Pramāṇa, Vol. 23, No. 2, August 1984, pp. 199–203. © Printed in India.

Which is heavier—*d* or *u* quark?

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MS received 18 August 1983; revised 10 April 1984

Abstract. For a large class of phenomenological potential models motivated by quantum chromodynamics, we have studied the behaviour of bound state masses as the constituent mass is increased and found that the mass of a quark-antiquark bound state increases when a constituent mass is increased. It appears, for these potentials, that d quark is heavier than u quark.

Keywords. u quark; d quark; bound state mass; quantum chromodynamics.

PACS No. 12·40 Qq

1. Introduction

The mass of a hadron increases when d quark replaces u quark as a constituent (Kelley *et al* 1980). This is easily understood if d quark is heavier than u quark. However, Carydas and Lichtenberg (1981) observed that under some special choice of the strong interaction between quarks, replacement of u by d can increase the mass of a hadron even if d is lighter than u. They consider the power law potential

$$V(r) = \lambda r^{\nu} \quad \lambda\nu > 0, \nu > -2 \tag{1}$$

where r is the distance between two particles of masses m and M ($m \leqslant M$) and λ, ν are constants. Using relativistic kinematics they show that for $\lambda > 0, \nu > 0$ the bound state mass increases as the constituent mass is increased and for $\lambda < 0, \nu < 0$ the mass of the bound state increases as the constituent mass decreases provided ν is in the range $-2 < \nu < M/(m+M) - 2$. They suggested that the colour-magnetic interaction (De Rujula *et al* 1975) taken together with colour-Coulomb interaction could lead to an effective ν value which lies in the range $-2 < \nu < M/(m+M) - 2$. However, the net result of varying the quark masses with such a potential was not calculated in their work.

We, on the other hand, demonstrate by considering the explicit expressions of bound state mass for a large number of potentials that replacement of u by d can increase the mass of a hadron only when d is heavier than u. The potentials for which the variations of the bound state mass with constituent mass are studied include arbitrary power law potentials ($ar^{\nu} + b$ with $a > 0$, $\nu > 0$), Coulomb plus simple harmonic oscillator potential, Coulomb plus linear potential and the quantum chromodynamics (QCD) motivated potential of De Rujula *et al* (1975). The demonstration with the last potential does throw new light on our understanding of strong interaction. The studies on these potentials are given in §§ 2·1–2·4. The results are discussed in § 3.

2. Theory

2.1 *An arbitrary power law potential*

For an arbitrary power law potential

$$V(r) = ar^{\nu} + b \qquad a > 0,\ \nu > 0 \tag{2}$$

the semi-classical solutions of the Schrödinger equation for the $n = 1, l = 0$ $q\bar{q}$ states would give the bound state mass in the form (Barik and Jena 1980)

$$W = 2m + C\,m^{-\nu/(\nu+2)} + b. \tag{3}$$

In (3), m is the quark mass,

$$C = a^{2/(\nu+2)}[A(\nu)(3/4)]^{2\nu/(\nu+2)}$$

with

$$A(\nu) = 2(\pi)^{1/2}\Gamma(3/2 + 1/\nu)/\Gamma(1 + 1/\nu).$$

We find that $\mathrm{d}W/\mathrm{d}m$ is positive provided

$$C(\nu/(\nu+2))m^{-2(\nu+1)/(\nu+2)} < 2 \tag{4}$$

Choosing the parameters a, ν from the potential (Martin 1982)

$$V = -8{\cdot}064 + 6{\cdot}87\,r^{0{\cdot}1} \tag{5}$$

it is found that for a meson composed of c and $\bar{c}$ quarks ($m = 1{\cdot}32$ GeV), the magnitude of left side of inequality (4) comes out to be 0·27.

2.2 *Covariant coulomb plus harmonic oscillator potential*

The (mass)^2 spectrum for mesons and baryons with such a potential can be written as (Mitra 1979)

$$M^2_{q\bar{q}} = \Omega_M[2n + \{(k+1)^2 - (16/3)\,\alpha_s(m_v^2)\pi^{-1}\}^{1/2} + 1] + C_M \tag{6}$$

and

$$M^2_{qqq} = \Omega_B[2N + \{(K+3)^2 - 24\,\alpha_s(m_v^2)\pi^{-1}\}^{1/2} + 1] + C_B \tag{7}$$

respectively. Here Ω_M (Ω_B) is the spring constant for meson (baryon), $n(N)$ is radial (super radial) quantum number for the $q\bar{q}(qqq)$ spectra and $k(K)$ is the global angular momentum in four (eight) dimensions respectively. Here $\alpha_s(m_v^2)$ is the strong coupling constant. C_M and C_B are constants having no explicit dependence on quark masses. The bound state mass increases with constituent mass provided the conditions satisfied for mesons and baryons are

$$(\mathrm{d}\Omega_M/\mathrm{d}m)[2n + \{(k+1)^2 - (16/3)\,\alpha_s(m_v^2)\pi^{-1}\}^{1/2} + 1] > 0 \tag{8}$$

and

$$(\mathrm{d}\Omega_B/\mathrm{d}m)[2N + \{(K+3)^2 - 24\,\alpha_s(m_v^2)\pi^{-1}\}^{1/2} + 1] > 0 \tag{9}$$

respectively. Since (6) and (7) are derived using the formalism of Feynman *et al* (1971), $\mathrm{d}\Omega_M/\mathrm{d}m$ and $\mathrm{d}\Omega_B/\mathrm{d}m$ are greater than zero. The conditions (8) and (9) are thus satisfied.

2.3 *Linear plus coulomb potential*

For the potential given by

$$V(r) = \alpha r - \beta/r \qquad \alpha, \beta \geqslant 0^+, \tag{10}$$

the bound state mass of quark and its antiquark may be written (Seetharaman *et al* 1983) as

$$W = 2m + \frac{3(n_r + 1/2)\pi 2^{-1/4}(m/2)^{-1/4}\alpha^{1/2}}{gcK(k) + g(a-c)E(k)}$$
$$-\frac{2^{3/2}\beta K(k)(m/2)^{1/2}}{cK(k)+(a-c)E(k)} + \frac{3L^2 K(k)}{c^2 K(k) + (a-c)cE(k)}$$
$$-\frac{3L^2(1/c - 1/b)\Pi(\alpha'^2, k)}{cK(k)+(a-c)E(k)}, \tag{11}$$

where m stands for the quark mass and K, E, Π *are* are complete elliptic integrals of the first, second and third kinds respectively. The quantities a, b, c are given by the roots of the equation

$$r^3 - \frac{W}{\alpha}r^2 - \frac{\beta}{\alpha}r + \frac{(l+1/2)^2}{2m\alpha} = 0$$

and

$$L^2 = (l+1/2)^2,\ k^2 = (a-b)/(a-c),\ \alpha'^2 = ck^2/b,\ g = 2/(a-c)^{1/2}.$$

It is easy to find that the bound state mass increases with the constituent mass if

$$\frac{3\times 2^{-13/4}(\alpha^{1/2}/g)(n_r + 1/2)\pi(m/2)^{-5/4} + (1/2)^{1/2}\beta K(k)(m/2)^{-1/2}}{cK(k)+(a-c)E(k)} < 2. \tag{12}$$

An approximate choice such as $\alpha = 0{\cdot}20$, $\beta = 0{\cdot}66$ and $m_c = 1{\cdot}32$ GeV gives the mass of $1^3S_1\, c\bar{c}$ state J/ψ equal to 3·481 GeV. With these parameters the left side of (12) comes out to be 0·229.

2.4 *The model of De Rujula et al*

The paper of De Rujula *et al* (1975) attempts to interpret the nonrelativistic quark model within the framework of quark dynamics described by QCD. Their model assumes a nonrelativistic SU(6) model, long-range flavour and spin-independent confining forces, SU(3) breaking *via* quark masses only and asymptotic freedom for the quark-gluon interactions to motivate a short-range, spin and flavor dependent force arising from the nonrelativistic reduction of one gluon exchange between quarks. Assuming nonrelativistic quark dynamics, the interaction between quarks 1 and 2 is the Breit interaction (Berestetskii *et al* 1971) given by

$$S_{12} = \frac{1}{r} - \frac{1}{2m_1 m_2}\left[\frac{\mathbf{p}_1\cdot\mathbf{p}_2}{r} + \frac{\mathbf{r}\cdot(\mathbf{r}\cdot\mathbf{p}_1)\mathbf{p}_2}{r^3}\right]$$
$$-\frac{\pi}{2}\delta^3(r)\left(\frac{1}{m_1^2} + \frac{1}{m_2^2}\right)$$
$$-\frac{1}{m_1 m_2}\left[\frac{8\pi}{3}\mathbf{s}_1\cdot\mathbf{s}_2\,\delta^3(\mathbf{r}) + \frac{1}{r^3}\left\{3(\mathbf{s}_1\cdot\hat{r})(\mathbf{s}_2\cdot\hat{r}) - \mathbf{s}_1\cdot\mathbf{s}_2\right\}\right]$$
$$-\frac{1}{2r^3}\left[\frac{1}{2m_1^2}\mathbf{r}\times\mathbf{p}_1\cdot\mathbf{s}_1 - \frac{1}{2m_2^2}\mathbf{r}\times\mathbf{p}_2\cdot\mathbf{s}_2\right.$$
$$\left.+\frac{1}{m_1 m_2}\{2\mathbf{r}\times\mathbf{p}_1\cdot\mathbf{s}_1 - 2\mathbf{r}\times\mathbf{p}_2\cdot\mathbf{s}_1\}\right] + \ldots \tag{13}$$

Here . . . denotes neglected relativistic corrections.

For a $q\bar{q}$ system 1^1S_0 state, the spin-orbit and tensor interactions vanish and $\mathbf{s}_1 \cdot \mathbf{s}_2 = -3/4$. In the centre of mass frame where $\mathbf{p}_1 = -\mathbf{p}_2 \equiv \mathbf{p}$, S_{12} may be written as

$$S_{12} = \frac{1}{r} + \frac{1}{2m^2}\left[\frac{\mathbf{p}^2}{r} + \frac{\mathbf{r}\cdot(\mathbf{r}\cdot\mathbf{p})\mathbf{p}}{r^3}\right] + \frac{\pi}{m^3}\delta^3(r). \tag{14}$$

At short distances, colour-Coulomb interaction is considered to be dominant and the colour-confining term may be neglected. Using S_{12} given in (14), the bound state mass of $1^1S_0 q\bar{q}$ state calculated perturbatively (by the method of determination of fine structure of para-positronium) is given by

$$W = 2m - \frac{4m\alpha_s^2}{9n^2} + \left(\frac{4\alpha_s}{3}\right)^4 \frac{m}{4n^3}\left[\frac{1}{2n} - \frac{3}{2}\right]. \tag{15}$$

The bound state mass increases with the constituent mass if

$$\frac{4\alpha_s^2}{9n^2} - \left(\frac{4\alpha_s}{3}\right)^4 \frac{1}{4n^3}\left[\frac{1}{2n} - \frac{3}{2}\right] < 2. \tag{16}$$

For the $1^1S_0 c\bar{c}$ bound state $\eta_c(\alpha_s = 0{\cdot}2, n = 1)$, the left side of (16) comes out to be $1{\cdot}78 \times 10^{-2}$, thus satisfying the condition $dW/dm > 0$ in this case too. At long distances the effective value of v is probably greater than zero, otherwise there would not be quark confinement. If quarks are confined within hadrons, the condition $dW/dm > 0$ and the conclusion that d quark is heavier than u quark is going to remain unchanged at long distances too.

3. Results and discussions

It is clear from our analysis with some QCD motivated potentials that the bound state mass increases as a constituent mass is increased. This behaviour of bound state mass is also expected according to the result obtained by Carydas and Lichtenberg (1981) for all the potentials considered here except the one given by (13). For this potential, the expectation of Carydas and Lichtenberg (1981) was opposite to what we obtained.

Again, the masses of u and d quarks being in question, it is logical to consider systems not containing any of these quarks. Hence we have taken parameters from $c\bar{c}$ spectroscopy. The analysis of Carydas *et al* (1981) is valid for $m \leqslant M$. We have chosen $m = M$ for simplicity.

The statement that d quark is heavier than u quark is model-dependent. However, certain QCD motivated potentials lay support in favour of a d which is heavier than u. If t exists, the empirical rule, that charge 2/3 member is more massive than the charge $-1/3$ member, satisfied by the doublets (c, s), (t, b) is violated by the doublet (u, d).

Acknowledgement

The author thanks Dr B K Talukdar for a critical reading of the manuscript.

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