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Thermal conduction through loose and granular two-phase materials at normal pressure

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Abstract. The integrated theory derived for the lattice-type dispersions is modified and extended to estimate the effective thermal conductivity of loose and granular two-phase materials at normal pressure assuming an effective continuous media approximation. A comparison of calculated values of λ_e with the reported experimental results over a wide range of loose and granular two-phase materials shows a good agreement.

Keywords. Random distribution; continuous media; cubic dispersion; effective continuous media; successive dispersion; neighbouring-interaction.

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1. Introduction

The effective thermal conductivity λ_e of homogeneous two-phase materials has been explored in the past using two distinct approaches. In the first approach λ_e is evaluated by considering a random distribution of dispersed phase (Reynolds and Hough 1957; Hashin and Shtrikman 1962; Hori 1973; Jefferey 1973; Kumar and Chaudhary 1980; Pande *et al* 1983). In the second approach λ_e is estimated by considering a homogeneous dispersion of interacting (Rayleigh 1892; Meredith and Tobias 1960; Mc Phedran and Mc Kenzie 1978; Bergman 1979; Sangani and Acrivos 1981) and non-interacting (Maxwell 1904) spherical particles in a continuous media. Further the latter approach (lattice model) has been explored (Pande and Chaudhary 1984a) to estimate the λ_e of different kinds of two-phase systems including loose materials. However, the common loose materials like beads, sand, soil etc have a variable porosity ranging from 0·35–0·65. The λ_e of such materials has been widely measured but the theoretical investigations (Maxwell 1904; Fricke 1924; De Vries 1952; Eucken 1932; Bruggeman 1935; Russel 1935; Sugawara and Hamada 1970; Böttcher 1945; Ratcliffe 1969; Kumar and Chaudhary 1980; Pande and Chaudhary 1984a) are disappointing. Any single theory is hardly helpful for more than one kind of specimen when there is a large variation in porosity.

The expression developed by Pande and Chaudhary (1984a) to estimate the λ_e of different kinds of two-phase materials such as powders and sands is applicable for a porosity variation of 0–0·4 and 0·6–1. Moreover for all sorts of loose materials like glass and metal shots, sands and soils, cement (loose), concrete, lime etc it does not estimate a reliable value of λ_e when there is a successive variation in porosity in the range 0·3–0·7.

In this paper we modify and extend the earlier theory to various kinds of loose and granular two-phase materials suitable to the porosity range 0·3–0·7.

2. Theory

2.1 *Effective thermal conductivity of two-phase system*

We consider a steady line source in a cylindrical sample container filled with a two-phase sample and assume that there is a homogeneous cubic dispersion of phase λ_2 in the continuous phase λ_1 within the two-phase sample. We consider a position-dependent conductivity function $\lambda(\mathbf{R})$ where

$$\lambda(\mathbf{R}) = \lambda_1 + \delta(\mathbf{R}, \mathbf{r}_i)(\lambda_2 - \lambda_1), \tag{1}$$

such that

$$\delta(\mathbf{R}, \mathbf{r}_i) = 1 \quad \text{for } |\mathbf{R} - \mathbf{r}_i| < a_i,$$
$$= 0 \quad \text{for } |\mathbf{R} - \mathbf{r}_i| > a_i.$$

Here a_i is the radius of the dispersed spheres of phase λ_2. In steady state, the Laplace equation transforms into the Poisson equation. The solution of the Poisson equation evaluated over the considered cubic geometry round the line source yields a field distribution at the source (Pande and Chaudhary 1984a) given as

$$\mathbf{E}(0) = \mathbf{E}_0(0) + \sum_i \left(\frac{\lambda_2 - \lambda_1}{4\pi\lambda_1}\right) \int_s \nabla \cdot \frac{1}{\mathbf{r}_i} [\nabla \phi(r_i) \cdot \mathrm{d}\mathbf{S}]. \tag{2}$$

Considering relevant interactions the field modification $\{\mathbf{E}_0 - \mathbf{E}(0)\}$ is estimated. The effective thermal conductivity of the media is evaluated as:

$$\lambda_e = \lambda_1 \left\{1 + \frac{\langle \mathbf{E} - \mathbf{E}(0) \rangle}{\mathbf{E}_0}\right\}. \tag{3}$$

As the thermal conductivity of air (0·026 $\mathrm{Wm^{-1} K^{-1}}$) is very small as compared to that of the solid phase the ratio $\lambda_s/\lambda_g \to \infty$ and $\lambda_g/\lambda_s \to 0$ for loose and granular two-phase materials. This consideration (when neighbouring interactions are very large) yields expression for the effective thermal conductivity λ_e (Pande and Chaudhary 1984a) as:

$$\lambda_e = \lambda_g\{1 + 3{\cdot}844\psi_s^{2/3}\} \quad (\text{for } 0{\cdot}5 > \psi_s > 0), \tag{4a}$$

and

$$\lambda_e = \lambda_s\{1 - 1{\cdot}545\psi_g^{2/3}\} \quad (\text{for } 0{\cdot}5 > \psi_g > 0), \tag{4b}$$

where ψ_s and ψ_g are the corresponding volume fractions of the solid and the gas phases.

2.2 *Effective continuous media approximation*

Equations (4a) and (4b) are derived considering small cubic dispersions in the continuous media. Moreover these dispersions are not very small as these account the field modification up to the fourth nearest neighbours (Pande and Chaudhary 1984a). The natural loose and granular two-phase materials are mixtures of solid and air phases where each of the phases occupy a large volume fraction (0·3–0·7) of the sample. In this situation none of the phases (solid or gas) provides the continuous media. Therefore (4a) and (4b) do not represent the thermal conductivity of the actual two-phase loose systems. The continuous media that prevails at this porosity ($\psi_s = \psi_g = 0{\cdot}5$) is an effective continuous media composed from both the phases. By allowing a small dispersion of solid or gas phase in the effective continuous media we can produce a porosity of actual loose and granular two-phase materials. Thus an actual two phase material is obtained as a result of very small dispersions (ξ) of solid or gas phase in the effective continuous media, composed by equal volume fractions of the solid and the gas phases. The effective thermal conductivity λ_e of a loose two-phase system formed by making a dispersion of ξ_s volume fraction of solid phase or ξ_g volume fraction of gas phase in the effective continuous media, is estimated through (1a) and (1b) as

$$\lambda_e = \lambda_{ec}(1 + 3{\cdot}844\xi_s^{2/3}) \quad (\text{for } \xi_s > 0), \tag{5a}$$

and

$$\lambda_e = \lambda_{ec}(1 - 1{\cdot}545\xi_g^{2/3}) \quad (\text{for } \xi_g > 0). \tag{5b}$$

Here $\xi_s = \psi_s - 0{\cdot}5$, $\xi_g = \psi_g - 0{\cdot}5$ and λ_{ec} is the thermal conductivity of the effective continuous media.

2.3 *Thermal conductivity of effective continuous media*

As (4a) and (4b) are valid at small dispersions, we consider a small dispersion of $\delta\psi_g$ volume fraction of the gas phase in the continuous solid phase (λ_s). The thermal conductivity of this media through (4b) is

$$\lambda = \lambda_s(1 - 1{\cdot}545\delta\psi_g^{2/3}). \tag{6}$$

Now consider a successive dispersion of gas phase for n times each of value $\delta\psi_g$ in the continuous solid phase. Under the limit $n\delta\psi_g \to 0{\cdot}5$, the resulting media will be the effective continuous media. The thermal conductivity λ_{ec} of this medium following (4b) and (6) is

$$\lambda_{ec} = \lambda_s(1 - 1{\cdot}545\delta\psi_g^{2/3})^n \quad (\text{for } n\delta\psi_g = 0{\cdot}5). \tag{7}$$

λ_{ec} can also be estimated by making n successive dispersions each of value $\delta\psi_s$ in the continuous gas phase. Under the limit $n\delta\psi_s \to 0{\cdot}5$, (4a) and (6) lead to λ_{ec} such that

$$\lambda_{ec} = \lambda_g(1 + 3{\cdot}844\delta\psi_s^{2/3})^n \quad (\text{for } n\delta\psi_s = 0{\cdot}5). \tag{8}$$

The self-consistency of (7) and (8) requires

$$\lambda_{ec}^2 = \lambda_g\lambda_s\{(1 + 3{\cdot}844\delta\psi_s^{2/3})(1 - 1{\cdot}545\delta\psi_g^{2/3})\}^n, \tag{9a}$$

which reduces to

$$\lambda_{ec}^2 = \lambda_g\lambda_s\{1 + 2{\cdot}299\delta\psi^{2/3} - 5{\cdot}939(\delta\psi^{2/3})^2\}^n, \tag{9b}$$

for the limit $\delta\psi_s = \delta\psi_g = \delta\psi$ and $n\delta\psi = 0{\cdot}5$.

Expression (9b) after expansion yields a rapidly converging series provided either $2{\cdot}299\delta\psi^{2/3}$ and $5{\cdot}939(\delta\psi^{2/3})^2$ are negligibly small or n lies in the vicinity of unity. Since $\delta\psi$ is responsible for the fourth nearest neighbour interactions (in the derivation of (4a) and (4b)), the terms $2{\cdot}299\delta\psi^{2/3}$ and $5{\cdot}939(\delta\psi^{2/3})^2$ have a sizable value to justify such a large overlapping of fields from nearest neighbours. Therefore expansion of (9b) is valid in the vicinity of n tending to unity only. As n cannot take values less than unity, the expansion of (9b) for $n \geqslant 1$ yields

$$\lambda_{ec}^2 \geqslant \lambda_g\lambda_s\left\{1 + 2{\cdot}299(n\delta\psi^{2/3}) - 5{\cdot}939\,\frac{(n\delta\psi^{2/3})^2}{n}\right\}. \tag{10}$$

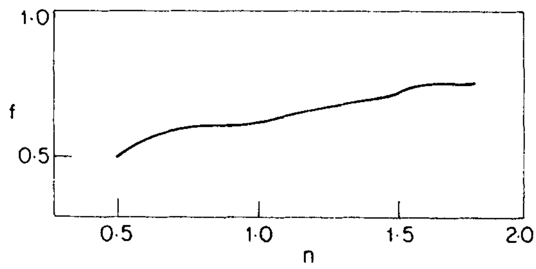

Figure 1. Variation of function f with n.

Solution of (10) is to be determined for $n\delta\psi = 0{\cdot}5$ and $n \geqslant 1$. Let $n\delta\psi^{2/3}$ be represented by a function f. The plot of f vs n (figure 1) indicates stepwise growth in f with n. The region of the curve from $n = 1$–$n = 1{\cdot}5$ is represented through a single step. Since n satisfies the condition $n \geqslant 1$ in this region the f value is estimated in this region only using

$$\bar{f} = \frac{\int f\,\mathrm{d}n}{\int \mathrm{d}n}. \tag{11}$$

Using (11) we find that

$$\bar{f} = 0{\cdot}6776763.$$

Substituting the $\bar{f}$ value in (10) we obtain the thermal conductivity of the effective continuous media as

$$\lambda_{ec} \geqslant 0{\cdot}6132(\lambda_s\lambda_g)^{1/2}. \tag{12}$$

2.4 *Effective thermal conductivity of loose material*

The λ_e value of two-phase loose and granular materials is found by substituting the λ_{ec} value from (12) in (5a) and (5b)

$$\lambda_e \geqslant 0{\cdot}6132(\lambda_g\lambda_s)^{1/2}\{1 - 1{\cdot}545\xi_g^{2/3}\} \quad (\text{for } \xi_g > 0), \tag{13a}$$

and

$$\lambda_e \geqslant 0{\cdot}6132(\lambda_g\lambda_s)^{1/2}\{1 + 3{\cdot}844\xi_s^{2/3}\} \quad (\text{for } \xi_s > 0). \tag{13b}$$

3. Comparison with experimental results and discussion

The effective thermal conductivity of loose and granular two-phase materials has been estimated and reviewed in literature (De Vries 1952; Sugawara and Hamada 1970; Chaurasia 1976; Chaudhary *et al* 1969; Kumar and Chaudhary 1980) using relevant expressions. The estimated λ_e values in the cited literature for sand-like systems using Kumar's expression (Kumar and Chaudhary 1980) resemble the reported experimental results. Table 1 gives the estimated λ_e values using present expressions (13a) and (13b), Maxwell's formula, Lichtnecker's formula and Kumar *et al*'s formula for a variety of loose and granular two-phase materials.

We find that the estimated λ_e values using the present expressions could be better adopted to experimental results as compared to the estimations using any other expression. The agreement is so excellent that in most of the cases the estimated and

Table 1. Comparison of calculated and experimental values of $\lambda_e(Wm^{-1}K^{-1})$ for various kind of loose materials.

Loose material system	λ_s	λ_g	ψ_g	λ_e using: Maxwell formula	Lichtnecker formula	Kumar *et al*'s formula	Present expression	λ_e (Experimental)
Zirconia powder/Air (Ratcliffe 1969)	1·998	0·021	0·53	0·073	0·178	0·134	0·115	0·119
Zirconia powder/Air (Godbee *et al* 1966)	1·998	0·0297	0·42	0·141	0·341	0·258	0·254	0·229
Zirconia Powder/Air (Godbee *et al* 1966)	1·998	0·0297	0·36	0·170	0·439	0·338	0·344	0·363
Glass beads/Air (Tanaeva and Domorod 1972)	1·200	0·0275	0·35	0·174	0·321	0·247	0·231	0·220
Glass beads/Air (Verschoor and Schuit 1950)	1·090	0·0290	0·40	0·137	0·255	0·195	0·198	0·188
Miami silt loam/Air (Smith 1942)	2·93	0·0234	0·448	0·072	0·746	0·254	0·222	0·221
Glass (loose)/Air (Chaurasia 1976)	1·129	0·0263	0·40	0·127	0·250	0·191	0·192	0·176–0·207
Dune sand/Air (Chaudhary *et al* 1969)	3·344	0·026	0·385	1·74	0·515	0·403	0·346	0·387
			0·3876	1·73	0·509	0·391	0·342	0·375
			0·4052	1·67	0·467	0·357	0·336	0·336
			0·4297	1·59	0·415	0·316	0·299	0·312
			0·4394	1·56	0·396	0·301	0·288	0·302
			0·4502	1·52	0·376	0·285	0·275	0·289
			0·460	1·50	0·358	0·278	0·262	0·274
Dune sand/Air (Pande *et al* 1984b)	3·344	0·026	0·480	1·43	0·325	0·244	0·232	0·239
			0·485	1·41	0·317	0·238	0·222	0·220
Mud powder/Air (Pande *et al* 1984b)	5·70	0·026	0·496	2·32	0·393	0·295	0·259	0·256

Table 1. (*Contd.*)

Loose material system	λ_s	λ_g	ψ_g	λ_e using Maxwell formula	Lichtnecker formula	Kumar *et al*'s formula	Present expression	λ_e (Experimental)
Brick sand/Air (Pande *et al* 1984b)	2·85	0·026	0·490	0·658	0·285	0·214	0·196	0·195
Concrete (stone/Air (Pande *et al* 1984b)	2·50	0·026	0·560	0·101	0·194	0·146	0·114	0·111
Cement (loose)/Air (Pande *et al* 1984b)	2·60	0·026	0·56	0·087	0·197	0·148	0·121	0·118
Lime (loose)/Air (Pande *et al* 1984b)	0·75	0·026	0·73	0·051	0·064	0·051	0·068	0·065
Glass/hydrogen (Prins *et al* 1950)	1·087	0·138	0·60	0·298	0·315	0·239	0·432	0·459
Glass/Air (Verschoor and Schuit 1950)	1·087	0·026	0·60	0·072	0·116	0·088	0·188	0·180
Lead shots/Helium (Woodside and Messmer 1961)	34·30	0·147	0·38	0·799	4·317	1·710	2·660	2·420
Selenium/Polypropylene (Bauxley and Coupes 1966)	5·184	0·14	0·60	0·389	0·592	0·451	0·407	0·421

reported values of λ_e differ (systems 2–7, 9–18) within 0–5 %. The average deviation between the reported and estimated values is $\pm 4 \cdot 5$ %.

The comparison also suggests that the assumption of an effective continuous media in loose two-phase systems is closer to reality.

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