Pramāṇa, Vol. 22, No. 6, June 1984, pp. 513–521. Printed in India.

Muon capture by ^{16}O – using a microscopic theory of particle-hole states

V N SRIDHAR, R PARTHASARATHY and Y R WAGHMARE*

Matscience, Institute of Mathematical Sciences, Madras 600 113, India

* Department of Physics, Indian Institute of Technology, Kanpur 208 016, India

MS received 16 January 1984; revised 10 March 1984

Abstract. The partial capture rates for the process, $\mu^- + {}^{16}O\ (g.s) \rightarrow {}^{16}N\ (2^-, 1^-, 0^-, 3^-) + \nu_\mu$ have been calculated using the particle-hole wavefunctions obtained using self-consistent procedure. In deriving these wavefunctions, the effective N-N interaction has been constructed from the bare Hamada-Johnston interaction. The terms in the muon capture Hamiltonian that depend on the momentum of the capturing proton have been included and their importance in $0^+ \rightarrow 0^-$ transition is exhibited. The agreement with the available experimental data is good. The need to incorporate meson exchange effects in $0^+ \rightarrow 0^-$ transition is pointed out.

Keywords. Landau-Migdal theory; muon capture; partial capture rates; induced pseudoscalar coupling; meson exchange effects.

1. Introduction

The study of muon capture by ^{16}O essentially serves as a powerful probe to examine the nuclear structure. Since the partial transition rates to the low-lying levels of ^{16}N, stable against the emission of nucleons, are highly nuclear model-dependent, such a study is not suitable to examine the muon capture coupling constants in general, and the induced pseudoscalar coupling g_p in particular. The long standing discrepancy between the measured and calculated capture rates in ^{16}O has been remedied to some extent by Rho (1967) by using the quasi-particle model of Migdal. The various theoretical results on partial capture rates for the PCAC value of g_p are given in table 1, along with a brief description of the nuclear models used and the available experimental data.

Table 1 reveals the nuclear model dependence of the capture rates, and a comparison with the experimental data certainly warrants more theoretical studies in this direction. Further, recently Guichon *et al* (1978, 1979) have pointed out the role of meson exchange effects on $0^+ \rightarrow 0^-$ transition in ^{16}O. This seems to have been confirmed by the recent Argonne National Laboratory and Stanford University experiment (Gagliardi *et al* 1982). On the contrary, Koshigiri *et al* (1979) claim that the nuclear structure effects when properly taken into account can explain the $0^+ \rightarrow 0^-$ transition rate.

From table 1, we notice that it is rather unfortunate that there is no consistent agreement among the theoretical estimates. Even in the most dominant $0^+ \rightarrow 2^-$ transition, the latest estimate of Graves *et al* (1980) disagrees with earlier estimates and

Table 1. Partial capture rates in ^{16}O in 10^3 sec^{-1}.

References	Model	0^-	1^-	2^-	3^-
Gillett and Jenkins (1965)	Elliot-flowers wavefunctions	2·66	4·25	19·8	0·163
	RPA	1·76	2·36	14·1	0·180
Rho (1967)	Migdal wavefunctions	1·01	2·16	6·36	0·121
Green and Rho (1969)	$2p$-$2h$ correlations in ^{16}O	1·06	3·18	10·40	0·170
Donnelly and Walecka (1972)	Serber-Yukawa-fitting e^- Scattering	0·86	1·42	7·54	0·060
Nalcioglu *et al* (1973)	Spherical and deformed ^{16}O	0·94	5·28	4·23	0·290
Rej (1975)	Up to $4p$-$4h$ in ^{16}O and $3p$-$3h$ in ^{16}N	1·30	2·10	10·90	—
Eramzhyan *et al* (1977)	Hamada-Johnston potential with screening and $2p$-$2h$ in ^{16}O	1·27	1·64	8·32	—
Koshigiri (1979)	RPA	1·86	—	—	—
	Brown-Green $2p$-$2h$ in ^{16}O	1·29	—	—	—
Graves *et al* (1980)	Helm model	—	1·60	5·07	0·040
Present result	Self-consistent p-h: Waghmare (1977)	0·89	2·31	9·69	0·200
Cohen *et al* (1964)	Experiment	$1·1 \pm 0·2$	$1·88 \pm 0·1$	$6·3 \pm 0·7$	—
Astbury *et al* (1964)	—	$1·6 \pm 0·2$	$1·40 \pm 0·2$	—	—
Deutsch *et al* (1967)	—	—	—	$7·9 \pm 0·8$	—
Deutsch *et al* (1969)	—	$0·85^{+0·15}_{-0·06}$	$1·85^{+0·36}_{-0·17}$	—	0·08
Palffy *et al* (1971)	—	—	—	—	$0·13 \pm 0·08$
Kane *et al* (1972)	—	$1·56 \pm 0·18$	$1·31 \pm 0·11$	$8·0 \pm 1·2$	0·09
Guichon *et al* (1979)	—	$1·57 \pm 0·13$	$1·36 \pm 0·13$	—	—

also with the experiment. The spherical and deformed co-existence approach of Nalcioglu *et al* (1973) underestimates $0^+ \rightarrow 2^-$ capture rate and overestimates $0^+ \rightarrow 1^-$ capture rate. These results clearly demonstrate the sensitivity of the partial capture rate towards the use of nuclear models. Thus it is worthwhile to examine them by a different nuclear model, hitherto not used and which is more realistic. Secondly the consensus among the various experimental measurements as displayed in table 1 is not good either.

This paper attempts to calculate the partial capture rates using the microscopic theory of particle-hole states developed by Waghmare (1977). A major departure from earlier theoretical calculations is the use of Hartree-Fock basis for the single particle states using a full effective interaction. The scheme becomes more palatable by the following additional feature, namely, the construction of an effective interaction from the bare Hamada-Johnston interaction. The success of this in explaining the total capture rates in ^{12}C and ^{16}O (Parthasarathy and Waghmare 1979) motivates the present study.

In §2, the relevant expression required to calculate the partial capture rate is given along with a discussion of MEC. Section 3 deals briefly with the nuclear model used in the present study and the results are discussed in §4.

2. Partial capture rates

Starting from the Fujii-Primakoff (1959) Hamiltonian for muon capture, the expression for the capture rate, λ to a particular low-lying level of the final nucleus J_f, in impulse approximation is given by,

$$\lambda(J_f) = \frac{v^2}{2\pi} |\phi_\mu|^2_{av} G^2 \{G_V^2 |\int 1_i|^2 + G_A^2 |\int \sigma_i|^2$$
$$+ (G_P^2 - 2G_P G_A) |\int \hat{v} \cdot \sigma_i|^2 - 2G_V \frac{g_V}{M} R \cdot P \cdot \{(\int 1_i)$$
$$\times (\int \hat{v} \cdot \mathbf{p}_i)^*\} + \frac{2}{M}(G_P - G_A) g_A R \cdot P \cdot \{(\int \hat{v} \cdot \sigma_i)(\int \sigma_i \cdot \mathbf{p}_i)^*\}$$
$$+ \frac{2}{M} G_V g_V R \cdot P \cdot \{i \int \sigma_i \cdot (\int \hat{v} \times \mathbf{p}_i)^*\}\}, \quad (1)$$

where $|\phi_\mu|^2_{av}$ is the square of the muon wavefunction in atomic K-orbit averaged over the nuclear volume and is given by $\pi^{-1}(z/a_0)^3 R_\mu$ with $z = 8$, a_0 the muonic Böhr radius and R_μ a correction factor for the finite size of the nucleus, v is the momentum carried away by the neutrino, G is the Fermi coupling constant, and M is the mass of the nucleon. G_V, G_A, G_P are the effective couplings and are given by

$$G_V = g_V(1 + v/2M),$$
$$G_A = g_A - (g_V + g_M)v/2M,$$
$$G_P = (g_P - g_A - g_V - g_M)v/2M, \quad (2)$$

with the following numerical values for the muon capture coupling constants. g_V the vector coupling is $0{\cdot}987G$, g_A the axial-vector coupling is $-1{\cdot}25\, g_V$, g_M the weak magnetism coupling is 3·7 g_V and g_P is varied around its PCAC value of 7·5 g_A. The second class current couplings are assumed to be absent. The various integrals in (1) are nuclear matrix elements, evaluated in the nuclear Hilbert space. The terms in (1) that are $O(1/M)$ are known as the momentum dependent terms and are of very much importance in the $0^+ \to 0^-$ transition as will be shown later. In fact they contribute as much as the momentum independent terms in the above transition and hence their omission in the calculation of Graves *et al* (1980) is not justified. To convert the capture rate from $\hbar = c = m_\mu = 1$ to cgs units, (1) must be divided by $\hbar/(m_\mu c^2) = 6{\cdot}22 \times 10^{-24}$ sec.

The expression for the various nuclear matrix elements occurring pairwise in (1) has been evaluated using the particle-hole model. As these derivations are standard, they are not given here. We briefly discuss the relevant changes when the meson exchange corrections (MEC) are incorporated. The use of soft-pion theorems and current algebra for muon capture and beta decay processes, demonstrate (Kubodera *et al* 1978) that the time part of the axial vector current gets considerably modified due to MEC, when compared to the space part. This can also be viewed as an important correction to impuse approximation (IA). The effect of this on processes that are dominated by the space part of the axial vector current are shown to be small (Parthasarathy and Sridhar 1981). In the present process, the $0^+ \to 0^-$ transition is dominated by the time part of the axial vector current. In order to take this into account, we note that the MEC to the time part of the axial vector current effectively changes g_A to Fg_A for those terms coming from the time part. Accordingly the expression for $\lambda(0^-)$ changes only in the third and fifth term in (1). Explicitly these two terms will read as $(G_P'^2 - 2G_P'G_A)|\int \hat{v}\cdot\sigma_i|^2$ and $2(G_P' - G_A)g_A'\ R{\cdot}P{\cdot}\{\int \hat{v}{\cdot}\sigma_i(\int \sigma_i{\cdot}\mathbf{p}_i)^*\}$, where G_P' is obtained with Fg_A. Note that there is no change in g_A contained in G_A as this comes from the space part of the axial vector current. It has been shown by Guichon *et al* (1979b) that the factor F which measures the change of the matrix element from its IA value is $\sim 1{\cdot}5$ and this factor practically does not depend upon the nuclear model. This MEC is considered for $0^+ \to 0^-$ transition in the last section.

3. Nuclear model

The nuclear model that has been used in this study is the self-consistent theory of particle-hole interactions by Waghmare (1977). The details of the various aspects of the model are contained in the above reference and so we briefly give the necessary steps only. The description for (i) the basis for particle-hole spectrum (ii) the *p-p*, *p-h*, *h-h* interactions and (iii) the mechanisms of invoking various excitations when energy is supplied to the nucleus, needed to study the excited states are summarized below. The single particle basis are obtained from a self-consistent approach. The HF energy and *p-h* interaction are derived from Landau (1958) theory of Fermi liquids. Here the first and second derivatives of the energy as a function of density, gives the HF energy and *p-h* interaction respectively. The detailed expressions for these are given by Waghmare (1977). The excited states of ^{16}O are described in Tamm-Dancoff approximation (TDA) and random phase approximation (RPA). Starting from the *N-N* interaction described by Hamada-Johnston potential, the Brueckner G matrix is evaluated after including the Coulombian part. Using the matrix elements of G between various combinations of neutron and proton states, the BHF procedure is used to obtain the ground state properties and HF basis. Landau's *p-h* interaction is obtained from *p-p* interaction using Pandya's theorem (Pandya 1956). In our application to muon capture by ^{16}O, we make use of the fact that the various final states $(2^-, 0^-, 1^-, 3^-)$ of ^{16}N are isobaric analogue states of $(T = 1)$ ^{16}O and consider TDA wavefunction. Waghmare (1977) has used these wavefunctions in his investigation of $B(E1, E2, E3)$ for the normal parity states of ^{16}O. In a way, the present study will provide further test to the nuclear wavefunctions of Waghmare (1977) as applied to muon capture process. For the sake of completion, the various configuration mixing coefficients used in this study are given in table 2. A striking feature is the expansion of the shell-model space to include higher single particle orbitals.

4. Numerical results and discussion

The transition rates to various low lying states of ^{16}N are evaluated using (1) and table 2. Their numerical values for $g_P/g_A = 7{\cdot}5$ are given in table 1 (present calculation). The $0^+ \to 1^-$ and $0^+ \to 3^-$ capture rates are independent of g_p while the variation of $0^+ \to 0^-$ and $0^+ \to 2^-$ capture rates with g_P are given in figures 1 and 2 respectively both for momentum-independent and momentum-dependent terms in (1). The calculations are done in IA. The effect of the finite size of the nucleus on the muon wavefunction is considered. Thus the calculation is complete. We proceed now to discuss the results.

It is to be seen from figure 1 that the momentum-dependent terms in $0^+ \to 0^-$ transition are very important and contribute as much as the momentum-independent terms. The most recent experiment of Guichon *et al* (1979) is projected in figure 1 by the shaded region. At first sight, it appears that the overlap with the complete calculation gives $g_P/g_A \sim 3$ which is just half the PCAC value. Using our earlier studies on average recoil nuclear polarization with γ-decay corrections in $A = 12$ system (Devanathan *et al* 1972; Parthasarathy and Sridhar 1979) and $\gamma - \nu$ angular correlation in $A = 28$ system (Parthasarathy and Sridhar 1981a) which are to a large extent free from the nuclear wave function uncertainties, for g_P/g_A as (13 ± 3), we find the $0^+ \to 0^-$

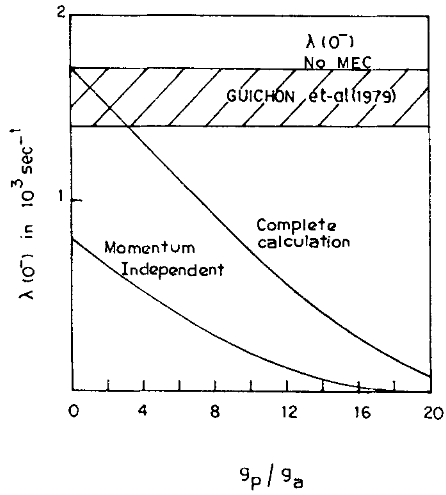

Figure 1. Variation of $\lambda(0^-)$ with g_P.

Table 2. Configuration mixing coefficients used.

J^π(MeV)	$(1d_{5/2})(1p_{3/2})^{-1}$	$(1d_{5/2})(1p_{1/2})^{-1}$	$(1d_{3/2})(1p_{3/2})^{-1}$	$(1d_{3/2})(1p_{1/2})^{-1}$	$(2s_{1/2})(1p_{3/2})^{-1}$	$(2s_{1/2})(1p_{1/2})^{-1}$	$(2p_{3/2})(1s_{1/2})^{-1}$	$(2p_{1/2})(1s_{1/2})^{-1}$	$(1f_{5/2})(1s_{1/2})^{-1}$	$(1f_{7/2})(1s_{1/2})^{-1}$
0^-(18·388)	—	—	−0·1437	—	—	−0·9894	—	−0·0421	—	—
1^-(18·318)	−0·1896	—	0·0606	−0·8150	0·1699	0·9621	0·0262	0·0180	—	—
2^-(18·515)	0·5552	0·6760	−0·3435	−0·0157	0·3516	—	−0·0007	—	−0·0232	—
3^-(18·715)	0·5882	0·7757	0·2162	—	—	—	—	—	0·0453	0·0638

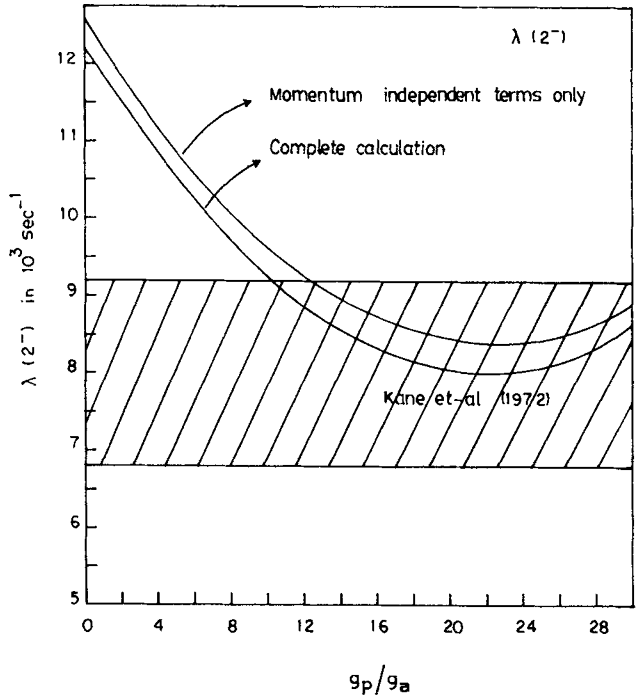

Figure 2. Variation of $\lambda(2^-)$ with g_P.

partial transition rate to be $(0{\cdot}6 \pm 0{\cdot}2) \times 10^3$ sec^{-1} which agrees only with the Louvain experiment (Deutsch *et al* 1969). In the analysis of ^{12}B recoil polarization and γ–ν angular correlation in $A = 28$, Parthasarathy and Sridhar (1981) have pointed out that any observable in muon capture that crucially depends on the momentum-dependent terms, will be suited to examine MEC. Hence the $0^+ \to 0^-$ transition is ideally suited to study the effect of MEC and this, in fact *enhances* the capture rate.

It has been shown by Kubodera *et al* (1978) that the time part of axial current $a_\mu(x)$ gets substantially modified by MEC. This should be viewed as an important correction or improvement over IA. Phenomenologically, the ratio of the matrix element with MEC to its IA value is a measure of such MEC. This value is nearly 1·5 for ^{16}O (Guichon *et al* 1979). As stated in §2, the $0^+ \to 0^-$ capture rate gets substantially *enhanced* now. Explicitly, the $\lambda(0^-)$ without momentum-dependent terms are $0{\cdot}2999 \times 10^3$ sec^{-1} in IA and $0{\cdot}3652 \times 10^3$ sec^{-1} with MEC, for $g_P = 7{\cdot}5\, g_A$. The contribution from the momentum-dependent terms are $0{\cdot}6115 \times 10^3$ sec^{-1} and $0{\cdot}9493 \times 10^3$ sec^{-1} in IA and with MEC respectively. One realises that momentum-dependent terms contribute twice the momentum independent terms. The full $\lambda(0^-)$ are $0{\cdot}9114 \times 10^3$ sec^{-1} in IA and $1{\cdot}3145 \times 10^3$ sec^{-1} with MEC. Thus MEC enhances $\lambda(0^-)$ and brings better agreement with experiment. The MEC corrected $\lambda(0^-)$ can be pushed up still further by appealing to $g_P \sim 13\, g_A$ in light nuclei. In view of the experience that other nuclear wavefunction modifications such as the inclusion of $2p$-$2h$ correlations in ^{16}O and $3p$-$3h$ correlations in ^{16}N will only decrease the capture rate, the only possible enhancement required, could be due to MEC. Thus our calculations unequivocally demonstrate the need for MEC in explaining $\lambda(0^-)$ and hence a strong evidence in favour of them.

The $\lambda(1^-)$ is in very good agreement with the Louvain measurement (Deutsch *et al* 1969). $\lambda(1^-)$ without and with momentum-dependent terms are $3{\cdot}14 \times 10^3\ \mathrm{sec}^{-1}$ and $2{\cdot}31 \times 10^3\ \mathrm{sec}^{-1}$ respectively. Quite contrary to $0^+ \to 0^-$ situation, the momentum-dependent terms decrease the capture rate. The decrease is about 25% compared to an increase of about 200% in $0^+ \to 0^-$ capture rate.

The $\lambda(3^-)$ without and with momentum-dependent terms are $0{\cdot}2602 \times 10^3\ \mathrm{sec}^{-1}$ and $0{\cdot}208 \times 10^3\ \mathrm{sec}^{-1}$ respectively. The momentum-dependent terms decrease the capture rate by about 20%. Our complete $\lambda(3^-)$ is $0{\cdot}208 \times 10^3\ \mathrm{sec}^{-1}$, which is in good agreement with the only measurement of Palffy *et al* (1971) which is $(0{\cdot}13 \pm 0{\cdot}08) \times 10^3\ \mathrm{sec}^{-1}$.

The $\lambda(1^-)$ and $\lambda(3^-)$ are independent of g_P.

The variation of $\lambda(2^-)$ with g_P is displayed in figure 2 along with the latest experimental measurement. The agreement with PCAC value of g_P is good. The nuclear model dependence of the capture rate prohibits us in drawing conclusions about g_P. From figure 2, for $g_P = (13 \pm 3) g_A$, the nuclear model insensitive estimate, $\lambda(2^-)$ is $(8{\cdot}7 \pm 0{\cdot}45) \times 10^3\ \mathrm{sec}^{-1}$ which is in excellent agreement with the experimental values of table 1. In this transition, the contribution from the momentum-dependent terms is about -5%.

5. Conclusions

(i) The use of BHF-Landau microscopic model of *p-h* states in ^{16}O explains the partial capture rates to ^{16}N better, (ii) The momentum-dependent terms contribute (by -5 to 20%) to the 0^+ to $1^-, 2^-, 3^-$ transitions while they contribute to 0^+ to 0^- transition by about $+200$% (the main reason for invoking MEC), (iii) The $0^+ \to 0^-$ capture rate can be explained only if the MEC are included. This indicates the existence of mesonic degrees of freedom in ^{16}O, (iv) For $0^+ \to 1^-, 2^-, 3^-$ transitions, the MEC are small, as they are dominated by the space part of $a_\mu(x)$, (v) The results are in better agreement with available data if we use the theoretical estimate of g_P obtained in a nuclear model insensitive way and hence a reliable estimate.

References

Astbury A, Auerbach L B, Cutts D, Esterling R J, Jenkins D A, Lipman N H and Shafer R E 1964 *Nuovo Cimento* **33** 1020
Cohen R C, Devons S and Kanaris A D 1964 *Nucl. Phys.* **57** 255
Devanathan V, Parthasarathy R and Subramanian P R 1972 *Ann. Phys.* **72** 291
Deutsch J P, Grenacs L, Igo-Kemenes P, Lipnik P and Macq P C 1967 *Nuovo Cimento* **B52** 557
Deutsch J P, Grenacs L, Lehmann J, Lipnik P and Macq P C 1969 *Phys. Lett.* **B29** 66
Donnelly T W and Walecka J D 1972 *Phys. Lett.* **B41** 275
Eramzhyan R A, Gmitro M, Sakaev R A and Tosunjan L A 1977 *Nucl. Phys.* **A290** 294
Fujii A and Primakoff H 1959 *Nuovo Cimento* **12** 327
Gagliardi C A, Garvey G T and Wrobel J R 1982 *Phys. Rev. Lett.* **48** 914
Gillet V and Jenkins D A 1965 *Phys. Rev.* **B140** 32
Graves R D, Lamers B A, Nagl A, Uberall H, Devanathan V and Subramanian P R 1980 *Can. J. Phys.* **58** 48
Green A M and Rho M 1969 *Nucl. Phys.* **A130** 112
Guichon P, Giffon M and Samour C 1978 *Phys. Lett.* **B74** 15
Guichon P, Bichoreau B, Giffon M, Gonclaves A, Julien J, Roussel L and Samour C 1979 *Phys. Rev* **C19** 987

Kane F R, Eckhause M, Miller G H, Roberts R L, Vislay M E and Welsh R E 1973 *Phys. Lett.* **B45** 292
Koshigiri K, Ohtsubo H and Morita M 1979 *Prog. Theor. Phys.* **62** 706
Kubodera K, Delorme J and Rho M 1978 *Phys. Rev. Lett.* **40** 755
Landau L D 1958 *Sov. Phys. JETP* **8** 70
Nalcioglu O, Goswami A and Graves R D 1973 *Lett. Nuovo Cimento* 7 897
Palffy L, Lehman J, Grenacs L and Subotowicz M 1971 *Nuovo Cimento* **A3** 505
Pandya S P 1956 *Phys. Rev.* **103** 956
Parthasarathy R and Waghmare Y R 1979 *Pramana* **13** 457
Parthasarathy R and Sridhar V N 1979 *Phys. Lett.* **B82** 167
Parthasarathy R and Sridhar V N 1981 *Phys. Lett.* **B106** 363
Parthasarathy R and Sridhar V N 1981a *Phys. Rev.* **C23** 861
Raj A K 1975 as quoted by Mukhopadhyay N 1977 *Phys. Rep.* **C30** 1
Rho M 1967 *Phys. Rev.* **161** 955
Waghmare Y R 1977 *Pramana* **9** 7