

Nonleptonic decay matrix elements in the variable pressure bag model

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Abstract. The consequences of the variable pressure bag model for the nonleptonic decays of hyperons and Ω^- are investigated. Though the order of magnitude and the relative sign for the various decay amplitudes are correctly reproduced, the overall results are small by a factor of 2 to 4, indicating that the theoretical predictions are strongly dependent on the model parameters.

Keywords. Nonleptonic hyperon decays; omega minus decays; bag model.

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1. Introduction

It has been hoped for long that the models of hadron-structure will add to our understanding of the physics underlying the weak nonleptonic decays. The advent of non-Abelian gauge theories in the early seventies activated considerable theoretical interest in the subject of the nonleptonic $\Delta S = 1$ interactions. Recently, several attempts (Donoghue *et al* 1980; Galic *et al* 1979; Tadic and Trampetic 1981; Ponce 1980; Colic *et al* 1982) have been made claiming to lead to a reasonable estimate of hyperon and Ω^- nonleptonic decays. The important ingredients in these efforts have been (a) current algebra and the partial conservation of axial vector current (PCAC) hypothesis, which relate a process involving a pion to the one with the pion removed, (b) QCD radiative corrections to the interactions of left-handed weak currents and (c) the MIT bag model (Chodos *et al* 1974; De Grand *et al* 1975) to describe the quark wave functions.

It is important to investigate whether the relatively satisfactory values of the weak nonleptonic amplitudes so obtained are the result of a particular model used. One of the possible tests would be to repeat calculations using a variant of the fixed sphere MIT model. Encouraged by the success of the variable bag pressure model (Joseph and Nair 1981; Chatley 1983; Chatley *et al* 1984) in reproducing mass spectra of low lying hadrons, and their certain electromagnetic properties, such as magnetic moments, the proton charge-radius and the $3/2^+ \rightarrow 1/2^+ \gamma$ and $1^- \rightarrow 0^- \gamma$ transitions, we are motivated to extend the success of this model to nonleptonic decay matrix elements.

Our main aim in this paper is, therefore, (i) to investigate the consequences of the variable pressure bag model (Joseph and Nair 1981) for the nonleptonic decays of hyperons and Ω^- , and (ii) to study whether the theoretical results for these decays are crucially influenced by variations in the bag parameters. The calculational scheme employed is the commonly adopted approach (Marshak *et al* 1969) to the problem,

which leads to the commutator approximation for the S -wave hyperon decays, and the ground-state pole-approximation for the P -wave hyperon and the Ω^- decays.

The paper is organised as follows: We write the operators which underlie $\Delta S = 1$ transitions in §2, and present the matrix elements in §3. In §4, we give numerical results of our calculations followed by a brief discussion.

2. Operators

The $\Delta S = 1$ effective nonleptonic weak Hamiltonian calculated originally by Shifman *et al* (1977) in the context of the four-quark model, and by Gilman and Wise (1979) in the context of the six-quark model, has been widely used to calculate the S - and P -wave amplitudes for hyperon decays (Donoghue *et al* 1980; Galic *et al* 1979; Tadic and Trampetic 1981). In such calculations, the effect of the short range gluons is considered by the use of QCD renormalisation methods and the confinement effects are incorporated using the bag model (Chodos *et al* 1974; De Grand *et al* 1975) wave-function for confined quarks.

The $\Delta S = 1$ weak nonleptonic Hamiltonian is given by (Shifman *et al* 1977)

$$H_W^{\text{eff}}(\Delta S = 1) = \sqrt{2} G_F \sin \theta_c \cos \theta_c \sum_i c_i 0_i \quad (1)$$

where 0_i are the following four-quark operators, the parenthesis indicating their flavour SU(3) and isospin properties:

$$\begin{aligned} 0_1 &= (\bar{d}_L s_L)(\bar{u}_L u_L) - (\bar{d}_L u_L)(\bar{u}_L s_L) & (\mathbf{8}_F, \Delta I = 1/2), \\ 0_2 &= (\bar{d}_L s_L)(\bar{u}_L u_L) + (\bar{d}_L u_L)(\bar{u}_L s_L) + \\ & \quad 2(\bar{d}_L s_L)(\bar{u}_L u_L) + 2(\bar{d}_L s_L)(\bar{s}_L s_L), & (\mathbf{8}_D, \Delta I = 1/2), \\ 0_3 &= (\bar{d}_L s_L)(\bar{u}_L u_L) + (\bar{d}_L u_L)(\bar{u}_L s_L) + \\ & \quad 2(\bar{d}_L s_L)(\bar{d}_L d_L) - 3(\bar{d}_L s_L)(\bar{s}_L s_L) & (27, \Delta I = 1/2), \\ 0_4 &= (\bar{d}_L s_L)(\bar{u}_L u_L) + (\bar{d}_L u_L)(\bar{u}_L s_L) - \\ & \quad (\bar{d}_L s_L)(\bar{d}_L d_L), & (27, \Delta I = 3/2), \\ 0_5 &= (\bar{d}_L \lambda_\alpha s_L)(\bar{u}_R \lambda^\alpha u_R) & (\mathbf{8}, \Delta I = 1/2), \\ 0_6 &= (\bar{d}_L s_L)(\bar{u}_R u_R + \bar{d}_R d_R + \bar{s}_R s_R) & (\mathbf{8}, \Delta I = 1/2), \end{aligned} \quad (2)$$

where, for example, $\bar{d}_L s_L$ is the shorthand for $\bar{d}^i \gamma_\mu (1 - \gamma_5) s^j / 2$ and $\bar{u}_R \lambda^\alpha u_R$ for $\bar{u}^i (\lambda^\alpha)_{ij} \gamma_\mu (1 + \gamma_5) u^j / 2$; the λ -matrices and the upper latin indices reflect the SU(3)-colour group with $\text{Tr}(\lambda^\alpha \lambda^\beta) = 2\delta_{\alpha\beta}$. All the operators in (2) are normal ordered. The operators $0_1, \dots, 0_4$ appear in any standard approach (Shifman *et al* 1977; Donoghue 1976; Galic *et al* 1976), and they belong to the $20''$ and 84 representations of the SU(4)-flavour group. The operators 0_5 and 0_6 are due to gluon radiative corrections and the SU(4) flavour symmetry breaking.

The coefficients appearing in (1) are calculated by studying the QCD renormalisation behaviour. They are functions of the charmed quark mass m_c (or t -quark mass m_t), the renormalisation mass μ , the intermediate vector boson mass m_w , and the running coupling constant $g^2(\mu^2/4\pi)$. A representative set of coefficients suggested by Shifman

et al (1977) is

$$\begin{aligned} c_1 &= -2.54, & c_2 &= 0.08, & c_3 &= 0.08, \\ c_4 &= 0.411, & c_5 &= -0.08, & c_6 &= -0.02, \end{aligned} \quad (3)$$

with $m_w = 100 \text{ GeV}$, $m_c = 2 \text{ GeV}$, $\mu = 0.5 \text{ GeV}$ and $g^2(\mu^2/4\pi) = 1$. Similarly, a set of coefficients calculated by Gilman and Wise (1979) for a six quark model is

$$\begin{aligned} c_1 &= -2.74, & c_2 &= 0.08, & c_3 &= 0.08, \\ c_4 &= 0.40, & c_5 &= -0.127, & c_6 &= -0.045, \end{aligned} \quad (4)$$

with $m_t = 30 \text{ GeV}$.

The values of these coefficients when the strong interactions are turned off are (Tadic and Trampetic 1980; Rai Choudhary 1980)

$$c_1 = -1, \quad c_2 = 0.5, \quad c_3 = 2/15, \quad c_4 = 2/3, \quad c_5 = c_6 = 0. \quad (5)$$

3. Matrix elements

3.1 Hyperon decays

We next give the matrix elements of the effective Hamiltonian described in §2. As already mentioned, the most satisfactory approach to calculate these matrix elements, at the present time, is the common approach (Marshak *et al* 1969) wherein soft-pion techniques are employed to reduce nonleptonic amplitudes $\langle B'\pi|H_w|B\rangle$ into matrix elements between single particle states. The latter can then be evaluated using quark-model methods.

The soft-pion techniques lead to the parametrisation of the amplitude involving a pion in the following way:

$$\langle B'\pi|H_w^{\text{eff}}(0)|B\rangle = -\frac{i}{f_\pi} \langle B'|[F_i^5, H_w^{\text{eff}}(0)]|B\rangle + P(q) + R(q), \quad (6)$$

where $f_\pi = 94 \text{ MeV}$ is the pion decay constant. In (6), $P(q)$ represents possible pole terms and $R(q)$ denotes any contribution not included in the first two terms. However, it is known (Ponce 1980) that $R(q) = 0$ as $q \rightarrow 0$, and therefore it can be neglected in the first approximation.

In the calculations made by Donoghue *et al* (1980, 1981) and Galic *et al* (1979) for hyperon decays, the commutator term at the right side of (6) gives the main contribution to the *pv* (parity-violating) *S*-wave amplitudes, and the pole terms $P(q)$ represents the main contribution to the *pc* (parity conserving) *P*-wave amplitudes. Since our primary aim in this paper is not to fit the nonleptonic data, but to extend the scope of the variable bag pressure scheme (Joseph and Nair 1981; Chatley 1983; Chatley *et al* 1984) as well as to test the stability of the theoretical results as obtained by Donoghue *et al* (1980, 1981) and Galic *et al* (1979, 1980), we concern ourselves here only with the contributions coming from the commutator term for the *S*-waves and from the pole-terms for the *P*-waves.

3.1a *S*-wave amplitudes: The current algebra commutator term contributing to the *S*-wave (*pv*) amplitude A involves the single-state weak matrix elements $a_{BB'} \equiv \langle B'|0_i|B\rangle$, where i runs from 1 to 6. In any model employing valence quarks

only, it can be shown (Donoghue *et al* 1980) that

$$\langle B' | 0_2 | B \rangle = \langle B' | 0_3 | B \rangle = \langle B' | 0_4 | B \rangle = 0, \quad (7)$$

in agreement with the Pati-Woo (1971) theorem. Thus the amplitudes $a_{BB'}$ receive contributions only from the operators 0_1 , 0_5 and 0_6 . The various $a_{BB'}$ needed for calculating the S -wave amplitudes for the nonleptonic decay processes can be obtained using table 1, where the quantities a , b , a' , b' are the radial overlap integrals defined as follows:

$$\begin{aligned} a &= \int_0^R d^3r [u_u^3(r)u_s(r) + v_u^3(r)v_s(r)], \\ b &= \int_0^R d^3r [u_u^2(r)v_u(r)v_s(r) + v_u^2(r)u_u(r)u_s(r)], \\ a' &= \int_0^R d^3r [u_s^3(r)u_u(r) + v_s^3(r)v_u(r)], \\ b' &= \int_0^R d^3r [u_s^2(r)v_s(r)v_u(r) + v_s^2(r)u_s(r)u_u(r)]. \end{aligned} \quad (8)$$

In (8), u_q and v_q denote the large and small components of the quark wavefunction, respectively.

Using the definitions (2) of the four-quark operators and table 1, we get the following expressions for the weak matrix elements $a_{BB'}$:

$$a_{\Lambda n} = \bar{G}_F \left\{ c_1 [2\sqrt{6}(a+b)] + \left(c_6 - \frac{8}{3}c_5 \right) [\sqrt{6}(a+b)] \right\}, \quad (9)$$

$$\begin{aligned} a_{\Sigma^0 \Lambda} &= \bar{G}_F \left\{ c_1 [-4\sqrt{6}(a+b)] + \left(c_6 - \frac{8}{3}c_5 \right) [\sqrt{6}(a-a')] \right. \\ &\quad \left. - (2/3)^{1/2}(9b-b') \right\}, \end{aligned} \quad (10)$$

$$a_{\Sigma^+ p} = \bar{G}_F \left\{ c_1 [-12(a+b)] + \left(c_6 - \frac{8}{3}c_5 \right) \left[\frac{2}{3}(3a-13b) \right] \right\}, \quad (11)$$

$$a_{\Sigma^0 n} = \bar{G}_F \left\{ c_1 [-6\sqrt{2}(a+b)] + \left(c_6 - \frac{8}{3}c_5 \right) \left[\frac{\sqrt{2}}{3}(3a-13b) \right] \right\}, \quad (12)$$

$$a_{\Sigma^- \Sigma^-} = \bar{G}_F \left\{ -\frac{2}{3} \left(c_6 - \frac{8}{3}c_5 \right) [(3a+3a') - (b+b')] \right\}, \quad (13)$$

$$\text{where } \bar{G}_F = \frac{G_F}{2\sqrt{2}} \cos \theta_c \sin \theta_c. \quad (14)$$

We choose for the weak interaction constants:

$$G_F = 1.026 \times 10^{-5} / m_p^2, \text{ and } \sin \theta_c \cos \theta_c \approx 1/4. \quad (15)$$

The S -wave hyperon decay amplitudes are now determined using the standard formulae (Marshak *et al* 1969).

Table 1. Matrix elements of the four-quark operators as functions of the overlap integrals a , b , a' and b' defined in (8).

$a_{BB'}$	$(\bar{d}s)(\bar{u}u)$			$(\bar{d}u)(\bar{u}s)$			$(\bar{d}s)(\bar{d}d)$			$(\bar{d}s)(\bar{s}s)$		
	VV	AA	AA	VV	AA	AA	VV	AA	AA	VV	AA	AA
$\langle \Lambda^0 0 n \rangle$	$-\frac{1}{\sqrt{6}}(3a+7b)$	$-\frac{1}{\sqrt{6}}(3a-b)$	$\frac{1}{\sqrt{6}}(8b)$	$\frac{1}{\sqrt{6}}(8b)$	$\frac{2}{\sqrt{6}}(3a-b)$	$-\frac{1}{\sqrt{6}}(3a-b)$	$-\frac{1}{\sqrt{6}}(3a-b)$	$\frac{1}{\sqrt{6}}(3a-b)$	$\frac{1}{\sqrt{6}}(3a-b)$	0	0	0
$\langle \Xi^0 0 \Lambda \rangle$	$-\frac{3}{\sqrt{6}}(a+5b)$	$-\frac{3}{\sqrt{6}}(3a-b)$	$\frac{3}{\sqrt{6}}(a+5b)$	$\frac{3}{\sqrt{6}}(a+5b)$	$\frac{3}{\sqrt{6}}(3a+b)$	0	0	0	$\frac{1}{\sqrt{6}}(3a-b)$	$\frac{1}{\sqrt{6}}(3a-b)$	$\frac{1}{\sqrt{6}}(3a-b)$	$\frac{1}{\sqrt{6}}(3a-b)$
$\langle \Sigma^+ 0 p \rangle$	$-\frac{2}{3}(3a+11b)$	$-\frac{4}{3}(3a-b)$	$\frac{1}{3}(3a+23b)$	$\frac{1}{3}(3a+23b)$	$\frac{5}{3}(3a-b)$	0	0	0	0	0	0	0
$\langle \Sigma^0 0 n \rangle$	$-\frac{1}{3\sqrt{2}}(3a+23b)$	$-\frac{5}{3\sqrt{2}}(3a-b)$	$\frac{2}{3\sqrt{2}}(3a+11b)$	$\frac{2}{3\sqrt{2}}(3a+11b)$	$\frac{4}{3\sqrt{2}}(3a-b)$	$-\frac{1}{3\sqrt{2}}(3a-b)$	$-\frac{1}{3\sqrt{2}}(3a-b)$	$\frac{1}{3\sqrt{2}}(3a-b)$	$\frac{1}{3\sqrt{2}}(3a-b)$	0	0	0
$\langle \Xi^- 0 \Sigma^- \rangle$	0	0	0	0	0	$-\frac{1}{3}(3a-b)$	$-\frac{1}{3}(3a-b)$	$\frac{1}{3}(3a-b)$	$\frac{1}{3}(3a-b)$	$-\frac{1}{3}(3a-b)$	$-\frac{1}{3}(3a-b)$	$\frac{1}{3}(3a-b)$
$\langle \Omega^- 0 \Xi^{*-} \rangle$	0	0	0	0	0	0	0	0	0	$-4\sqrt{3}(3a-b)$	$4\sqrt{3}(3a-b)$	$4\sqrt{3}(3a-b)$
$\langle K^- 0 \pi^- \rangle$	$-(a+5b)$	$(3a-b)$	0	0	$6(3a-b)$	0	0	0	0	0	0	0
$\langle K^0 0 \pi^0 \rangle$	0	$3\sqrt{2}(a-3b)$	$-\frac{1}{\sqrt{2}}(a+5b)$	$-\frac{1}{\sqrt{2}}(a+5b)$	$\frac{1}{\sqrt{2}}(3a-b)$	$\frac{1}{\sqrt{2}}(3a-b)$	$\frac{1}{\sqrt{2}}(a+5b)$	$-\frac{1}{\sqrt{2}}(a+5b)$	$-\frac{1}{\sqrt{2}}(a+5b)$	0	0	0

3.1b *P-wave amplitudes*: The standard treatment for the *P*-waves is to calculate baryon poles (Marshak *et al* 1969). These consist of $\Delta S = 1$ baryon to baryon transitions accompanied by strong pion emission. The baryon pole-terms contributing to the various *P*-wave amplitudes are

$$\begin{aligned}
 B^P(\Lambda_0^0) &= g(N + \Lambda) \left[\frac{a_{\Lambda n}}{(\Lambda - N)(2N)} - \frac{\sqrt{2/3} da_{\Sigma^+ p}}{(\Sigma - N)(\Sigma + \Lambda)} \right], \\
 B^P(\Xi_0^0) &= \sqrt{2}g(\Lambda + \Xi) \left[\frac{a_{\Xi^- \Sigma^-} d}{\sqrt{3}(\Xi - \Sigma)(\Sigma + \Lambda)} + \frac{a_{\Xi^0 \Lambda}(f - d)}{\sqrt{2}(\Xi - \Lambda)(2\Xi)} \right], \\
 B^P(\Sigma_0^+) &= -g(N + \Sigma) \left[\frac{a_{\Sigma^+ p}}{(\Sigma - N)(2N)} - \frac{2fa_{\Sigma^+ p}}{(\Sigma - N)(2\Sigma)} \right], \\
 B^P(\Sigma_+^+) &= 2g(\Sigma + N) \left[-\frac{a_{\Sigma^+ p}(f + d)}{\sqrt{2}(\Sigma - N)(2N)} - \frac{a_{\Sigma^0 n} f}{(\Sigma - N)(2\Sigma)} \right. \\
 &\quad \left. + \frac{a_{\Lambda n} d}{\sqrt{3}(\Lambda - N)(\Sigma + \Lambda)} \right], \tag{16}
 \end{aligned}$$

where g is the πNN coupling constant ($g^2/4\pi = 14.6$), f and d reflect the SU(3) structure of the strong coupling ($f + d = 1$), and the particle symbols represent their experimental masses. For d/f , we use the best fit value (Gronau 1972), $d/f = 1.8$. The weak baryon to baryon matrix elements $a_{\Lambda n}$, $a_{\Sigma^+ p}$, $a_{\Xi^0 \Lambda}$, $a_{\Xi^- \Sigma^-}$ are the same as given in (9) to (13).

It is known that kaon-poles also contribute to the *P*-wave hyperon decays, in addition to the baryon-poles. The strength of the *K*-poles is determined by evaluating the $K - \pi$ transition matrix elements $a_{K\pi}$. Their forms for the various *P*-wave decay amplitudes are:

$$\begin{aligned}
 B^K(\Lambda_0^0) &= -g \frac{d + 3f}{\sqrt{3}} \frac{a_{K^0 \pi^0}}{m_K^2 - m_\pi^2} (2m_K^2)^{1/2}, \\
 B^K(\Xi_0^0) &= -2g \frac{d - 3f}{\sqrt{3}} \frac{a_{K^0 \pi^0}}{m_K^2 - m_\pi^2} (2m_K^2)^{1/2}, \\
 B^K(\Sigma_0^+) &= -\sqrt{2} g(d - f) \frac{a_{K^0 \pi^0}}{m_K^2 - m_\pi^2} (2m_K^2)^{1/2}, \\
 B^K(\Sigma_+^+) &= 0. \tag{17}
 \end{aligned}$$

Here the strong baryon-kaon vertex is parametrised through f and d in the usual way. The matrix elements $a_{K^0 \pi^0}$ and $a_{K^- \pi^-}$ as obtained from table 1 and definitions (2) for the four-quark operators are

$$\begin{aligned}
 a_{K^0 \pi^0} &= \bar{G}_F \left\{ [c_1 - 2(c_2 + c_3 - 2c_4)](a - 3b) \right. \\
 &\quad \left. + \left(c_6 + \frac{16}{3}c_5 \right)(a + b) \right\}, \tag{18}
 \end{aligned}$$

$$a_{K^-\pi^-} = \overline{G}_F \left\{ [-c_1 + 2(c_2 + c_3 + c_4)](a - 3b) - \left(c_6 + \frac{16}{3}c_5 \right) (a + b) \right\}. \quad (19)$$

In (17), the factor $(2m_K^2)^{1/2}$ occurs because of the invariant normalisation term $(4m_K E_\pi)^{1/2}$ (Donoghue *et al* 1980).

3.2 Ω^- -decays

We restrict ourselves to the three nonleptonic decay modes of Ω^- seen experimentally:

$$\Omega^- \rightarrow \Lambda K^-, \quad \Omega^- \rightarrow \Xi^0 \pi^-, \quad \Omega^- \rightarrow \Xi^- \pi^0.$$

It is known (Galic *et al* 1980; Tadic and Trampetic 1981; Finjord 1978; Ponce 1980 and references therein) that Ω^- decay probability is determined largely by the parity conserving P -wave amplitude B . All B amplitudes receive contributions from pole diagrams. The contributions coming from these pole diagrams are

$$\begin{aligned} B^P(\Omega^- \rightarrow \Lambda K^-) &= g_{\Lambda \Xi^* \pi^-} \frac{a_{\Omega^- \Xi^* \pi^-}}{\Omega^- - \Xi^* \pi^-} - g_{\Xi^0 \Omega^- K^-} \frac{a_{\Xi^0 \Lambda}}{\Xi^0 - \Lambda}, \\ B^P(\Omega^- \rightarrow \Xi^0 \pi^-) &= g_{\Xi^0 \Xi^* \pi^-} \frac{a_{\Omega^- \Xi^* \pi^-}}{\Omega^- - \Xi^* \pi^-}, \\ B^P(\Omega^- \rightarrow \Xi^- \pi^0) &= g_{\Xi^- \Xi^* \pi^0} \frac{a_{\Omega^- \Xi^* \pi^-}}{\Omega^- - \Xi^* \pi^-}, \end{aligned} \quad (20)$$

where the particle symbols represent their experimental masses, the matrix element $a_{\Xi^0 \Lambda}$ is determined from (10), and the element $a_{\Omega^- \Xi^* \pi^-}$ is given by

$$a_{\Omega^- \Xi^* \pi^-} = \overline{G}_F \left\{ -8\sqrt{3} \left(c_6 - \frac{8}{3}c_5 \right) (3a' - b') \right\}. \quad (21)$$

Equation (21) can be easily obtained from table 1 and using the definitions (2). The strong coupling constants in (20) can be determined using SU(3) relations (Nagels *et al* 1979)

$$\begin{aligned} g_{\Xi^0 \Omega^- K^-} &= -g_{\Xi^- \Omega^- \pi^0} = \sqrt{2} g_{\Lambda \Xi^* \pi^-} \\ &= \sqrt{3} g_{\Xi^0 \Xi^* \pi^-} = \sqrt{6} g_{\Xi^- \Xi^* \pi^0} = 13.01 \text{ GeV}^{-1} \end{aligned} \quad (22)$$

The processes $\Omega^- \rightarrow \Xi^0 \pi^-$ and $\Omega^- \rightarrow \Xi^- \pi^0$ may also receive contributions from K -pole terms:

$$\begin{aligned} B^K(\Omega^- \rightarrow \Xi^0 \pi^-) &= g_{\Xi^0 \Omega^- K^-} - \frac{a_{K^-\pi^-}}{m_K^2 - m_\pi^2} (2m_K^2)^{1/2}, \\ B^K(\Omega^- \rightarrow \Xi^- \pi^0) &= -\frac{g_{\Xi^0 \Omega^- K^-}}{\sqrt{2}} - \frac{a_{K^0 \pi^0}}{m_K^2 - m_\pi^2} (2m_K^2)^{1/2}, \end{aligned} \quad (23)$$

where $a_{K\pi}$ are given by (18) and (19).

4. Numerical results and discussion

Our choice of input parameters is (Joseph and Nair 1981) $m_u = m_d = 0.114$ GeV, $m_s = 0.302$ GeV, $R = 8.88$ GeV⁻¹ for baryons, and $R = 7.75$ GeV⁻¹ for mesons. Using these values, we evaluate the overlap integrals (equation (8)) and list them in table 2. For the QCD enhancement factors c_i 's, we use the values given in (3), and then calculate the nonleptonic hyperon and Ω^- decay amplitudes through relations (16), (17), (20) and (23). Numerical results are presented in tables 3 and 4 for hyperon decays, and in table 5 for Ω^- decays.

We find that, in our bag model calculations, though the relative sign and the relative magnitude of all non-leptonic decay amplitudes can be correctly reproduced, our overall results are too small by a factor of 2 to 4 when compared with those obtained by Donoghue *et al* (1981), Galic *et al* (1979), and Tadic and Trampetic (1981). The decrease in the amplitudes is readily understandable if we realize that our results crucially depend upon the parameters of the model. It is evident that overlap integrals (equation (8)) transform as $1/R^3$. Our model has $R = 8.88$ GeV⁻¹, while the model of De Grand *et al* (1975) employs $R = 5.00$ GeV⁻¹, roughly accounting for the decrease. Obviously, the radius sets the scale for the overlap integrals.

It is worth mentioning here that in literature, one finds two sets of ground state parameters for the MIT bag model. The set predicted by De Grand *et al* (1975), which gives $2\mu_p M_p = 1.9$ (about 30% less than the experimental value of 2.79), and $\langle r_p^2 \rangle^{1/2}$

Table 2. Numerical values of bag overlap integrals (Each value is to be multiplied by 10^{-3} GeV⁻³).

Overlap Integral	Baryons	Mesons
a	0.37	0.54
b	0.08	0.13
a'	0.45	—
b'	0.06	—

For baryons $R = 8.88$ GeV⁻¹, $w_u = w_d = 0.294$ GeV, $w_s = 0.427$ GeV and for mesons $R = 7.75$ GeV⁻¹, $w_u = w_d = 0.321$ GeV, $w_s = 0.448$ GeV.

Table 3. S-wave hyperon decay amplitudes in units of $10^5 m_\pi^{-1/2} \text{sec}^{-1/2}$.

Decay mode	Present analysis	Donoghue <i>et al</i> (1981)	Galic <i>et al</i> (1979)	Experimental
Σ_0^+	-0.54	-1.27	-1.53	-1.48
Σ_0^-	0.75	1.80	2.13	1.93
Ξ_0^0	0.43	1.03	1.21	1.54
Λ_0^0	-0.15	-0.50	-0.62	-1.07

Table 4. *P*-wave hyperon decay amplitudes in units of $10^5 m_\pi^{-1/2} \text{sec}^{-1/2}$.

Decay mode	Present analysis			Tadic and Trampetic (1980)	Experimental
	Baryon poles	Kaon pole	Total		
Σ_0^+	1.92	0.08	2.00	7.98	12.04
Σ_1^+	3.14	0	3.14	11.22	19.06
Ξ_0^0	-1.17	-0.06	-1.23	-4.62	-5.56
Λ_0^0	-0.52	0.12	-0.40	-2.0	-7.07

For the strong coupling constants f and d , we use the best fit value (Gronau 1972) $d/f = 1.8$.

Table 5. Ω^- decay amplitudes in GeV^{-1}

Decay mode	Present analysis			Tadic and Trampetic (1981)
	Baryon poles	Kaon poles	Total	
$10^6 B(\Omega_K^-)$	0.92	0	0.92	4.01
$10^6 B(\Omega_\pi^-)$	-0.20	0.04	-0.16	0.84
$10^6 B(\Omega_\sigma^-)$	-0.14	0.02	-0.12	0.31

$= 0.73 \text{ fm}$ (which is about 20% smaller than the experimental value of 0.88 fm) is the only one which gives a satisfactory estimate of nonleptonic decay amplitudes. The other set (Donoghue *et al* 1975) with $R = 7.33 \text{ GeV}^{-1}$ (yielding $2\mu_p M_p = 2.63$, $\langle r_p^2 \rangle^{1/2} = 1.03 \text{ fm}$), which is nearer our values leads to much poorer predictions of the decay amplitudes.

Lastly, we point out certain uncertainties inherent in such calculations. First is the calculation of *K*-pole terms, which involves the evaluation of the matrix elements a_{K^0, π^0} and a_{K^-, π^-} . In a bag scheme, these matrix elements depend upon the combination $(a - 3b)$ of the overlap integrals. It has to be pointed out (Colic *et al* 1982) that due to helicity suppression, the exact magnitude and even sign of the particular combination $(a - 3b)$ is uncertain. It may be positive, zero or even negative. With the MIT model parameters of De Grand *et al* (1975), it is $-1.20 \times 10^{-3} \text{ GeV}^3$, whereas with our parameters, it is $0.16 \times 10^{-3} \text{ GeV}^3$. Its precise value is thus very strongly dependent on the values of the bag model parameters. There are problems also with the theoretical enhancement coefficients c_i . Though the values of the coefficients c_1, c_2, c_3, c_4 are said to be reasonably secure, the same is not true for the coefficients c_5 and c_6 . These (c_5 and c_6) arise due to SU(4) symmetry breaking, and cannot be estimated theoretically to an impressive level of accuracy (Vainshtein *et al* 1977; Donoghue *et al* 1980).

Our conclusion is that the present model is not as successful for nonleptonic decays as for predicting static properties of hadrons. As already pointed out, one of the reasons for the discrepancy may be the large bag size. Another reason for the poor results may be the soft-pion approximation, which may not work as well, in the present context, as it does in other low-energy processes.

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