Pramāṇa, Vol. 23, No. 2, August 1984, pp. 257–261. © Printed in India.

$1/f$ noise and resistivity of spin glasses

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MS received 27 January 1984

Abstract. Electrical resistivity $\rho(T)$ of spin glasses within the framework of Mookerjee and Chowdhury's percolation model where there is a distribution of relaxation times (DRT) is calculated. $\rho(T)$ thus calculated is in better qualitative agreement with experimental results than that in the single relaxation time model.

Keywords. Electrical resistivity; spin glasses; percolation model.

PACS No. 75·40 – s; 72·15 – v

1. Introduction

It is now experimentally well established that there exists a distribution of relaxation times (DRT) in spin glasses (SG) (Mydosh 1983). This has also been theoretically confirmed for the Sherrington-Kirkpatrick (SK) model (Sompolinsky 1981; Hertz 1983a; Sommers 1983). However, in the SK model an essential ingredient is the infinitely weak, infinitely long-ranged interaction. This is an ideal situation which may not be realizable in practice. In the dynamical formulation of the SK model all the relaxation times diverge in the thermodynamic limit (Mackenzie and Young 1982). Therefore, the SK model exhibits a genuine phase transition from the paramagnetic to the SG phase. The fundamentally crucial feature that leads to this cooperative phase transition in SK model is that the corresponding random exchange matrix J does not have any localized eigenstate (Young and Stinchcombe 1976). On the other hand, realistic models of the interaction matrix J, *e.g.* damped RKKY, do have localized eigenstates (Anderson 1970, 1978; Hertz 1983b), and hence the nature of the transition may differ from that in the SK model. It is now generally believed (Anderson 1978; Chowdhury and Mookerjee 1984) that it is the localized eigenstates which so convincingly mimic the clusters of the Tholence-Tourmier-Wohlfarth (TTW) phenomenological model (Tholence and Tournier 1974; Wohlfarth 1977). We have developed a percolation model (Mookerjee and Chowdhury 1983) which describes SG transition as a percolation of clusters of dynamically correlated spins. The percolating spin clusters have been shown to be closely related to the TTW model and the localization model (Chowdhury and Mookerjee 1983a, 1984). In this paper we shall derive an expression for electrical resistivity $\rho(T)$ in canonical SG, *e.g.* $\underline{\text{Au}}$Fe based on the percolation model, which is in qualitative agreement with experimental results. So far as $\rho(T)$ is concerned, it has been calculated by Fischer (1979) for $T \gg T_g$ and $T \ll T_g$ where T_g corresponds to the cusp in the low field low frequency a.c. susceptibility. Our work is complimentary to that of Fischer in the limited sense that we shall be concerned mainly with T near T_g.

2. Resistivity

The system considered is modelled by a collection of n spins randomly distributed over N lattice sites and interacting *via* indirect damped RKKY interaction (De Gennes 1962). We shall calculate resistivity assuming only two sources of conduction electron scattering: (a) potential fluctuations V and (b) s-d exchange interaction J both of which are assumed to be isotropic. The Hamiltonian can be written as

$$H = H_0 + (1/N)\sum_{\mathbf{k}}\sum_{\mathbf{k}'}\sum_{\sigma}\sum_{\mathbf{r}_i} n_i V_{\mathbf{k}\mathbf{k}'} \exp\{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}_i\}\, a^{+}_{\mathbf{k}'\sigma}a_{\mathbf{k}\sigma}$$

$$-(1/N)\sum_{\mathbf{k}}\sum_{\mathbf{k}'}\sum_{\gamma_i} n_i J_{\mathbf{k}\mathbf{k}'}[(a^{+}_{\mathbf{k}'\uparrow}a_{\mathbf{k}\uparrow} - a^{+}_{\mathbf{k}'\downarrow}a_{\mathbf{k}\downarrow})S_i^z + a^{+}_{\mathbf{k}'\downarrow}a_{\mathbf{k}\uparrow}S_i^{+}$$

$$+ a^{+}_{\mathbf{k}'\uparrow}a_{\mathbf{k}\downarrow}S_i^{-}]\exp\{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{r}_i\}, \qquad (1)$$

where H_0 is the Hamiltonian of the s-electrons in the host, n_i is the occupation probability *i.e.* $n_i = 1$ if the ith site is occupied by a magnetic atom and zero otherwise. We shall calculate the spin flip and non-spin flip scattering contributions separately neglecting any short range ferro or antiferromagnetic order.

2.1 *Non-spin flip scattering* ($\sigma = \sigma'$)

In the Born approximation, the probability of transition from state $\mathbf{k}\sigma$ to $k'\sigma$ is given by

$$\sum_{\mathbf{k}'\sigma}\Gamma(k\sigma \to k'\sigma) = A + B + C, \qquad (2)$$

where

$$A = (J^2/N)\sum_i n_i \int_{-\infty}^{\infty} \mathrm{d}\omega\, R_i^{zz}(\omega)\left\{\frac{f(\varepsilon_k) - f(\varepsilon_k - \omega)}{1 - e^{\beta\omega}}\right\}$$

$$\sum_{\sigma} N(\varepsilon_{k\sigma} + \omega), \qquad (3)$$

$$B = (V^2/N)[-\beta^{-1}(\mathrm{d}f/\mathrm{d}\varepsilon_k)\sum_i n_i \sum_{\sigma} N(\varepsilon_{k\sigma})], \qquad (4)$$

$$C = (2JV/N)[-\beta^{-1}(\mathrm{d}f/\mathrm{d}\varepsilon_k)\sum_i n_i \langle S_i^z \rangle \sum_{\sigma}\hat{\sigma} N(\varepsilon_{k\sigma})], \qquad (5)$$

where f is the Fermi function, $\hat{\sigma}$ is the unit vector along σ and $N(\varepsilon_{k\sigma})$ is the density of states of electrons with energy ε_k and spin σ,

$$R_i^{zz}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \exp(i\omega t)\langle S_i^z(t) S_i^z(0)\rangle\, \mathrm{d}t.$$

The spin autocorrelation $\langle S_i^z(t) S_i^z(0)\rangle$ decays exponentially to its equilibrium value as $\exp(-t/\tau_i)$ with a characteristic relaxation time τ_i. In the percolation model spins belonging to the same clusters have the same relaxation time, equal to the relaxation time of the cluster, but spins belonging to different clusters have different relaxation times. Therefore, there is a DRT in SG given by $g(\tau)$. It has been shown by Chowdhury and Mookerjee (1983b) that the distribution

$$g(\tau) = \begin{cases} \dfrac{1}{\ln(\tau_2/\tau_1)}\dfrac{1}{\tau} & \text{for } \tau_2 > \tau > \tau_1 \\ 0 \text{ otherwise} \end{cases} \tag{6}$$

holds near T_g. This distribution will be utilized later in this paper.

2.2 *Spin flip scattering* ($\sigma \neq \sigma'$)

In this case

$$\sum_{k'\sigma'} \Gamma(\mathbf{k}\sigma \to \mathbf{k}'\sigma') = (J^2/N)\sum_i n_i \int_{-\infty}^{\infty} d\omega$$
$$\{R_i^{xx}(\omega) + R_i^{yy}(\omega)\}\left[\frac{f(\varepsilon_k) - f(\varepsilon_k)}{e^{\beta\omega} - 1}\right]$$
$$\sum_\sigma N(\varepsilon_{k\sigma} + \omega). \tag{7}$$

Now onward we shall consider $\varepsilon = \varepsilon_F$, the Fermi energy, because only the electrons near the Fermi surface contribute dominantly to resistivity. This is a good approximation to our problem because we are concerned only with canonical SG near T_g which is usually quite a low temperature. We shall also utilize the statistical isotropy of SG i.e. $R^{xx} = R^{yy} = R^{zz}$.

Before calculating $\rho(T)$ in the percolation model we consider a *special case*. Suppose that

(a) all the spins are frozen cooperatively at T_g, so that $1/N \sum_i n_i R_i^{zz}(\omega) = cR^{zz}(\omega)$ where c is the concentration of magnetic impurities, and
(b) all the spins have the same relaxation time τ_1, *i.e.* $g(\tau) = \delta(\tau - \tau_1)$. In this case,

$$\rho(T) \propto c\left[V^2 + 3\beta J^2 \int d\omega \frac{\omega R(\omega, \tau_1)}{(e^{\beta\omega} - 1)}\right], \tag{8}$$

where

$$R(\omega, \tau_1) = Q\,\delta(\omega) + \left[\frac{S(S+1)}{3} - Q\right]\frac{\tau_1/\pi}{\omega^2\tau_i^2 + 1}. \tag{9}$$

Carrying out the integration in (8) we get

$$\rho(T) \propto \left\{cV^2 + 3\beta J^2\left[\frac{Q}{\beta} + \left\{\frac{S(S+1)}{3} - Q\right\}\frac{\phi(T, \tau_1)}{\pi\tau_1}\right]\right\},$$

where

$$\phi(T, \tau_1) = \ln y - (2y)^{-1} - \psi(y),$$

with $y = \beta/2\pi\tau_1$ and $\psi(y)$ is the digamma function. Ramakrishnan (1974) has given an approximate form for $\phi(y)$ valid over a large range of y (except at very high temperatures):

$$\phi(y) = (2y + 12y^2)^{-1}. \tag{10}$$

A careful analysis (Chowdhury 1983) shows that $\rho(T)$ exhibits a cusp at T_g, in the single relaxation time model. No such cusp has been observed experimentally (Ford and Mydosh 1976; Campbell *et al* 1982).

In the percolation model, (8) gets modified to

$$\rho(T) \propto c\left[V^2 + 3\beta J^2 \int d\omega \int d\tau \frac{\omega R(\omega,\tau) g(\tau)}{(e^{\beta\omega}-1)} \right]. \tag{11}$$

For mathematical simplicity we interchange the integrations over ω and τ in (11) to get

$$\rho(T) \propto c\left\{ V^2 + 3\beta J^2 \left[\frac{Q(T)P(T)}{\beta} + \left\{ \frac{S(S+1)}{3} - Q(T)P(T) \right\} \int \frac{\phi(T,\tau)}{\pi\tau} g(\tau) d\tau \right] \right\}, \tag{12}$$

since in the percolation model, only a fraction $P(T)$ of the spins is included in the infinite cluster and have non-zero long time ($t \to \infty$) autocorrelation $Q(T)$. Finally, putting (6) into (12)

$$\rho(T) \propto c\left\{ V^2 + 3J^2 \left[QP + \left\{ \frac{S(S+1)}{3} - QP \right\} \frac{\ln\{\pi k_B T\tau_2 + 3)/(\pi k_B T\tau_1 + 3)\}}{\ln(\tau_2/\tau_1)} \right] \right\}. \tag{13}$$

Since $Q(T=0) = S^2/3$, $P(T=0) = 1$, and since both $Q(T)$ and $P(T)$ vanish for $T \geqslant T_g$, (13) approaches the correct Yoshida limits (Yoshida 1957):

$$\rho(0) \propto c[V^2 + J^2 S^2],$$

and

$$\rho(\infty) \propto c[V^2 + J^2 S(S+1].$$

Also, notice that $\rho(T_g) < \rho(\infty)$. Differentiating (13) one can check that $d\rho/dT > 0$ at all T. However, one should not use (13) for $T \gg T_g$. Therefore, (13) implies that $d\rho/dT > 0$ for all T near T_g. The behaviour of $d\rho/dT$ at high temperatures can be found from a different consideration. Since all the clusters break up at very high temperatures, $g(\tau) \to \delta(\tau - \tau_1)$ as $T \to \infty$, where τ_1 is the relaxation time of a single spin. Therefore, for $T \gg T_g$ (say $T > 6T_g$),

$$3\beta \int_{-\infty}^{\infty} \frac{\omega\, d\omega}{(e^{\beta\omega}-1)} \int R(\omega,\tau) g(\tau) d\tau \simeq 3\beta \int_{-\infty}^{\infty} \frac{\omega R(\omega,\tau_1)}{(e^{\beta\omega}-1)} d\omega. \tag{14}$$

It is straightforward to check that the right side of (21) approaches its asymptotic value $S(S+1)$ for $T \to \infty$, *with a positive slope*. Therefore, $d\rho/dT > 0$ for both $T \gg T_g$ (from (14)) and $T \simeq T_g$ (from (13)). Moreover, it is very unlikely that $\rho(T)$ will exhibit any maxima or minima between these regimes of temperature. So, finally, $\rho(T)$ is a smooth function that approaches correct Yoshida limits for $T \to 0$ and $T \to \infty$, has a positive slope at all T, and does not exhibit any cusp at T_g. Such a behaviour is in qualitative agreement with experiments (Ford and Mydosh 1976; Campbell *et al* 1982).

Experimental data on $\rho(T)$ sometimes show a maximum at a finite temperature $T_m \gg T_g$. This does not follow from (13) because our analysis goes only up to the order J^2; the decrease of $\rho(T)$ beyond T_m is "reminiscent of Kondo effect" (Fischer 1981) for which at least terms of the order J^3 have to be taken into account.

3. $1/f$ Noise

In our calculation of resistivity we have utilized the relation

$$r(\omega) = \int r(\omega, \tau) g(\tau) \mathrm{d}(\tau), \tag{15}$$

where

$$r(\omega, \tau) = (\tau/\pi)/(\omega^2\tau^2 + 1) = R(\omega, \tau) - R_{eq}\delta(\omega).$$

Using (6) in (15) we get

$$r(\omega) = \frac{[\tan^{-1}(\omega\tau_2) - \tan^{-1}(\omega\tau_1)]}{\ln(\tau_2/\tau_1)}$$

which can be approximated as follows:

for $\omega < \tau_2^{-1}$, $r(\omega) \sim$ constant

for $\tau_2^{-1} < \omega < \tau_1^{-1}$, $r(\omega) \sim 1/\omega$

and for $\tau_1^{-1} < \omega$, $r(\omega) \sim 1/\omega^2$.

In other words, we get $1/f$ noise in the time scales $\tau_1 < t < \tau_2$. Thus $1/f$ noise is intimately related to the logarithmic relaxation of magnetization in SG (Kogan 1981; Chowdhury and Mookerjee 1983b). Thus, we conclude that the smooth behaviour of $\rho(T)$ near T_g and the logarithmic relaxation of magnetization have the common origin in the DRT in SG and hence the $1/f$ noise.

References

Anderson P W 1970 *Mat. Res. Bull.* **5** 549
Anderson P W 1978 in *Ill condensed matter* (eds) R Balian, R Maynard and G Toulouse (North Holland)
Campbell I A, Ford P J and Hamzic A 1982 *Phys. Rev.* **B26** 5195
Chowdhury D 1983 Ph.D. Thesis, IIT, Kanpur
Chowdhury D and Mookerjee A 1983a *J. Phys.* **F13** L19
Chowdhury D and Mookerjee A 1983b *Solid State Commun.* **48** 887
Chowdhury D and Mookerjee A 1984 *Phys. Lett.* **A** (in press)
Fischer K H 1979 *Z. Phys.* **B34** 45
Fischer K H 1981 *Z. Phys.* **B42** 27
Ford P J and Mydosh J A 1976 *Phys. Rev.* **B14** 2057
Hertz 1983a *J. Phys.* **C16** 1219, 1233
Hertz J A 1983b NORDITA preprint, (see also *Phys. Rev. Lett.* **51** 1880)
Mackenzie N D and Young A P 1982 *Phys. Rev. Lett.* **49** 301
Mydosh 1983 in *Proc. Heidelberg colloq. on spin glasses* (Berlin: Springer Verlag)
Mookerjee A and Chowdhury D 1983 *J. Phys.* **F13** 431
Ramakrishnan T V 1974 *Phys. Rev.* **B10** 4014
Sommers H J 1983 *Z. Phys.* **B50** 97
Sompolinsky H 1981 *Phys. Rev. Lett.* **47** 935
Tholence J L and Tournier R 1974 *J. Phys.* **35** C4-229
Wohlfarth E P 1977 *Physica* **B86–88** 852
Young A P and Stinchcombe R B 1976 *J. Phys.* **C9** 4419
Yoshida K 1957 *Phys. Rev.* **107** 396