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Origin of fine structures in solar radio bursts

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Abstract. The radiation, resulting from the nonlinear interaction of whistler solitons, which act as localized antennae, with the upperhybrid waves in the coronal loop, is shown to give rise to fine structures in solar radio bursts. All the observed features of microwave spikes in radio flares, *e.g.* their frequency, polarization and short duration can be explained by the presence of about 10^6 solitons occupying a volume of $\sim 10^8$ m^3, provided this interaction takes place at low altitudes. However, if this interaction takes place close to the top of the coronal loop, it gives rise to the isolated tadpole 'eyes' features in the dynamic spectra. About 10^9 solitons are needed to account for the observed flux of these 'eyes'.

Keywords. Solar radio bursts; radiation mechanism; whistler solitons; microwave spikes; radio flares.

1. Introduction

The study of fine structures in continuum solar radio emissions gives important information about the plasma processes occurring in the source region and, thus is relevant as a diagnostic tool to explore the solar radio flares. Repeated bursts of microwave emissions during the solar flares of April 11 and April 28, 1978 have been observed by Slottje (1978, 1980) at a frequency of 2·65 GHz. The microwave bursts contained several spikes having durations ranging from 20 ms to 300 ms. The maximum radio flux of several thousands solar flux units (SFU) was associated with some of the microwave spikes. These fine structures were circularly polarized and had high brightness temperatures ($T_b \gtrsim 10^{13}$ K), which indicate that a coherent mechanism must be responsible for the observed bursts. Slottje (1978) had suggested that spiky emission was due to plasma radiation at the fundamental plasma frequency. Holman *et al* (1980), however, suggested a gyrosynchrotron instability mechanism for the occurrence of such microwave bursts.

The dynamic spectrum of type IV decimetric solar bursts between the frequencies 160 MHz to 320 MHz is also found to contain various fine structures like zebra patterns, fiber bursts and tadpole emission (Slottje 1972). Tadpole bursts consist normally of an absorption body, a bright 'eye' or dot at the high frequency side and a weak emission tail at the low frequency side. Zheleznyakov and Zlotnik (1975) tried to explain the excitation of tadpole emission in terms of cyclotron instability of the Bernstein modes. However, there are a number of cases when only separate bright dots ('eyes') have been recorded on the dynamic spectra (Elgaroy 1961; Slottje 1972). The typical duration of such bursts is about 0·1 sec. The appearance of the isolated tadpole 'eyes' (or dots) cannot be explained satisfactorily in terms of Bernstein mode instability as it requires a very large number of relativistic electrons.

Bernold and Treumann (1983) interpreted the fiber fine structure in type IV bursts in terms of the interaction of whistler solitons with plasma waves. They consider both the whistler solitons and the plasma waves to be propagating parallel to the magnetic field.

Following Buti and Lakhina (1984), who discussed the coherent generation mechanism for the auroral kilometric radiations in the earth's magnetosphere, in this paper, we propose a coherent radiation mechanism involving the interaction between whistler solitons and the upper hybrid waves (both the solitons and the upper hybrid waves propagating nearly transverse to the magnetic field) to explain the origin of fine structure in the microwave bursts and of isolated tadpole 'eye' in type IV decimetric bursts.

2. Source model

Let us consider a coronal loop where the fast electrons with energies of few tens of keV are trapped during a solar flare. As the fast electrons stream down towards the base of the loop, their pitch angles increase and they eventually mirror back at some point near the base of the loop. Electrons having their mirror points at a sufficiently small altitude are quickly lost due to collisions with the dense background plasma. As a result only fast electrons, with sufficiently large pitch angles, are trapped and consequently the electron distribution function develops temperature anisotropy and a loss-cone. Such electron distributions can excite large amplitude whistler waves (Kennel and Petschek 1966; Kuijpers 1975, 1980). Moreover the upper hybrid waves can also be excited by fast electrons having loss-cone or beam type distributions (Ashour-Abdalla and Kennel 1978; Zheleznyakov and Zlotnik 1975; Kuijpers 1980). Since near the mirror points the loss cone is the strongest, the large amplitude whistler waves in this region can become modulationally unstable and in turn produce whistler solitons (Karpman and Washimi 1977; Hasegawa 1970; Shukla *et al* 1975). The interaction, between nearly transverse propagating whistler solitons and the upper hybrid waves, generates a nonlinear current $j = -en\mathbf{v}$ (n and $\mathbf{v}$ being the density and the velocity perturbations) which has three components, namely, $j_{\parallel}$, j_L and j_R. All the three components radiative at fundamental ($\omega_0 = \omega_u + \omega_s$, where ω_s and ω_u are respectively the whistler and upper-hybrid wave frequency) as well as the second harmonic whistler sideband of ω_u ($\omega_0 = 2\omega_u \pm \omega_s$). The radiations originating from $j_{\parallel}$ (which are plane-polarized) and j_L (which are left handed (L) circularly polarized) are much weaker than the right handed (R) circularly polarized emission from j_R current component (Buti and Lakhina 1983). The j_R current also contains additional components at frequencies $\omega_0 \simeq 2\omega_u$, $\omega_u \pm 2\omega_s$ and $2\omega_u \pm 2\omega_s$. Amongst all the components of j_R current, the one at the fundamental whistler side band ($\omega_0 = \omega_u + \omega_s$) is the strongest and this emits intense coherent radiation which would show up as fine structure in the continuum in the dynamic bursts spectra. We presume that the continuum radiation exists in the coronal loop; the cause of which is not specified.

The frequency and duration of the fine structure emission would depend on the characteristics of the interaction or source region whose location is decided by the distribution function of energetic electrons injected at the top of the flaring coronal loop. For example, if the injected energetic electrons' population is such that the mirror points lie low on the legs of the loop, the emitted radiation would have a large frequency whereas if the mirroring takes place rather high up on the coronal loop, its frequency would be lower. In our model, the duration of the fine structure is related to the time $t_A = d/V_A$ (d and V_A being respectively the diameter of the loop and the Alfvén velocity at the place where emission occurs), taken by the solitons, moving with Alfvén velocity, to go out of the coronal loop.

3. Radiation emitted

Here we shall describe the radiation mechanism very briefly, the details can be found in Buti and Lakhina (1984).

Let the electric field trapped in the whistler soliton be given by $E_R = (E_x - iE_y)$ where E_x and E_y are the components of a slowly (s) varying amplitudes E_s of the right-hand (R) polarized whistler wave, namely,

$$\mathbf{E}_s = [E_x(\mathbf{x}, t)\hat{x} + E_y(\mathbf{x}, t)\hat{y}]\exp(ik_s z - i\omega_s t).$$

The propagation is taken parallel to the magnetic field $\mathbf{B}_0 = B_0\hat{z}$. Here ω_s is the frequency of the whistler wave and $k_s = k_s\hat{z}$ its wavevector. The functional form of the whistler soliton amplitude is given by Karpman and Washimi (1977):

$$E_R(r) = E_{R0}\,\mathrm{sech}(\kappa r)\exp(ikr + i\tilde{\omega}\tau), \tag{1}$$

where $E_{R0} = (AB_0/\varepsilon^{1/2})$ is the maximum amplitude of the soliton field, A is a constant and $\varepsilon = C^2k_s^2/\omega_s^2$ is the dielectric constant which satisfies the whistler dispersion relation, namely,

$$\varepsilon = 1 + \frac{\omega_p^2}{\omega_s(\Omega - \omega_s)}, \tag{2}$$

where $\omega_p = (4\pi n_0 e^2/m_e)^{1/2}$ is the plasma frequency and $\Omega = (eB_0/m_eC)$ is the electron cyclotron frequency. Equation (1) has been written in the frame moving with the soliton velocity $\mathbf{u}$. Moreover, $r = \mathbf{q}\cdot\mathbf{x} = (\xi\cos\theta_s + \eta\sin\theta_s)$, and $\mathbf{q} = (\sin\theta_s, 0, \cos\theta_s)$ is the unit vector in the direction of the soliton, θ_s is the angle between the soliton velocity and the magnetic field $\mathbf{B}_0$, $\tilde{\omega} = (\omega - \mathbf{k}\cdot\mathbf{u})$ and $\tau = t$, $\xi = (Z - u\cos\theta_s t)$ and $\eta = (x - u\sin\theta_s t)$. Also $\kappa = (\alpha^{1/2}A)$ denotes the inverse width of the soliton. The condition for the existence of the soliton is that $\alpha > 0$ i.e., the soliton must have a real width, κ^{-1}. For the case of nearly transverse propagating 'i.e. $\cos\theta_s \sim (m_e/m_i)^{1/2}$ resonant soliton ($u \simeq V_A$), α is given by Buti and Lakhina (1984),

$$\alpha = \frac{V_A k_s^3 q_x^4 \omega_p^4 (1+\beta_p)^{-1}}{16\kappa V_g q_z^4 Q^2 \varepsilon^2 (\Omega - \omega_s)^4}. \tag{3}$$

Equation (3) is valid provided $\kappa/k_s \ll (m_i/m_e)^{1/2}$ and the following condition is satisfied:

$$1 + \tfrac{1}{2}\omega_p^2\Omega(\Omega - \omega_s)^{-1}(\omega_s^2 - \omega_p^2) \simeq 0. \tag{4}$$

In Equation (3), $\beta_p = (4\pi n_0 T/B_0^2)$ and

$$Q = 1 - \varepsilon\left[1 + \frac{\omega_p^2\Omega}{(\Omega - \omega_s)^3}\right]\left[1 + \frac{\omega_p^2\Omega}{2\omega_s(\Omega - \omega_s)^2}\right]^{-2}. \tag{5}$$

Near the resonance cone, the upper hybrid waves become electrostatic with frequency

$$\omega_u = \left[\omega_p^2 + \Omega^2 - \frac{2\omega_p^2\Omega^2\cos^2\theta_u}{(\omega_p^2 + \Omega^2)}\right]^{1/2}, \tag{6}$$

provided $(\omega_p^2+\Omega^2)^2 \gg 4\omega_p^2\Omega^2\cos^2\theta_u$. Here $\cos\theta_u = \mathbf{k}_u\cdot\mathbf{B}_0/(k_u B_0)$ with $\mathbf{k}_u$ being the wave-vector of the upper hybrid wave which is given by $k_u = (k_{ux}\hat{x}+k_{uz}\hat{z}) = \mathbf{k}_u(\sin\theta_u\hat{x} + \cos\theta_u\hat{z})$.

In the soliton rest frame, the electric field of the upper hybrid wave can be written as

$$\mathbf{E}_u = \mathbf{E}_0\cos(\varepsilon k_{uz}+\eta k_{ux}-\tilde{\omega}_u\tau), \tag{7}$$

where $\mathbf{E}_0 = (E_{0x}\hat{x}+E_{0z}\hat{z})$ is taken as real and $\tilde{\omega}_u = (\omega_u = \mathbf{k}_u\cdot\mathbf{u})$.

The total electric field felt by the plasma electrons is the sum of the soliton and upper hybrid wave fields. The electrons experience large accelerations due to the combined action of these electric fields and are able to radiate much more efficiently (Treumann and Bernold 1981; Buti and Lakhina 1984).

The nonlinear current $\mathbf{j}$ can be obtained by solving for n and $\mathbf{v}$, the equation of continuity and momentum conservations of electrons. On using the Fourier transformation,

$$f(\sigma,\rho,\mu) = \iiint \mathrm{d}\tau\,\mathrm{d}\xi\,\mathrm{d}\eta\, f(\tau,\xi,\eta)\cdot\exp i(\sigma z-\xi\rho-\eta\mu), \tag{8}$$

the fundamental whistler sideband ($\sigma = \omega_u+\omega_s$) of j_R current component turns out to be (Buti and Lakhina 1984):

$$j_R^F(\sigma,\rho,\mu) = \frac{\pi^3 n_0 e^3 E_{R0}}{2\kappa m_e^2\lambda_0}\left[\frac{k_{uz}E_{0z}}{\omega_u^2}+\frac{k_{ux}E_{0x}}{(\omega_u^2-\Omega^2)}+\frac{E_{0x}(\mu-k_{ux})}{(\omega_u+\Omega)(\lambda_0+\Omega)}\right]\cdot$$
$$\cdot\,\delta(\sigma-\tilde{\omega}_u-\tilde{\omega}_s+\tilde{\omega})\delta(\lambda_1)\,\mathrm{sech}\,(\pi\lambda_2/2\kappa), \tag{9}$$

where

$$\lambda_0 = [\sigma-\tilde{\omega}_u+u(\rho-k_{uz})\cos\theta_s+u(\mu-k_{ux})\sin\theta_s-\Omega], \tag{10}$$

$$\lambda_1 = [(\mu-k_{ux})\cos\theta_s+(k_s+k_{uz}-\rho)\cdot\sin\theta_s], \tag{11}$$

and

$$\lambda_2 = [k+(k_s+k_{uz}-\rho)\cos\theta_s-(\mu-k_{ux})\sin\theta]. \tag{12}$$

The last term in square bracket in (9) arises from the coupling of density perturbations (produced by the soliton propagating at an angle to $\mathbf{B}_0$) with the R component of velocity perturbations. For $\theta_s = 0$, from (10) to (12), we see that this term vanishes.

The power emitted per unit solid angle is simply given by

$$\frac{\mathrm{d}P}{\mathrm{d}\phi} = \frac{cR_0^2k_0^2}{16\pi}(1+\cos^2\theta)|A_R^F|^2, \tag{13}$$

where $k_0 = (\omega_0/c)$ is the wavenumber of the emitted radiation at distance R_0 from the source, $\omega_0 \simeq (\omega_u+\omega_s)$ is the frequency of radiation, θ is the angle of emission with respect to $\mathbf{B}_0$ and A_R^F is the vector potential corresponding to the current j_R^F. Under the far field approximation, A_R^F is given by

$$A_R^F(\omega_0) = \frac{\exp(ik_0R_0)}{cR_0}\int_0^\infty \mathrm{d}r\; r.$$
$$\times\int_{-\infty}^{\infty}\mathrm{d}\mu\, j_R^F(\tilde{\omega}_0,\,k_0\cos\theta,\,\mu)\cdot J_0[r(k_0\sin\theta-\mu)], \tag{14}$$

where J_0 is the Bessel function of order zero. On taking the functional dependence of κ as $\kappa = \kappa_0\,[\exp(-x^2)]$, with $X = r/R$ and $\kappa_{0R} \gg 1$ (R being the transverse length of the soliton), it is found that the maximum power output occurs when the following conditions for the wavevectors are fulfilled:

$$(\omega_0/c)\cos\theta \simeq k_s + k_{uz} + k\cos\theta_s \tag{15a}$$

and

$$(\omega_0/c)\sin\theta \simeq k_{ux} + k\sin\theta_s. \tag{15b}$$

Making use of the matching conditions (15a) and (15b) and integrating (13) over the solid angle ϕ, we get:

$$P_{\max} = \frac{3\pi^5\omega_p^6 k_{ux}^2\omega_0^2 n_0 T^2 R^4 W_u W_R}{128\, m_e c^3 \kappa_0^2 \cos^2\theta_s(\Omega-\omega_s)^2} \cdot \times \left[1 + \frac{k^2}{k_{ux}^2\omega_s^2}(\omega_u+\Omega)^2\sin^2\theta_s\right]\cdot(\omega_u^2-\Omega^2)^{-2}, \tag{16}$$

with $W_u = E_{0x}^2/(8\pi n_0 T)$ and $W_R = E_{R0}^2/(8\pi n_0 T) = \kappa_0^2/(2\beta_p\varepsilon\alpha)$. Since by definition $W_R/\kappa_0^2 = (2\varepsilon\beta_p\alpha)^{-1}$, (16) shows that $P_{\max}$ depends on the properties of the whistler solitons contained in the parameter α which in turn is defined completely (as given by (3)) by the plasma parameters. In writing (16), we have used the restrictions that $(E_{0z}k_{uz}/k_{ux}E_{0x}) \ll 1$ and $\cos\theta_s \neq 0$.

In the next two sections, we will show the potential applications of our model to interpret the observed solar flare microwave spikes as well as the isolated tadpole 'eyes'.

4. Solar flare microwave spikes

We consider the source region of microwave spikes, observed at a frequency of about 2·65 GHz (c.f. §1), to lie far down on the legs of a flaring loop (Slottje 1978, Holman *et al* 1980). This region of the loop is characterised by the parameters: $B_0 \simeq 800$ G, $n_0 \simeq 10^{10}$ cm^{-3} and $T \simeq 10^6$ K; correspondingly $\omega_p/\Omega \simeq 0{\cdot}4$ and $\beta_p = 2{\cdot}7 \times 10^{-5}$. In this case, (4) is satisfied for $\omega_s \simeq \omega_p/\sqrt{2}$. So the frequency of the emitted radiation would be $\omega_0 = (\omega_u + \omega_s) \simeq \Omega(1 + 0{\cdot}7\,\omega_p/\Omega) \simeq 2{\cdot}8$ GHz. Let us take $\cos\theta_s = (m_e/m_i)^{1/2}$; in which case the parameter α is appropriately defined by (3). Further we take $R\kappa_0 = 10$, $k/k_{ux} = 0{\cdot}01$ and $\kappa_0/k_s = 0{\cdot}1$, then the values of κ_0/k_{ux}, W_R and the soliton volume $V_s = \pi R^2/k_0$ can be found self-consistently in terms of the known parameters. For the parameters stated above, we find that $\kappa_0/k_{ux} \simeq 0{\cdot}03$, $V_s = 6{\cdot}1 \times 10^7$ cm^3 and $W_R = 1{\cdot}1 \times 10^{-4}$. We consider the saturation level of upper hybrid waves to be $W_u \simeq 10^{-6}$, which is about one order of magnitude higher than the thermal fluctuation level expected at the number density and temperature considered here. Then, from (16), we obtain $P_{\max} \simeq 3{\cdot}78 \times 10^{13}$ ergs sec^{-1} and consequently, the radio flux at 1 AU from a single soliton would be $F = 5{\cdot}8 \times 10^{-4}$ SFU (1 SFU $= 10^{-19}$ ergs cm^{-2} sec^{-1} Hz^{-1}). Observationally, the maximum flux associated with microwave spike events is $\sim 10^3$ SFU (Slottje 1978, 1980) which is about (2×10^6) times higher than the flux emitted by a single soliton. In our model, the soliton acts as a localized antenna occupying a small volume V_s. A total number ($N_s = 2 \times 10^6$) of solitons, occupying a volume $V_R = N_s V_s \simeq 10^8$ m^3, are, therefore, needed to explain the observed peak radio flux. Further, the wavenumber matching conditions given by (15) require that the radiation be emitted nearly transverse to $\mathbf{B}_0$. Since the solitons propagate nearly transverse to $\mathbf{B}_0$ with speed $U \simeq V_A$, the emission would only last as long as the solitons remain in the emission region i.e., for the time $t_A = d/V_A$ (d being the diameter of the loop). For $B_0 \simeq 800$ G and $n_0 = 10^{10}\,\mathrm{cm}^{-3}$, $V_A \simeq 1{\cdot}74 \times 10^4\,\mathrm{km\,sec}^{-1}$; correspondingly, $t_A \simeq$ (22–220) msec for d = (400–4000) km. It is interesting to note that the observed duration of spikes ranges between 20–300 msec. The existence of more than one spike in the dynamic spectra can be attributed to either the generation of fresh whistler solitons due to the modulational instability or to the injection of energetic electrons into the loop several times during the flare event.

5. Interpretation of isolated tadpole 'eyes'

The observations (cf. §1) suggest that the isolated tadpole 'eyes' bursts occur high up *i.e.*, near the top of the coronal loop, where $n_0 \simeq 5 \times 10^8\,\mathrm{cm}^{-3}$, $B_0 \simeq 15\,\mathrm{G}$, $V_T = (K_B T/m_e)^{1/2} = 10^{-2}\,\mathrm{c}$, $\beta_p = 2{\cdot}5 \times 10^{-3}$, $V_A = 1{\cdot}5 \times 10^3\,\mathrm{km\,sec}^{-1}$ and $\omega_p/\Omega = 5$. For these parameters, we see that (4) is satisfied for $\omega_s \simeq \Omega/2$. Thus the frequency of emission in this case is $\omega_0 = (\omega_u + \omega_s) \simeq 1{\cdot}1\ \omega_p \simeq 220$ MHz. For the parameters $R\kappa_0 = 10$, $k/k_{ux} = 0{\cdot}1$, $\kappa_0/k_s = 1{\cdot}0$, we find that $V_s = 5 \times 10^6\,\mathrm{cm}^3$, $\kappa_0/k_{ux} = 0{\cdot}9$ and $W_R = 2 \times 10^{-4}$. We take the level of upper hybrid wave to be $W_u \simeq 10^{-6}$, then (16) gives $P_{\max} = 5{\cdot}9 \times 10^8\,\mathrm{erg\,sec}^{-1}$. The radio flux at 1 AU is then $F = 1{\cdot}3 \times 10^{-7}$ SFU. The flux associated with the tadpole 'eyes' is typically of the order of 10^2 SFU. Therefore, to explain the observed flux, about 10^9 solitons are needed. The corresponding volume of emission region is about $V_s = 5 \times 10^9\,\mathrm{m}^3$. Moreover, the maximum duration of the 'eye' according to our model would be equal to $t_A \simeq 0{\cdot}25$ sec for coronal loop of diameter 400 km. This is comparable to the observed duration $\sim 0{\cdot}1$ sec.

6. Discussion and conclusions

From §4, it is clear that the coherent, high intensity radiation resulting from the interaction of nearly transverse propagating whistler solitons and upper hybrid waves on the coronal loops would manifest themselves as fine structures on the continuum in the dynamic spectra of the bursts. According to our model, when the interaction between solitons and upper hybrid waves is favoured at low altitudes, the emission produces microwave spikes of duration 20 msec to 200 msec. All the features of the microwave spikes like their frequency, polarization and the observed flux can be very well explained by the presence of about 10^6 solitons occupying a volume of $10^8\,\mathrm{m}^3$ or so in the coronal loop. We may point out that gyrosynchrotron instability mechanism of Holman *et al* (1980) could explain the frequency and polarization of the microwave spikes, but its efficiency to produce the observed radio flux has not been proven.

The model proposed here is also suited to explain the origin of isolated tadpole 'eyes' or dots in type IV bursts which could not be explained by the Bernstein mode instability mechanism of Zheleznyakov and Zlotnik (1975). Choosing the parameters characteristics of the interaction region lying close to the top of the loop, the predicted values the frequency, radio flux and the duration of the isolated 'eyes' agrees very well with the observations.

Our model may also be relevant to explain some of the fine structures observed in type I and III bursts as well as in x-ray bursts.

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