{P}{2} - m\right)^{-1}, \end{aligned} \tag{32}$$

one obtains the following expression for the decay amplitude:

$$A(Z^\circ \to l\bar{l}) = \frac{i}{(2\pi)^4}\int d^4p\,\mathrm{Tr}\{\psi(p)N_\nu\}L_\nu,$$

where $\quad N_\nu = f\gamma_\nu(a' - \gamma_5) \quad$ and $\quad L_\nu = \bar{v}_l\gamma_\nu(a - \gamma_5)v_l. \qquad (33)$

The decay rate for $Z^\circ \to l\bar{l}$ then becomes

$$\Gamma(Z^\circ \to l\bar{l}) = \int d\phi[i(2\pi)^{-4}\int d^4p\,\mathrm{Tr}\{\psi^V(p)N_\nu\}L_\nu]^2. \tag{34}$$

Here $\int d\phi$ stands for an appropriate phase space integral and $\psi^V(p)$ is the vector meson Bethe-Salpeter wavefunction discussed in (15). Since (34) gives the $Z^\circ \to l\bar{l}$ decay rate, the weak coupling f must have the same dimensions as the coupling g in the standard electroweak model. If we fit the decay rate given by (34) with that obtained in the

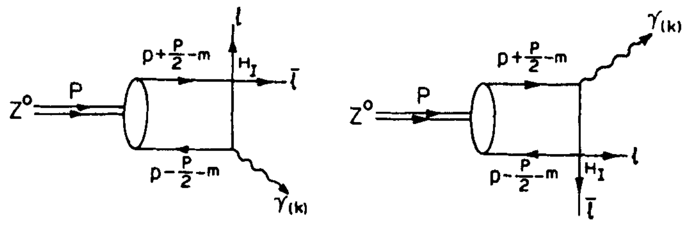

Figure 4. $Z^\circ \to l\bar{l}\gamma$ in the composite vector meson model.

standard model, we may relate the parameters f and a' with those in the standard model. We have, however, suppressed the normalization constant of the Bethe-Salpeter wavefunction since we are only interested in the ratio $\Gamma(Z^\circ \to l\bar{l})|\Gamma(Z^\circ \to l\bar{l}\gamma)$. For the latter process, *viz* $Z^\circ \to l\bar{l}\gamma$ we have the following Feynman diagrams:
Following the same method as for the decay $Z^\circ \to l\bar{l}$, the decay rate for this process may be written as

$$\Gamma(Z^\circ \to l\bar{l}\gamma) = \int \mathrm{d}\phi |i(2\pi)^{-4} \int \mathrm{d}p \,\mathrm{Tr}\{\psi^V(p)\varepsilon_\mu^{Z^\circ}\gamma_\mu \varepsilon_\nu^\gamma \gamma_\nu N_\rho\} L_\rho|^2, \tag{35}$$

with $N_\rho = \left(p + \dfrac{P}{2} - k - m\right)^{-1} \gamma_\rho(a' - r_5) +$ contributions from the second diagram.
From (34) and (35) we get for the branching ratio

$$\frac{\Gamma(Z^\circ \to l\bar{l}\gamma)}{\Gamma(Z^\circ \to l\bar{l})} \simeq \frac{\alpha}{\pi}\frac{1}{a'^2}(M_Z/\Lambda_H)^4. \tag{36}$$

If $\Lambda_H \simeq G_F^{-1/2}$, one obtains 20 % γ-emission in lepton decays of the Z° only for values of $a' \ll 1$. The differential energy spectrum of the emitted photon has not been studied here. If subsequent experimental studies reveal the emitted photon to be monochromatic, this particular bound state model of radiative Z° decays would require modifications; however, the one described earlier in §5 in which the decay proceeds through a scalar particle U° would continue to be acceptable. In either case, a picture of weak vector bosons as composite structures may not be ruled out.

We turn next to a discussion of Renard's model (Renard 1982) for $Z^\circ \to l\bar{l}\gamma$ decays which is based on higher order electromagnetic processes.

7. Renard's model for the decay $Z^\circ \to l\bar{l}\gamma$

Renard assumes that parity violation leads to a $^3S_1 - {}^3P_1$ configuration mixing and that only the 1^{++} component of the Z° undergoes the decay $Z^\circ \to l\bar{l}\gamma$ which proceeds as a higher order electromagnetic decay. Thus, for example, the axial vector component of the Z° decays into a photon and an off-shell photon which subsequently materializes into a lepton pair

$$Z^\circ \to \gamma + \gamma^* \quad (\gamma^* \dashrightarrow l\bar{l})$$

In a composite model for the Z°, the relevant Feynman diagrams for the process are as follows:

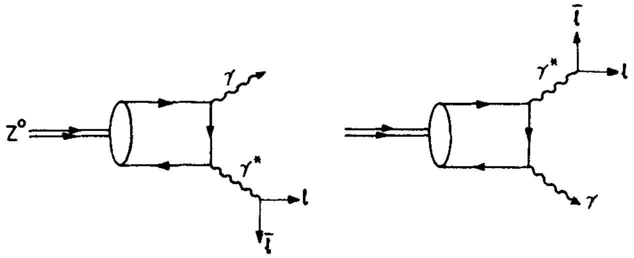

Figure 5. $Z^\circ \to l\bar{l}\gamma$ as a purely electromagnetic decay in the composite model.

This model is similar to that originally considered by Barbieri *et al* (1976) for the decay of a $\mu^+\mu^-$ bound state into $e^+e^-\gamma$. Adapting their model to the present case, Renard observed that in the bound state model, the process is peaked for low $l\bar{l}$ masses, whereas in an external bremsstrahlung process, the peak occurs at higher masses of the $l\bar{l}$ system. The differential branching ratio for $Z^\circ \to l\bar{l}\gamma$ as compared to $Z^\circ \to l\bar{l}$ has been estimated by Renard for k^2/M_Z^2 ($\equiv z$) $\ll 1$ (k being the momentum of the emitted photon); his result is

$$\mathrm{d}B/\mathrm{d}z \simeq 0(10^{-3}), \qquad z \ll 1.$$

This result is to be compared with those obtained when one considers the decays as higher order weak processes; one should then expect this differential branching ratio to be 10^{-3} times the estimate given by the bound state model. If one assumes the photon to appear from external bremsstrahlung, one obtains (for $z \ll 1$) $\mathrm{d}B/\mathrm{d}z \simeq 3 \times 10^{-5}$. It may be noted that in this model, the absolute decay rate $\Gamma(Z^\circ \to l\bar{l}\gamma) \propto \alpha^3$ whereas in the models discussed in §§5 and 6 the rate is proportional to G_F^2. Renard's model thus ascribes a lower rate of decay for the process.

As a further test of the compositeness of the Z°, Renard (1982) suggested a search for the decay mode $Z^\circ \to 3\gamma$. This mode has a branching ratio $0(10^{-4})$ in a composite model whereas in the standard model the ratio is $0(10^{-10})$.

It is necessary to note that if the Z° has coloured subconstituents, then one would expect Z° production through the process $p + \bar{p} \to Z^\circ + g$, g being a gluon (Renard 1983). The rate of Z° production through this mechanism is expected to be of the same order of magnitude as may be expected in a Drell-Yan process in the standard model. However, Renard also finds that if the Z° were composite, large PT Z° production would be seen in $p\bar{p}$ collider experiments contrary to what results from a Drell-Yan process.

Composite models of weak interactions in which leptons and quarks are composite and in which their weak interactions result from residual interactions of their constituents have also been studied, in particular by Abbott and Farhi (1981a, b). We describe in the following the important features of their work.

8. Composite model

The need for such composite models (Abbott and Farhi 1981a, b; Barbieri *et al* 1981) is best seen in the context of the notorious gauge hierarchy problem which has been alluded in §1. In this model, the weak interaction scale is determined not by the magnitude of the vacuum expertation value of a scalar field but by Λ_2, the constant which determines the energy at which the $SU(2)_L$ gauge coupling constant becomes large. This is exactly similar to the situation obtained in QCD in relation to the exact $SU(3)_c$ gauge symmetry. To achieve this result, the standard model is taken at the QHD level, removing the scalar potential so that no spontaneous symmetry breaking occurs. the $SU(2)_L$ coupling value is increased until the parameter Λ_2 is $\sim$ 300 GeV. Recall that in the absence of spontaneous symmetry breaking, $SU(2)_L$ and U(1) continue to be exact symmetries; the U(1)symmetry in this model is the gauge symmetry of electromagnetism. Since $SU(2)_L$ is now an unbroken symmetry and its coupling constant is large at the weak interaction scale $G_F^{-1/2}$ ($\simeq$ 300 GeV) we expect the constituent preons of this model (for which, in the context of weak interactions, we have been using the term haplons) are all confined at the energy scale $G_F^{-1/2}$. The observed quarks and leptons must then be composed of these confined preons. The bound state fermions (quarks and leptons) would then interact through their residual gauge interactions in much the same way in which nucleons interact through the residual nuclear force.

This model of weak interactions then has the symmetry structure $SU(2)_L \times U(1)$ with $SU(2)_L$ playing the role of a global confining gauge symmetry similar to the role of $SU(3)_c$, the colour gauge group in QCD. Thus there are N left-handed $SU(2)_L$ doublets ψ_L^a ($a = 1, \ldots, N$, $N = 4 \times$ number of generations) acting as preons, one complex scalar doublet ϕ and right singlets ψ_R^b ($b = 1, \ldots$). These have the same U(1) quantum number assignments as in the standard theory. For the right-handed singlets, the U(1) hypercharge is the electric charge since, in this model, U(1) is exact and identified with electromagnetism. Since the right-handed particles ψ_R^b are $SU(2)_L$ singlets, they are not confined and correspond to the right-handed quarks and leptons at the observed level. The SU(2) doublets ψ_L^a and ϕ are, however, confined at a scale of Λ_2^{-1}. It is, therefore, necessary to construct the left-handed quarks and leptons out of these confined doublets. One expects most of these bound states to have masses $\sim$ 300 GeV as the interaction scale of this strong $SU(2)_L$ theory is at $\Lambda_2 \cong G_F^{-1/2} = 300$ GeV. Nevertheless, since we expect an underlying chiral symmetry, some of these bound states will have lower masses and may be identified with the light quarks and the leptons.

The quarks and leptons are then constructed as bound states of the left-handed fermions ψ_L^a and a scalar ϕ in the following manner:

$$\left.\begin{aligned} (\nu, u, c, \ldots) &= \phi_i^* \psi_{Lj}^a \\ (e, d, s, \ldots) &= \varepsilon^{ij} \phi_i \psi_{Lj}^a \end{aligned}\right\} \qquad a = 1 \ldots N$$

One easily checks that the charges of the composites are merely the sum of the charges of the constituents and agree with observed quark and lepton charges. The masses of these particles may be imagined as being generated in the following fashion. Once the bound states of the left-handed fermions are formed, they are no longer confined (in this dynamics) and hence one can calculate the overlap of the wave functions of these bound states with those of the right-handed particles, generating a Yukawa type interaction of the form

$$\lambda \bar{\psi}_R \phi_i^* \psi_{Li}, \quad \lambda \bar{\psi}_R \phi_i \psi_{Lj} \varepsilon^{ij}$$

These are just the terms one obtains in the standard model for the generation of quark and lepton masses once the scalar field ϕ develops a nonvanishing expectation value. In the present case, transitions between the fundamental right-handed fermions and the composite left-handed ones will generate masses. On dimensional grounds, these masses $\sim \lambda\Lambda_2$. If λ is chosen to have a value $\sim 10^{-6}$ as in the standard theory, one obtains the correct order of magnitude for the masses of the fermions.

For the description of weak interactions at low energies ($E < G_F^{-1/2}$), this model is identical in content to that of Fritzsch and Mandelbaum described in §2. For example, one can easily construct $\mathrm{SU}(2)_L$ singlet, bound state spin-one bosons of the following form:

$$\phi^* D_\mu \varepsilon \phi^*, \quad \phi D_\mu \varepsilon \phi, \quad \phi^* \bar{D}_\mu \phi$$

having charges $+1$, -1 and 0 respectively where D_μ is the covariant derivative. These can be identified with the W^+, W^- and W^0 b vector bosons. The mass of these W bosons should be around Λ_2, the scale set by the $\mathrm{SU}(2)_L$ coupling constant. The masses of the three W's are equal, on account of the global $\mathrm{SU}(2)_L$ symmetry of the $\mathrm{SU}(2)_L \times \mathrm{U}(1)$ Lagrangian at the QHD level. SU(2) symmetry then demands that the effective fermion-fermion-vector interaction must have the form $g_\mu^{\mathbf{J}} \cdot \mathbf{W}_\mu$, g being the weak coupling constant and $\mathbf{J}_\mu$ the standard left-handed isotriplet current. It is worth noting that the coupling of these vector bosons is only to the left-handed fermions because only these fermions "feel" the strong $\mathrm{SU}(2)_L$. The coupling constant g cannot be calculated but presumably its relation to the $\mathrm{SU}(2)_L$ gauge coupling is similar to the relationship of the ρNN coupling to the $\mathrm{SU}(3)_c$ gauge coupling in QCD. Since g is related to the underlying strong coupling gauge constant, it may turn out to be large *i.e.* of order unity; in such a situation the relation $g^2/8m_W^2 = G_F/\sqrt{2}$ would require a somewhat higher mass for the W boson than that obtained in the standard electroweak theory. Clearly, in this theory there is no possibility of achieving electroweak unification; it is necessary to assume a strong γ–W^0 mixing to generate the neutral current phenomena. This is similar to the situation (in this regard) in the model of Fritzsch and Mandelbaum.

From the above it follows that a strong $\mathrm{SU}(2)_L$ unbroken gauge theory can provide a viable substratum theory of weak interactions. There exist in literature several variants of this model, pointing to the possibility of viable alternatives to the present electroweak gauge theory. Casalbuoni and Gatto (1980) constructed a composite model with scalar preons and noted that they satisfy the t'Hooft anomaly condition. Barbieri *et al* (1981) constructed a left-right symmetric version of such a composite model, elevating parity to the status of an exact symmetry, to be broken only by the vacuum. The key to a decision as to which of these versions is closer to nature lies in the masslessness or otherwise of the neutrino. Mohapatra and Sejanovic (1980) have succeeded in obtaining a relation for the small neutrino mass m_ν in terms of the mass scale at which the right-handed interactions will be manifest which is of the form

$$m_{W_R} = m_{q^2}/m_\nu,$$

where m_{W_R} is the mass of the W-meson associated with the weak right-handed currents and m_q is the quark mass. In the model constructed by Barbieri *et al* (1981) the basic unbroken interaction of the theory is based on the gauge group $\mathrm{SU}(4)_H \times \mathrm{SU}(3)_c \times \mathrm{U}(1)_{e.m.}$. The theory has two sets of fundamental preons which transform under the gauge group as under:

$$F_u^\alpha \sim (4, 1, 1/2),$$

$$F_d^\alpha \sim (4, 1, -1/2).$$

There are also, in the theory, four scalar bosons with the following transformation properties:

$$\phi_i^\alpha \sim (\bar{4}, 3, 1/6),$$

$$\phi_4^\alpha \sim (\bar{4}, 1, -1/2).$$

The interaction Lagrangian involving these fields is locally invariant under the gauge group and is a renormalizable theory. The hypercolour group $SU(4)_H$ is expected to have properties similar to these resulting from $SU(3)_c$ in QCD *e.g.* confinement of nonsinglet hypercolour states etc.

The appearance of scalar bosons in models of this type distinguishes them from QCD like theories in which no scalar bosons appear at the fundamental level. The construction of quarks and leptons in this model proceeds as in the model of Abbott and Farhi (1981). Thus one has

$$u_i = (F_u^\alpha \phi_{\alpha, i}); \quad \nu = (F_u^\alpha \phi_{\alpha, 4}),$$

$$d_i = (F_d^\alpha \phi_{\alpha,i}); \quad e^- = (F_d^\alpha \phi_{\alpha,4}).$$

The mass scale should, as before, be $\sim \Lambda_H$; one also expects a chiral symmetry at the $SU(4)_H$ level to allow the light quarks and leptons to have small masses. To obtain consistency withe the value of $(g-2)$ for the muon, the preon masses have to be vanishingly small. Indeed, this requirement sets a very stringent limit on the compositeness scale, since one expects $(g-2)_{\text{muon}} \simeq (\mu/\Lambda_H)^2$, μ being the preon mass. Thus a scale of $\Lambda_H = 1$ TeV gives rise to $(g-2)_{\text{muon}} \sim 10^{-8}$ (Barbieri and Mohapatra 1983). The W-mesons which mediate the weak interactions are again composite; thus $m_W \simeq \Lambda_H \sim G_F^{-1/2}$. This therefore provides a basis for the smallness of the Fermi coupling constant. As in other models of this type, one uses γ–W° mixing to obtain neutral current phenomena.

9. Conclusion

We have examined the composite model scenario for vector bosons and weak interactions with one principal aim which may be stated as follows: is it possible to establish the point of view that the fundamental interactions of nature are associated with unbroken local Yang-Mills gauge symmetries? It would then follow that the observed particle interactions are the consequences of the residual interactions of the subconstituents appearing in the local gauge theories. If successful, this description would put QHD as the basis of weak interactions in the same way in which QED provides the foundation for the physics of atoms and QCD for the physics of nuclei.

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References

Abbott L F and Farhi E 1981a *Phys. Lett.* **B101** 69
Abbott L F and Farhi E 1981b *Nucl. Phys.* **B189** 547
Arnison G *et al* UA1 Collab. 1983a *Phys. Lett.* **B122** 103
Arnison G *et al* UA1 Collab. 1983b *Phys. Lett.* **B126** 398
Arnison G *et al* UA1 Collab. 1983c *Phys. Lett.* **B129** 273
Bagnaia P *et al* UA2 Collab. 1983 *Phys. Lett.* **B129** 130
Banner M *et al* UA2 Collab. 1983 *Phys. Lett.* **B122** 476
Barbieri R, Gatto R and Remiddi E 1976 *Phys. Lett.* **B61** 465
Barbieri R, Mohapatra R N and Masiero A 1981 *Phys. Lett.* **B105** 369
Barbieri R and Mohapatra R N 1983 *Phys. Lett.* **B120** 198
Baur U, Fritzsch H and Faissner H 1984 *Phys. Lett.* **B135** 313
Biswas S N, Choudhury S R, Datta K and Goyal Ashok 1984 *Delhi University preprint* (to be published)
Bjorken, J D 1979 *Phys. Rev.* **D19** 335
Böhm M and Kramer M 1977 *Nucl. Phys.* **B120** 113
Buchmüller W, Peccei R D and Yamagida T 1983 MPI-PAE/P.Th. 41/83.
Casalbuoni M and Gatto R 1980 *Phys. Lett.* **B93** 47
Chen P and Sakurai J J 1982 *Phys. Lett.* **B110** 481
deGroot E and Schildknecht D 1981 *Z F Phys.* **C10** 55
Fritzsch H and Mandelbaum G 1981a *Phys. Lett.* **B102** 319
Fritzsch H and Mandelbaum G 1981b *CERN* preprint TH 3219
Fritzsch H, Schildknecht D and Kögerler R 1982 *Phys. Lett.* **B114** 157
Fritzsch H and Mandelbaum G 1981c CERN preprint TH 3203
Fritzsch H and Mandelbaum G 1982 *Phys. Lett.* **B109** 224
Greenberg O W and Sucher J 1981 *Phys. Lett.* **B99** 339
Gounaris G, Kögerler R and Schildknecht D 1983 University of Bielefeld preprint BI-TP83/11.
Hung, P Q and Sakurai J J 1978 *Nucl. Phys.* **B143** 81
Jackson J D 1976 LBL preprint 3500
Koruda M and Schildknecht D 1982 *Phys. Lett.* **B121** 173
Mohapatra R N and Senjanovic D 1980 *Phys. Rev. Lett.* **44** 912
Peccei R D 1984 *Phys. Lett.* **B136** 121
Peskin M 1981 *Proc. Int. Symp. on Lepton and Photon Interactions* (Bonn: Univ. of Bonn) (ed) W Pfell
Renard F M 1982 *Phys. Lett.* **B116** 269
Renard F M 1983 *Phys. Lett.* **B132** 449
Schrempp B and Schrempp F 1983 *DESY* 83-24.
Shifman M A, Vainstein A I and Zakharov V I 1979 *Nucl. Phys.* **B147** 385