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Regionwise absorption in nuclear optical model

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Abstract. A mathematical procedure to calculate the contribution to the reaction cross-section from a shell of radius r and thickness Δ around the scattering centre within the frame work of a nuclear optical model is presented. The method is illustrated by describing graphically the regionwise absorption in nucleon-nucleus and nucleus-nucleus optical scattering. It is demonstrated that unlike in nucleon-nucleus scattering, in the nucleus-nucleus scattering volume absorptive optical potential, in general, does not imply that absorption is taking place in the entire nuclear volume; it is confined to mostly the surface region.

Keywords. Optical model; surface absorption; volume absorption; nuclear scattering.

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1. Introduction

The optical model of elastic scattering is a widely used method in the analysis of collision processes having a large number of inelastic and reaction channels open in addition to the elastic channel. The nuclear optical model is used to study the nucleon-nucleus and nucleus-nucleus collisions. The complex many-body processes involved in these collisions are globally described in terms of the scattering of a particle by a semi-absorptive complex potential. In wave optics a medium having a complex refractive index is used to describe the absorption, reflection, refraction and diffraction of the incident wave by the medium. Similarly, the optical model can describe the absorption or reaction cross-section, elastic scattering dominated by diffractive and refractive processes etc. This model confines its domain to describe the totality of absorption or reaction processes in addition to elastic scattering. The attractive complex potential used in the optical model has a non-unitary scattering matrix which takes into account both the elastic scattering and absorption processes. The imaginary part of the complex potential has the primary role in generating the reaction cross-section. The most commonly used nuclear optical potential has the form

$$U(r) = U_R f(r, R, a) + iU_I g(r, R', a') + U_c(r, R_c) + U_s \lambda_\pi^2 \frac{1}{r}\left(\frac{df}{dr}\right) \bar{l}\cdot\bar{\sigma} \qquad (1)$$

with

$$f(r, R, a) = [1 + \exp(r - R)/a]^{-1}$$

$$g(r, R', a') = -4a' \exp -[(r - R')/a']^2 \quad \text{for surface absorptive potential} \qquad (2)$$

$$= f(r, R', a') \quad \text{for volume absorptive potential} \qquad (3)$$

For the surface absorption form factor one can also use

$$g(r, R', a') = -4a' \frac{\mathrm{d}}{\mathrm{d}r} f(r, R', a').$$

U_R, U_I and U_s are the potential strengths and are negative. $\lambda\!\!\!^{-}_{\pi}$ is the Compton wavelength of the pion. R, R', R_c are given in terms of the radius parameter r_0, r'_0, r_c:

$$R = r_0 A^{1/3},\ R' = r'_0 A^{1/3},\ R_c = r_c A^{1/3};$$

where A is the mass number of the target in nucleon-nucleus optical model. In nucleus-nucleus scattering instead of $A^{1/3}$, $A_1^{1/3} + A_2^{1/3}$ is generally used where A_1, A_2 are the mass numbers of incident and target nuclei respectively. U_c is the electrostatic potential corresponding to a uniformly charged sphere of radius R_c:

$$U_c(r) = \frac{Z_1 Z_2 e^2}{2R_c^3}(3R_c^2 - r^2) \qquad r \leqslant R_c, \tag{4}$$

$$= Z_1 Z_2 \frac{e^2}{r} \qquad r \geqslant R_c, \tag{5}$$

U_R, U_I, U_s, R, R', R_c, a, a' are the optical potential parameters.

The last term in (1) is the spin-orbit term. It generally is taken to be non-absorptive (real). In the present paper dealing primarily with the absorption process we confine our calculations to optical potentials which do not have the spin orbit term.

The present paper describes a method to analyse in detail the absorption taking place in different regions around the scattering centre due to the imaginary potential and illustrates it with typical calculations for the nucleon-nucleus and nucleus-nucleus optical models. This will clarify the meaning of surface absorptive and volume absorptive potentials. It will be clear from this paper that the volume absorptive optical potential need not necessarily generate significant absorption in the regions interior to the surface region. This is particularly true in nucleus-nucleus collisions. The approach described in this paper was developed in connection with the analysis of the potential ambiguity in nucleus-nucleus collisions (Shastry and Gambhir 1983). It was then realised that the same method could be used to provide a conceptual and detailed description of optical model suitable for pedagogical purposes.

Section 2 summarises a method to evaluate the contribution to the reaction cross-section σ_r from the region $r_i \leqslant r \leqslant r_i + \Delta$. In §3 we describe calculations carried out in nucleon-nucleus and nucleus-nucleus optical model which bring out the process of absorption in different regions around the scattering centre.

2. Mathematical formalism

Consider the Schrödinger equation for a complex potential:

$$\left[\frac{-\hbar^2}{2m}\nabla^2 + U(r) + iU_I(r) + \frac{Z_1 Z_2 e^2}{r}\right]\Psi(\bar{r}) = E\Psi(\bar{r}), \tag{6}$$

where $U(r) = U_R(r) + U_c(r) - Z_1 Z_2 \dfrac{e^2}{r}$.

The asymptotic behaviour of scattering solution is

$$\Psi(\bar{r}) = \left[1 - \frac{\eta^2}{ik(r-z)}\right] \exp\{i[kz - \eta \ln(kr - kz)]\}$$
$$+ f(\theta) \exp i[kr - \eta \ln 2kr]/r \tag{7}$$

where $k^2 = 2mE/\hbar^2$, $\eta = Z_1 Z_2 e^2 m/\hbar^2 k$ and $f(\theta)$ is the scattering amplitude. Using the general expression

$$\mathbf{j} = \frac{\hbar}{2im}(\psi^* \nabla\psi - \psi \nabla\psi^*) \tag{8}$$

one can evaluate the reaction cross-section σ_r, that is, the number of particles absorbed per unit incident flux:

$$\sigma_r = \frac{-m}{\hbar k} \int \mathbf{j} \cdot \hat{n}\, ds = -\frac{m}{\hbar k} \int \nabla \cdot \mathbf{j}\, d\mathbf{r} = -\frac{2m}{\hbar^2 k} \int \psi^* \psi U_I(r)\, d\bar{r} \tag{9}$$

$$\text{Defining } \sigma_r^{(i)}(r_i, \Delta) = -\frac{2m}{\hbar^2} \int_{r_i}^{r_i + \Delta} \psi^* \psi\, U_I(r) r^2\, dr\, d\Omega, \tag{10}$$

we can write

$$\sigma_r = \sum_{i=0}^{\infty} \sigma_r^{(i)}(r_i, \Delta),\ r_0 = 0,\ r_{i+1} = r_i + \Delta, \tag{11}$$

$\sigma_r^{(i)}(r_i, \Delta)$ is the contribution of the region $r_i \leqslant r \leqslant r_i + \Delta$ to σ_r and is a measure of the flux absorbed inside a shell of thickness Δ and inner radius r_i. Thus $\sigma_r^{(i)}(r_i, \Delta)$ is well suited in analysing the relative importance of different regions in the absorption process and can be expected to provide more insight into the physical process rather than *e.g.*, the imaginary part of the optical potential alone. Equation (10) indicates the role of both the imaginary part, and the real part of the interaction (through ψ) in the absorption process. The result corresponding to (10) for the partial wave reaction cross-section can be deduced in terms of a properly normalised regular solution of the radial Schrödinger equation:

$$\frac{d^2}{dr^2}\phi(\lambda, k, r) + \left(k^2 - \frac{\lambda^2 - \frac{1}{4}}{r^2} - V_N(r) - \frac{2\eta k}{r}\right)\phi(\lambda, k, r) = 0, \tag{12}$$

where $\lambda = l + \frac{1}{2}$, $V_N(r) = \frac{2m}{\hbar^2}[U(r) + iU_I(r)]$.

The result is

$$1 - \eta_l^2 = -4k \int_0^{\infty} \operatorname{Im} V_N(r) \left|\frac{\phi(\lambda, k, r)}{F(\lambda, -k)}\right|^2 dr, \tag{13}$$

$$= \sum_{i=0}^{\infty} \chi_r(\lambda, r_i, \Delta),\ r_0 = 0,\ r_{i+1} = r_i + \Delta. \tag{14}$$

The regular solution $\phi(\lambda, k, r)$ behaves as $r^{\lambda + 1/2}$ near origin, $F(\lambda, -k)$ denotes the Jost function (see DeAlfaro and Regge 1965; Newton 1964; Mukherjee and Shastry 1967). The S-matrix is given by

$$S(\lambda, k) = \exp(i\pi l)\, F(\lambda, k)/F(\lambda, -k), \tag{15}$$

and $\eta_l = |S(\lambda, k)|$. It can be shown that

$$\chi_r(\lambda, r_i, \Delta) = -4k \operatorname{Im}\left[\frac{\phi'(\lambda, k, r)}{F(\lambda, -k)}\frac{\phi^*(\lambda, k, r)}{F^*(\lambda, -k)}\right]_{r_i}^{r_i+\Delta} \tag{16}$$

The term $(1-\eta_l^2)$ is a measure of the partial wave reaction cross-section $\sigma_{r,l}$ given by

$$\sigma_{r,l} = \frac{\pi}{k^2}(2l+1)(1-\eta_l^2), \tag{17}$$

and

$$\sigma_r = \sum_{l=0}^{\infty} \sigma_{r,l}. \tag{18}$$

Therefore, $\chi_r(\lambda, r_i, \Delta)$ is a measure of the contribution to $\sigma_{r,l}$ from the region $r_i \leqslant r \leqslant r_i + \Delta$. Consistent with (10), (14) indicates that the maximum contribution to $\sigma_{r,l}$ will be from the region where $|\phi(\lambda, k, r)|^2 \operatorname{Im} V_N(r)$ is maximum. It is not surprising that $\chi_r(\lambda, r_i, \Delta)$ occurs as a function of $\psi(\lambda, k, r) = \phi(\lambda, k, r)/F(\lambda, -k)$. In fact it is known that the radial part of the full scattering solution consistent with the scattering wave boundary condition is $\psi(\lambda, k, r)/(kr)$ *i.e.* the importance of different regions around the scattering centre depend crucially on the 'normalised' wavefunction $\psi(\lambda, k, r)$. If absorption or scattering takes place predominantly from the surface region one can expect $\psi(\lambda, k, r)$ to be small in the interior. One can physically visualise this as follows: From the time evolution point of view, ψ develops from a state when the target and the projectile are well apart from the scattering centre and the interaction around the exterior region of the interacting potential takes place prior to the development of the state to the regions in the interior. From this point of view, one can say that ψ in the interior region close to the scattering centre should depend on the physical process occurring in the surface region.

Since absorption as defined here depends on the wave function the present approach can be utilised to relate the nature of absorption from the behaviour of ψ (Shastry and Gambhir 1983). Why the wavefunction plays a crucial role can also be understood from the JWKB approximation to the incoming wavefunction which has the form (see *e.g.* Green *et al* 1968)

$$\exp[-ip(r)],$$

where

$$p(r) = kr + \int_{\infty}^{r} [k(r) - k]\, dr;\ k(r) = [k^2 - V(r) - \lambda^2/r^2]^{1/2}.$$

Because of the imaginary part of $k(r)$ this wave damps out for small r, particularly so if the potential is surface absorptive. The term $p(r)$ as a function of energy determines approximately the energy dependance of $\sigma_r(r, \Delta)$. In fact $\eta_l \sim |\exp[-2ip(r_0)]|$ where r_0 is the WKB turning point.

From (14) it is easy to obtain the contribution $\sigma_r^{(i)}(r_i, \Delta)$ from the region $r_i \leqslant r \leqslant r_i + \Delta$ to the total reaction cross-section. We write

$$\sigma_r = \sum_{i=0}^{\infty} \sigma_r^{(i)}(r_i, \Delta),\ r_{i+1} = r_i + \Delta,\ r_0 = 0, \tag{19}$$

$$\sigma_r^{(i)}(r_i, \Delta) = \sum_{l=0}^{\infty} \frac{\pi}{k^2}(2l+1)\chi_r(\lambda, r_i, \Delta). \tag{20}$$

The contribution to σ_r from the region $0 \leqslant r \leqslant r_N$ is given by

$$[\sigma_{r_N}]_0^{r_N} = \sum_{i=0}^{N} \sigma_r^{(i)}(r_i, \Delta). \tag{21}$$

By appropriately modifying the optical model codes it is straightforward to compute $\chi_r(\lambda, r_i, \Delta)$ and $\sigma_r^{(i)}(r_i, \Delta)$. In the standard optical model codes the S-matrix is computed in terms of the ratio $\phi(\lambda, k, r)/\phi'(\lambda, k, r)$ for large r and it is not essential to keep track of what happens in different regions. However for computing $\chi_r(\lambda, r_i, \Delta)$ a uniformly normalised $\phi(\lambda, k, r)$ and hence $\psi(\lambda, k, r)$ is essential. Therefore the calculation of $\sigma_r(r, \Delta)$ needs some subtle changes in the standard optical model code due to the very large numbers involved. We have modified the optical model code to evaluate the Jost function and uniform normalisation and have used it in our calculation. In the optical model calculations $\phi(\lambda, k, r)$ increases very rapidly with r due to the complex potential and one resorts to renormalisation to manageable numbers at different stages. It is essential to keep track of these to obtain an uniformly normalised $\phi(\lambda, k, r)$ behaving as $r^{\lambda+1/2}$ near the origin.

In §3 based on numerical calculations we analyse the behaviour of $[\sigma_r]_0^r$ and $\sigma_r(r, \Delta)$ which give respectively the contribution to the reaction cross-section from the region $(0, r)$ and $(r, r+\Delta)$.

3. Numerical calculations of regionwise absorption

In order to give a comprehensive understanding of the regionwise absorption in nuclear optical model we consider typical cases of (i) neutron-nucleus (ii) proton-nucleus (iii) alpha-nucleus (iv) light nucleus-light nucleus and (v) nucleus-nucleus scattering systems. We have chosen those cases of optical potentials which do not have spin orbit term for the reason specified earlier. In the nucleon-nucleus systems we consider both volume absorptive and surface absorptive optical potentials. The various optical potential parameters are given in table 1.

For the systems listed in table 1 $\sigma_r(r, \Delta)$ and $[\sigma_r]_0^r$ are evaluated numerically using (20) and (21). $[\sigma_r]_0^r$ indicates the contribution of the spherical volume of radius r around the scattering centre to the reaction cross-sections σ_r. Similarly, $\sigma_r(r, \Delta)$ indicates the contribution to σ_r from the spherical shell having inner radius r and outer radius $r+\Delta$. In the present calculations we have used $\Delta = 0.09$ fm. As one can expect, the maximum slope of $[\sigma_r]_0^r$ as a function of r occurs when $\sigma_r(r, \Delta)$ is maximum. $[\sigma_r]_0^r$ saturates to the value σ_r, reaction cross-section as r becomes large: $r \geqslant 2R$, implying negligible absorption in the regions further exterior.

Figures 1–2 depict $[\sigma_r]_0^r$ and $\sigma_r(r, \Delta)$ for n-Bi scattering at energies $E_{\text{lab}} = 7{\cdot}0$ MeV and 14·6 MeV respectively. The imaginary part of the optical potential used in figure 1 is volume-absorptive; and in figure 2 it is surface-absorptive. Accordingly $\sigma_r(r, \Delta)$ has significant contribution from regions $r > 1$ fm in the former case whereas in the second case $\sigma_r(r, \Delta)$ is negligible for $r < 6{\cdot}5$ fm. As one expects in the surface absorption case, $\sigma_r(r, \Delta)$ is peaked around the surface region of the target nucleus Bi. Thus in these two cases of n-nucleus scattering the volume absorptive potential generates absorption

Table 1. Optical model parameters for various nucleon-nucleus and nucleus-nucleus systems used in the analysis of the regionwise absorption. (The parameter $r_0' = r_c = r_0$. In $^{18}O + ^{58}Ni$ case $R = R_0(A_1^{1/3} + A_2^{1/3})$. In the remaining cases $R = r_0 A_2^{1/3}$. The form factor for the real part is $f(r, R, a)$ in all cases).

System	E_{lab} (MeV)	Form factor for the imaginary part of the optical potential	U_R (MeV)	U_I (MeV)	r_0 (fm)	a (fm)	a' (fm)	References
$n + Bi$	7·0	$f(r, R', a')$	44·0	3·3	1·30	0·5	0·5	*a, b*
$n + Bi$	14·6	$g(r, R', a')$	40·3	8·0	1·308	0·6	0·978	*a, c*
$p + Ni$	5·25	$f(r, R', a')$	52·5	0·9	1·33	0·5	0·5	*a, d*
$p + Ni$	6·8	$g(r, R', a')$	55·0	3·0	1·35	0·65	0·98	*a*
$\alpha + ^{27}Al$	28·0	$f(r, r', a')$	68·3	13·34	1·58	0·56	0·56	*a, e*
$^{6}Li + ^{12}C$	5·8 / 59·8	$f(r, R', a')$	$153 - 0{\cdot}46E_{lab}$	$2{\cdot}2 + 0{\cdot}25E_{lab}$	1·45	0·67	0·97	*f*
$^{18}O + ^{58}Ni$	60·0	$f(r, R', a')$	90·1	42·9	1·22	0·5	0·5	*g*

a. Hodgson P (1963); *b.* Beyster *et al* (1956); *c.* Bjorklund *et al* (1956); *d.* Melkanoff *et al* (1956); *e.* Kemper *et al* (1972); *f.* Polling *et al* (1976); *g.* Videbaek, *et al* (1976).

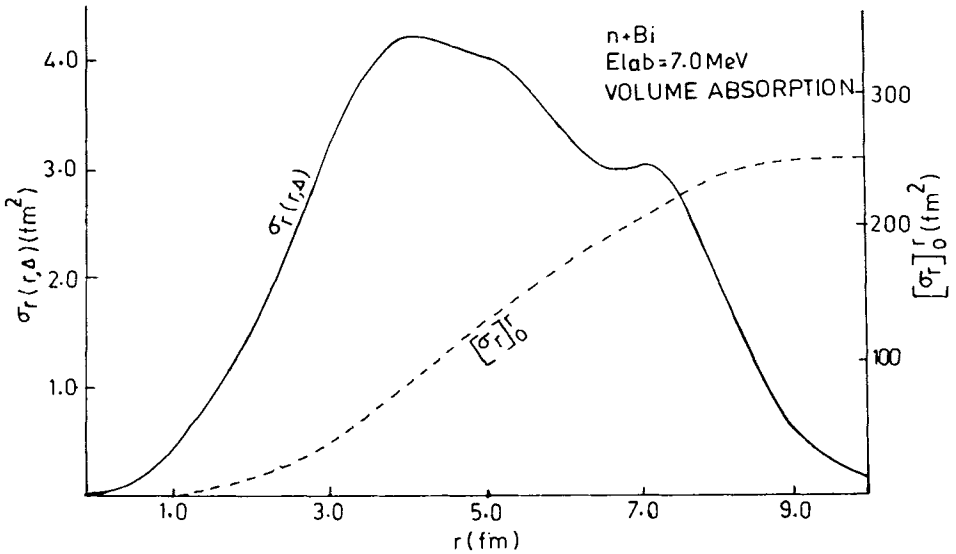

Figure 1. Variation of $\sigma_r(r, \Delta)$ and $[\sigma_r]_0^r$ as a function of r for n-Bi scattering with $E_{lab} = 7$ MeV. Volume absorption optical potential specified in table 1 is used

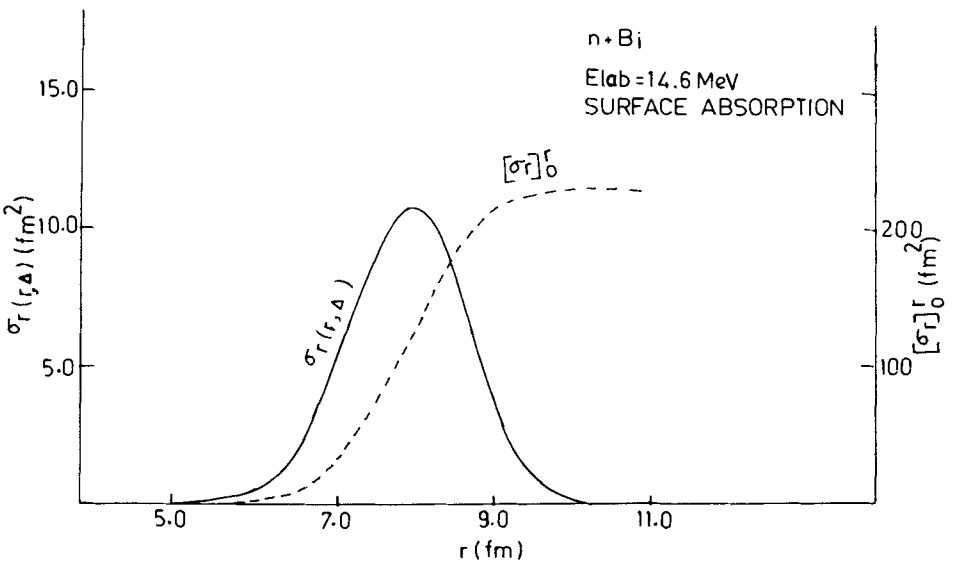

Figure 2. Same as figure 1 for n-Bi scattering with $E_{lab} = 14{\cdot}6$ MeV. Surface absorptive optical potential specified in table 1 is used.

more or less in the entire volume of the nucleus, and surface absorptive potential restricts most of the absorption to the surface region. In figures 3 and 4 we depict a similar analysis for p-Ni scattering at $E_{lab} = 5{\cdot}25$ and $6{\cdot}8$ MeV respectively. Interestingly, in figure 3 one sees oscillatory behaviour of $\sigma_r(r, \Delta)$ indicating that the absorption taking place may be more dominant in some regions and comparatively less dominant in some other regions. The flatness of the curve $[\sigma_r]_0^r$ around $r = 3{\cdot}8$ fm is due to the minima of $\sigma_r(r, \Delta)$ around the same region. From figures 3 and 4 also one gets

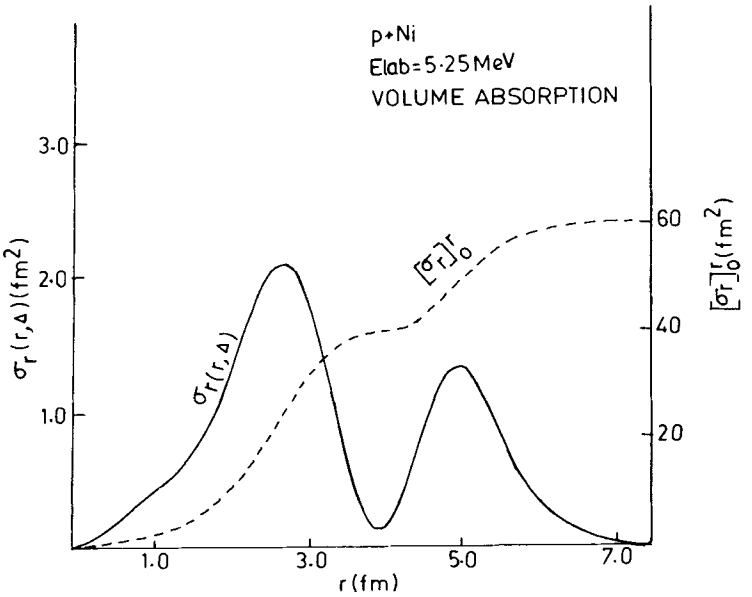

Figure 3. Same as in figure 1 for p-Ni scattering with $E_{lab} = 5{\cdot}25$ MeV. Volume absorptive optical potential specified in table 1 is used.

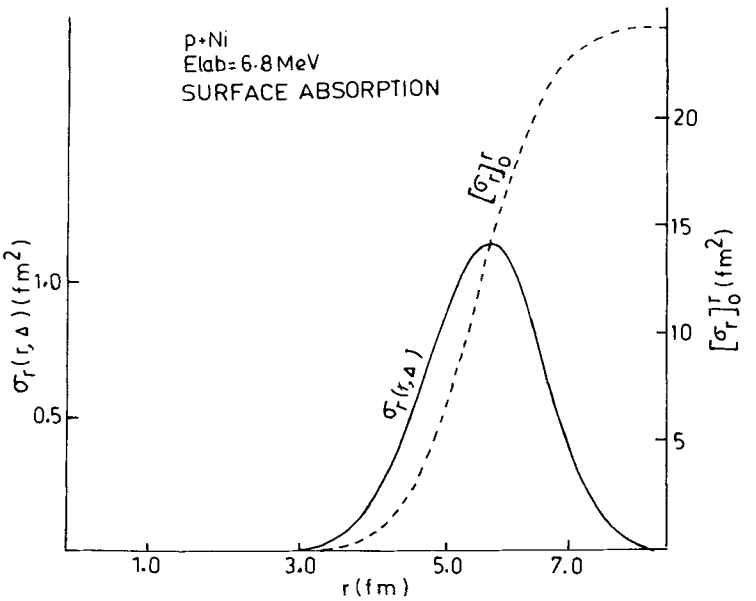

Figure 4. Same as in figure 1 for p-Ni scattering with $E_{lab} = 6{\cdot}8$ MeV. Surface absorptive optical potential specified in table 1 is used.

the conclusion similar to that deduced from figures 1 and 2. Thus figures 1–4 are well suited to explain the significance of volume absorptive and surface absorptive nucleon-nucleus optical potential and the subtlities involved in its structure.

In figures 5–7 we analyse the regionwise absorption for α-^{27}Al, and ^{6}Li-^{12}C systems. These can be termed light nucleus-light nucleus scattering systems. All these systems are studied with volume absorptive optical potentials. However unlike the *n*-Bi or *p*-Ni cases studied in figures 1–4, we find that most of the absorption process in these cases happen in and around the surface region despite the fact that we have used volume absorption optical potential. However, in spite of this, a significant amount of absorption takes place in the interior regions also. The behaviour of $\sigma_r(r, \Delta)$ for the ^{6}Li-^{12}C system at $E_{lab} = 5{\cdot}8$ MeV (figure 6) shows interesting structures and explores more or less the entire region around the scattering centre. In figure 6 the oscillations are not prominent in $[\sigma_r]_0^r$ because of the smoothening of oscillations in integration and also due to the much smaller scale used in plotting $[\sigma_r]_0^r$. The oscillations can be interpreted as due to the interference of internal wave and the barrier wave (Shastry and Gambhir 1983). The details of these are not relevant for our purpose here and hence will be omitted. From figures 5–7 we can conclude that for light nucleus-light nucleus $\sigma_r(r, \Delta)$ and $[\sigma_r]_0^r$ are significant in all regions, even though most of the absorption is generated in the surface region. Thus, in these cases the use of volume absorption optical potential does not imply that most of the absorption process is taking place in the interior volume.

Figure 8 depicts the absorption process in a typical nucleus-nucleus scattering or what is more commonly known as heavy ion scattering. It is known for quite sometime that the scattering data in typical heavy ion scattering is not sensitive to the interior regions of the potential. This can be attributed to the sizes of the colliding nuclei and the

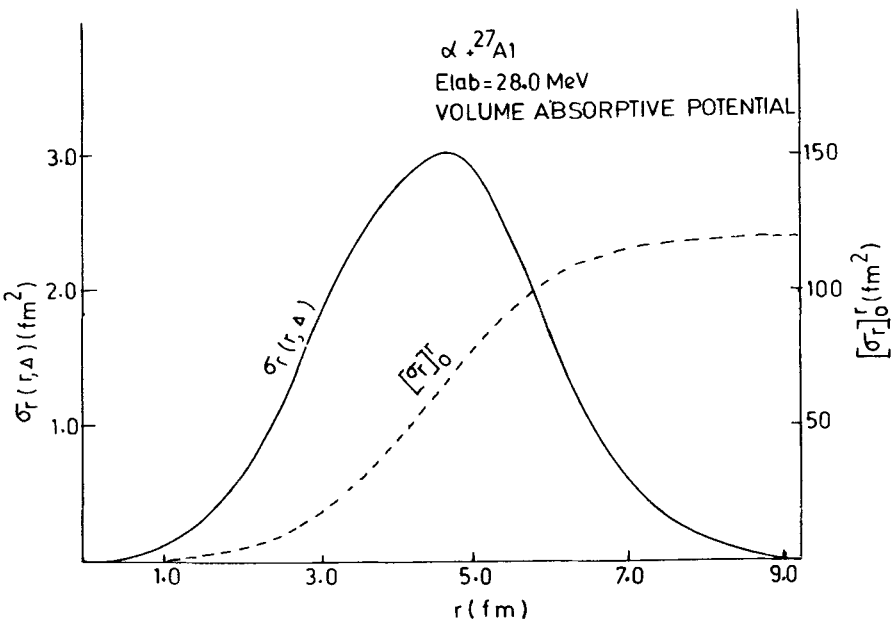

Figure 5. Same as figure 1 for α-^{27}Al scattering at $E_{lab} = 28$ MeV.

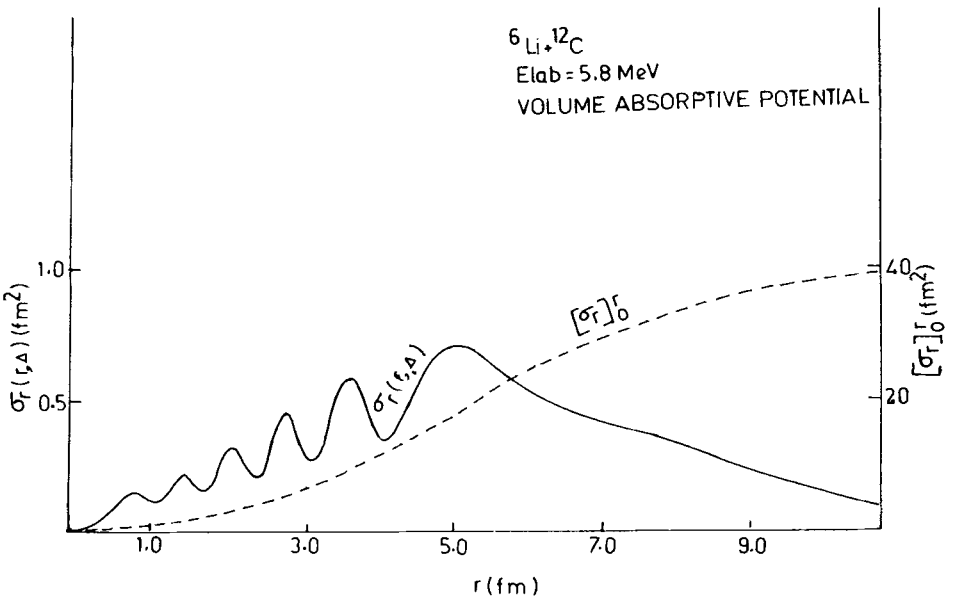

Figure 6. Same as figure 1 for ^{6}Li-^{12}C scattering at E_{lab} = 5·8 MeV.

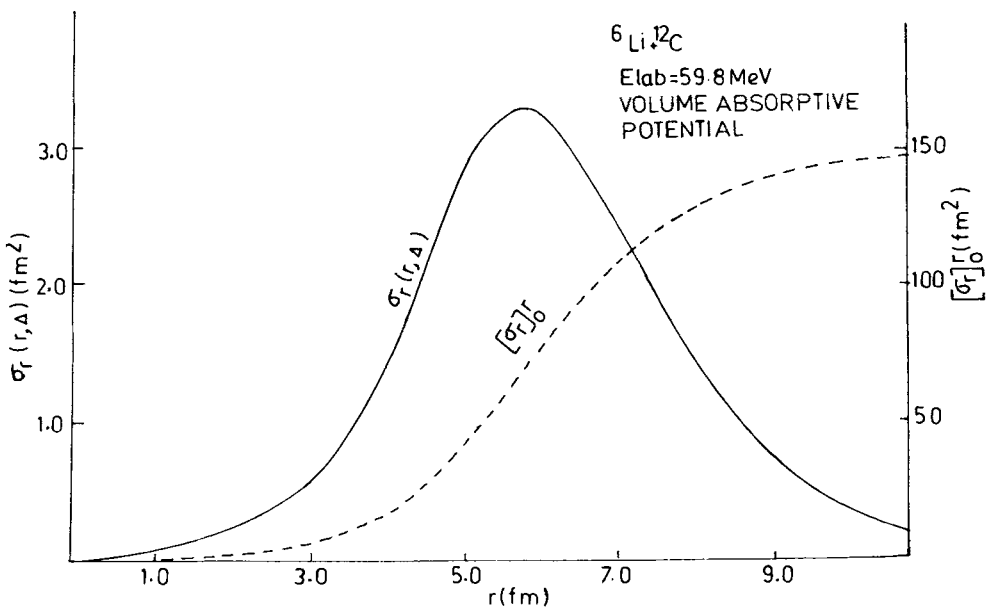

Figure 7. Same as figure 1 for ^{6}Li-^{12}C scattering at E_{lab} = 59·8 MeV.

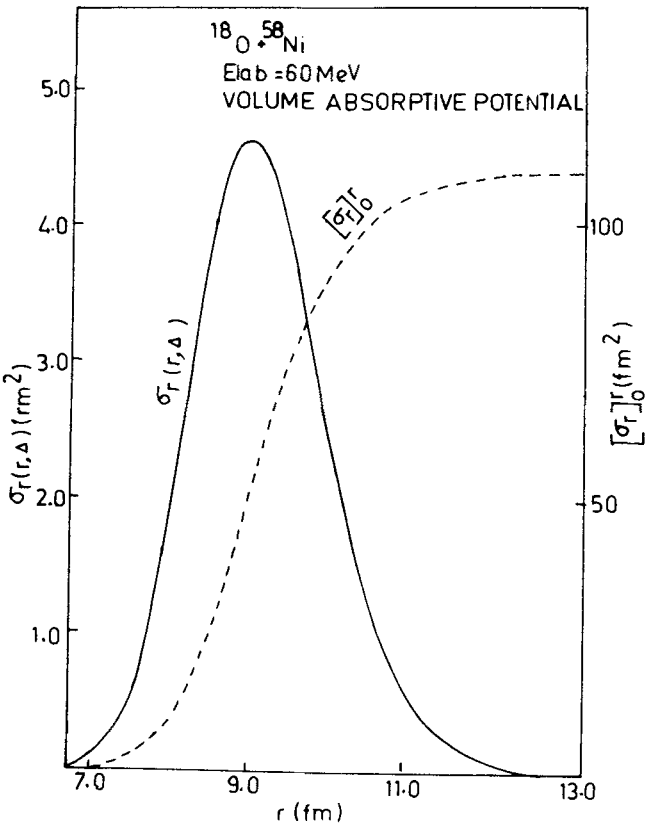

Figure 8. Same as figure 1 for ^{18}O-^{58}Ni scattering at $E_{lab} = 60$ MeV.

comparatively large Coulomb barrier which makes the colliding nuclei spend more time in the surface region. In these cases the absorption is predominantly on the surface and even if one uses volume absorptive optical potential, the interior regions of the potential are left unexplored in the scattering data. From ^{18}O–^{58}Ni shown in figure 8 it is clear that in spite of volume absorptive optical potential, there is negligible contribution to $[\sigma_r]_0^r$ or $\sigma_r(r, \Delta)$ from the regions with $r < 7$ fm. Thus in a typical heavy ion scattering, volume absorption optical potential does not generate absorption in the interior.

From a global point of view, in contradistinction with the present approach, Austern (1961) has studied the absorption phenomena in alpha-nucleus scattering using the JWKB approximation. As in the present approach in this work also the absorption or attenuation process is analysed from the behaviour of the wavefunction and the potential. The variation of absorption for different partial waves is analysed semi-classically in terms of the reflection from the centrifugal barrier, the potential barrier and the phase averaging. In contrast the present approach gives a completely quantum mechanical prescription to analyse the contribution of various regions to both partial wave reaction cross-section $\sigma_{r,l}$ and the total reaction cross-section σ_r. The results obtained by Austern (1961) for the attenuation of the radial wavefunction in the

interior regions of the nucleus are similar to that obtained using the present approach (Shastry and Gambhir 1983).

From our detailed analysis we find that the nature of absorption in different regions has to be studied by carrying out explicit calculations and in general volume absorption optical potential does not guarantee significant absorption in all regions of the potential. The quantum mechanical reason for this is discussed in §2 where it was shown that it is $|\Psi|^2 \operatorname{Im} V_N(r)$ which is important in deciding the dominance or otherwise of absorption process in a given region. Our results show that the nucleon-nucleus optical model volume absorption and surface absorption potential imply the nature of absorption and this is not true in other cases. It should however be stressed that if an optical potential is surface absorptive it is unlikely that significant amount of absorption will take place in the interior region.

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