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Deuteron form factors and the tensor force

S S MEHDI* and V K GUPTA

Department of Physics and Astrophysics, University of Delhi, Delhi 110007, India

* Department of Physics, Jamia Millia Islamia, New Delhi 110025, India

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Abstract. Results of a non-relativistic calculation of deuteron form factors are presented for separable potentials with and without tensor force. The tensor term in triplet state is added in such a way as to keep the values of deuteron binding energy, a_t and r_{0t} unaltered, so that the difference in the form factors can be regarded as the effect of tensor force only. The calculation has been performed for two different shapes of separable potentials and for three different D-state probabilities to study their comparative effect.

Keywords. Electromagnetic form factors; separable potentials; tensor force; deuteron.

1. Introduction

The electromagnetic form factors of deuteron provide a convenient probe of the nucleon-nucleon (NN) interaction. The NN interaction is fairly complicated and is well-known to have an attractive central, repulsive core, tensor and $\mathbf{L}\cdot\mathbf{S}$ components. Many phenomenological forms for NN interaction have been proposed and studied in the literature over the past two decades. The parameters of the various components are first obtained from a comparison of two-body data—scattering in singlet and triplet states at various energies, properties of deuteron, including binding energy, quadrupole moment, magnetic moment, etc. These forces are then used in studying various more complicated systems—three-body bound state and scattering systems, electromagnetic form factors of $3N$ systems, few-body reactions and so on. Since most of these calculations are forbiddingly difficult, requiring solution of many-dimensional integral equations, eigen value problems and quadratures, which in turn require large computing times and programming and calculational skills, one often includes only those terms which are likely to make a reasonable contribution to the quantity under calculation.

In the triplet state the most important term after the attractive central term, is the tensor term. (The effect of the repulsive core (in the triplet state) and the $\mathbf{L}\cdot\mathbf{S}$ term can usually be ignored for many calculations; there are situations however where these terms could be crucial). One measure of the tensor term is the deuteron D-state probability P_D. It is usually taken to be anywhere between 3 % and 7 %, though it is not amenable to direct experimental measurement (Amado 1981). Another, and perhaps more reliable, measure of the tensor force is the asymptotic D/S ratio, η. This quantity has been estimated both theoretically and experimentally. Till recently its average value was quoted as $\eta = (0{\cdot}0264 \pm 0{\cdot}0004)$ which agreed rather well with the theoretical estimate of Ericson and Rosa-Clot (1982). However two more recent measurements have both given a somewhat higher value for η. Borbély *et al* (1982) quote a value $\eta = 0{\cdot}0272 \pm 0{\cdot}0004$ and Goddard *et al* (1982) $\eta = 0{\cdot}0271 \pm 0{\cdot}0008$, which are consistent with the theoretical estimate $\eta = 0{\cdot}0271 \pm 0{\cdot}0007$ by Klarsfeld *et al* (1981). This has pushed up the average η value to $0{\cdot}0271 \pm 0{\cdot}0004$ (Ericson and Rosa-Clot 1983). Therefore, for the present even this quantity remains somewhat uncertain.

The deuteron and triton form factors, most properties of ^{3}H and ^{3}He and of scattering state have been calculated with extremely elaborate potentials containing a large number of terms. Many four-body, n-body ($n > 4$) ground-state calculations and many few-body calculations of vertex functions are too complicated to be handled with the complete potential; in the first instance such calculations are carried out with the simplest of potentials including only the central term hoping that the effect of the tensor term will be small. Nevertheless, it is important to have an estimate of the contribution of tensor term on various quantities of interest. In the three-body bound state problem its contribution is well documented; the addition of tensor force alters the binding energy by $\sim 10\%$ at best (Kharchenko *et al* 1968; Schrenk and Mitra 1967; Schrenk *et al* 1970). The n-d doublet scattering length is a more sensitive parameter, whose experimental value (van Oers and Seagrave 1967; Dilg *et al* 1971) is still not agreed upon with absolute certainty and therefore is not particularly suited for estimating the contribution of tensor term. The 2-body and 3-body radii and form factors are usually believed to be quite sensitive to the tensor term. One expects the radii to be reduced by a few percent and the form factors at high momentum transfer to be reduced quite significantly. Though there are quite a few calculations of the form factors with tensor term present, a clear-cut comparison of the form factors with and without the tensor term is lacking. Further, the contribution of tensor force to a particular quantity could be different for different central potentials. Kharchenko *et al* (KPS) (1968) developed a series of separable potentials with shapes of the type $(p^2 + \beta^2)^{-n}$, $n = 1, 2, \ldots$, which could be fitted to give the same two-body low energy data but differed in their off-shell behaviour. In a series of calculations (Mehdi and Gupta 1974, 1976, 1979, 1980) using these potentials for $n = 1, 2$ we found that the $n = 2$ potential gave better fit to the triton form factors and ^{3}H–^{3}He binding energy difference. The effect of addition of a tensor term to each of these potentials on the triton binding energy and the n–d doublet scattering length reduces as n is increased progressively (Kharchenko *et al* 1968). It is therefore likely that the effect will reduce progressively with increasing n even for other quantities of interest, such as the 2-body and 3-body form factors, ^{3}H–^{3}He binding energy difference etc. For the deuteron form factors this can be seen most clearly because analytic expressions for these form factors can be written down.

In this paper we report results of such a calculation of the electromagnetic form factors of deuteron for potentials with and without a tensor term. The tensor term is added in such a way as to keep the values of binding energy (BE), a_t and r_{0t} unaltered, so that the difference in the form factors can be regarded as the "true" effect of tensor force. Though deuteron form factor calculations with tensor force have been reported earlier (Gourdin 1964, 1965; Mehrotra and Gupta 1970), no such comparative study exists in the literature to the best of our knowledge. We have performed the calculation with potentials $(p^2 + \beta^2)^{-n}$ corresponding to $n = 1$ and $n = 2$ to study their comparative effect.

In § 2, we present the well-known formalism of deuteron form factors. We also give the exact analytic expressions for the form factors for $n = 1$ and 2 potential shapes. In

§ 3, we present the results of our numerical calculation of the form factors and end with a brief discussion.

2. Formalism

We consider a separable potential of the form

$$V(\mathbf{p}, \mathbf{p}') = -\frac{\lambda}{M} g(\mathbf{p}) g(\mathbf{p}'), \tag{1}$$

in the triplet state. $g(\mathbf{p})$ may be parametrized in the manner of Yamaguchi and Yamaguchi (1954)

$$g(\mathbf{p}) = C(p) + \frac{1}{\sqrt{8}} S(\mathbf{p}) T(p), \tag{2}$$

where

$$S(\mathbf{p}) = \frac{3}{p^2} (\sigma_1 \cdot \mathbf{p})(\sigma_2 \cdot \mathbf{p}) - \sigma_1 \cdot \sigma_2, \tag{3}$$

is the usual tensor operator. $C(p)$ represents the pure central part and $T(p)$ the tensor part of the potential. Following KPS, we use the following parametrization for the central part $C(p)$:

$$C(p) = (\beta^2 + p^2)^{-n}, \quad n = 1, 2, \tag{4}$$

and the usual Yamaguchi form for $T(p)$

$$T(p) = -tp^2(\gamma^2 + p^2)^{-2}. \tag{5}$$

Potential shapes corresponding to $n = 1, 2$ are called shape-1 and shape-2 respectively.

Using these potential shapes one can derive the expressions for BE, scattering length, effective range, quadrupole moment of the deuteron and the D-state probability. These expressions are given in the appendix both for shape-1 and shape-2 for the sake of completeness. Using the experimental values of these quantities one can then work out the parameters λ, β, γ and t occurring in the potential. Tables 1 and 2 list various sets of these parameters for shapes 1 and 2 respectively. They are all adjusted to give deuteron

Table 1. The triplet potential parameters β, γ, t and λ for potential sets of shape-1 and the two-body parameters a_t, r_{0t}, P_D and Q to which the former are fitted.

Set potential	β/α	γ/α	t	λ/α^3	a_t (fm)	r_{0t} (fm)	Q (fm^2)	P_D (%)
I $C_Y^{\text{eff}\,a}$	6·255	—	—	33·36	5·378	1·716	—	—
II $(C+T)_{MG_1}$	5·97	6·0	0·8605	26·24	5·384	1·716	0·1978	2
III $(C+T)_{MG_2}$	5·7	6·05	1·3615	20·58	5·390	1·716	0·2744	4
IV $(C+T)_{MG_3}$	5·4	7·5	3·019	14·49	5·392	1·716	0·2745	6

α is the deuteron BE parameter. C^{eff} represents the effective $^3S_1$ force. C and T represent, respectively, the central and tensor parts of $^3S_1$ force. Suffix Y denotes the Yamaguchi parametrization and suffix MG, the Mehdi and Gupta parametrization.

[a] Yamaguchi (1954).

Table 2. The triplet potential parameters for potential sets of shape-2 and the corresponding two-body parameters to which the former are fitted.

Set potential	β/α	γ/α	$t\alpha^2$	$\lambda(\times 10^{-6})/\alpha^7$	a_t (fm)	r_{0t} (fm)	Q (fm^2)	P_D (%)
I C_{MG}[a]	9·0	—	—	1·083	5·405	1·76	—	—
II $(C+T)_{MG_1}$	8·6	6·0	0·0056	0·7031	5·412	1·76	0·2252	2
III $(C+T)_{MG_2}$	8·25	6·7	0·0121	0·4471	5·416	1·76	0·2804	4
IV $(C+T)_{MG_3}$	7·94	8·0	0·0265	0·2638	5·418	1·76	0·2813	6

[a] Mehdi and Gupta (1980).

binding energy of 2·225 MeV. The Q, P_D, a_t and r_{0t} values to which these sets are fitted are also listed in the tables.

With these potentials of the Yamaguchi type, the deuteron structure is described by a nonrelativistic wavefunction in momentum space of the form

$$\psi(\mathbf{p}, \text{spin}) = \left[u(p) + \frac{1}{\sqrt{8}} S(\mathbf{p})w(p)\right]\chi_1^m, \tag{6}$$

where χ_1^m, is the triplet spin function, and $u(p)$ and $w(p)$ are the radial wavefunctions associated with the S- and D-states

$$u(p) = NC(p)(\alpha^2+p^2)^{-1}; \quad w(p) = NT(p)(\alpha^2+p^2)^{-1}. \tag{7}$$

The expression for N, the normalization constant, is given in the appendix.

For the deuteron, using impulse approximation and nonrelativistic wavefunctions, one usually introduces three form factors, (i) a charge form factor $F_{\text{ch}}(q^2)$, (ii) a quadrupole form factor $F_Q(q^2)$ obtained as a result of non-central distribution of charge, and (iii) a magnetic form factor $F_{\text{mag}}(q^2)$ as a result of distribution of currents. In the impulse approximation, these are expressible in terms of iso-scalar nucleon form factors, G_{ch}^S and G_{mag}^S and certain integrals involving the deuteron wavefunction (Gourdin 1964, 1965):

$$F_{\text{ch}}(q^2) = 2G_{\text{ch}}^S(q^2)C_E(q^2), \tag{8}$$

$$F_Q(q^2) = 2G_{\text{ch}}^S(q^2)C_Q(q^2), \tag{9}$$

$$F_{\text{mag}}(q^2) = 2(M_d/M)G_{\text{mag}}^S(q^2)C_S(q^2), \tag{10}$$

where M_d and M are the masses of the deuteron and the nucleon respectively, and

$$C_E(q^2) = \int_0^\infty [u^2(r)+w^2(r)]\, j_0(\tfrac{1}{2}qr)\mathrm{d}r, \tag{11}$$

$$C_Q(q^2) = \frac{3}{\eta\sqrt{2}}\int_0^\infty \left[u(r)w(r) - \frac{w^2(r)}{2\sqrt{2}}\right] j_2(\tfrac{1}{2}qr)\mathrm{d}r, \tag{12}$$

$$C_S(q^2) = \int_0^\infty [u^2(r) - \tfrac{1}{2}w^2(r)]\, j_0(\tfrac{1}{2}qr)\mathrm{d}r + \frac{1}{\sqrt{2}}\int_0^\infty \left[u(r)w(r) + \frac{w^2(r)}{\sqrt{2}}\right] j_2(\tfrac{1}{2}qr)\mathrm{d}r, \tag{13}$$

with

$$\eta = q^2/4M_d^2.$$

Here $u(r)$ and $w(r)$ are the radial S- and D-state wavefunctions of the deuteron in configuration space, normalized so that

$$\int_0^\infty [u^2(r) + w^2(r)]\,\mathrm{d}r = 1. \tag{14}$$

The expressions for $u(r)$ and $w(r)$ both for shape-1 and shape-2 are once again relegated to the appendix.

The functions C_E, C_Q and C_S are associated with the charge, quadrupole and magnetic form factors of the deuteron, and are normalized at $q^2 = 0$, to the charge, quadrupole moment and magnetic moment of the deuteron respectively.

The integrals involved in (11) to (13) can be handled analytically. For some of the terms one can use the well-known integral (Watson 1944)

$$\int_0^\infty \exp(-ax) J_\nu(bx) x^{\mu-1}\,\mathrm{d}x = \frac{(b/2a)^\nu \Gamma(\nu+\mu)}{a^\mu \Gamma(\nu+1)} \times F\left(\frac{\nu+\mu}{2}, \frac{\nu+\mu+1}{2}; \nu+1; -b^2/a^2\right) \tag{15}$$

$$[\mathrm{Re}(\nu+\mu) > 0,\ \mathrm{Re}(a+ib) > 0,\ \mathrm{Re}(a-ib) > 0],$$

where $F(a, b; c; z)$ is the hypergeometric function. This integral for $\nu = 1/2$, $\mu = 1/2, 3/2$ and $5/2$ respectively reduces to

$$\int_0^\infty \exp(-ax) J_{1/2}(bx) x^{-1/2}\,\mathrm{d}x = \frac{2}{(2\pi b)^{1/2}} \tan^{-1}(b/a), \tag{16}$$

$$\int_0^\infty \exp(-ax) J_{1/2}(bx) x^{1/2}\,\mathrm{d}x = \frac{1}{(2\pi b)^{1/2}} \cdot \frac{2b}{(a^2+b^2)}, \tag{17}$$

and

$$\int_0^\infty \exp(-ax) J_{1/2}(bx) x^{3/2}\,\mathrm{d}x = \frac{1}{(2\pi b)^{1/2}} \cdot \frac{4ab}{(a^2+b^2)^2}, \tag{18}$$

For $\nu = 5/2$, $\mu = 5/2$ also the integral can be reduced to

$$\int_0^\infty \exp(-ax) J_{5/2}(bx) x^{3/2}\,\mathrm{d}x = \frac{2}{b^2(2\pi b)^{1/2}} \left[3\tan^{-1}(b/a) - \frac{3ab}{a^2+b^2} - \frac{2ab^3}{(a^2+b^2)^2}\right]. \tag{19}$$

a result we could not locate in any standard text. Integrals corresponding to $\nu = 5/2$, $\mu = 3/2$ and $1/2$ can also be derived analytically from the above integral by standard integration techniques. Some other terms corresponding to $\nu = 1/2$ or $5/2$, $\mu = -1/2$, $-3/2$, $-5/2$ and $-7/2$ give rise to divergent integrals but all such divergences of course cancel. After a laborious calculation of these integrals, cancelling all the divergences, the final analytic expressions for the structure functions C_E, C_Q and C_S were obtained which are given, for the sake of completeness, in the appendix.

3. Results and discussion

The results are usually analysed in terms of the combinations

$$F_{el}^2(q^2) = F_{ch}^2(q^2) + \tfrac{8}{9}\eta^2 F_Q^2(q^2), \tag{20}$$

and

$$A(q^2) = F_{el}^2(q^2) + \tfrac{2}{3}\eta F_{mag}^2(q^2). \tag{21}$$

F_{el} and F_{mag}/μ_d are normalized to unity at $q^2 = 0$. Our results for the quantities $F_{el}(q^2)$, F_{mag}/μ_d and $A(q^2)$ are shown in figures 1–3, respectively, for various potential sets. A notable feature is that though tensor force reduces the form factors, the effect is rather negligible up to $q^2 \sim 10$–$15\ \mathrm{fm}^{-2}$. However, the surprising fact is that beyond this value of q^2 inclusion of tensor term increases the form factors. In fact for the magnetic form factor, $F_{mag}(q^2)$, this is true right from $q^2 = 0$ onwards. The asymptotic expressions for the form factors (for $q^2 \to \infty$) can easily be obtained from the formulae given in §2 and one can check that numerically the asymptotic values of the form factors are greater for potentials with a tensor term. By this we are neither suggesting that these calculations are suitable up to asymptotically large q^2 values in this rather simple model without meson exchange and other relativistic effects, nor that a comparison with data is meaningful. In fact the relevant region of momentum transfer up to which such nonrelativistic calculations are expected to be meaningful is only up to say $q^2 \sim 15$–$20\ \mathrm{fm}^{-2}$. The point being emphasized is that, quite contrary to the usual belief,

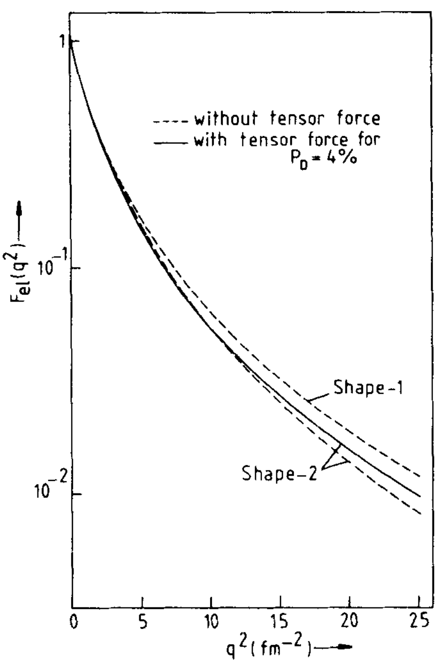

Figure 1. Electric form factor of deuteron for potential sets without and with tensor force ($P_D = 4\%$). For shape-1, the effect of tensor force ($P_D = 4\%$) on $F_{el}(q^2)$ is negligible and hence not shown in figure.

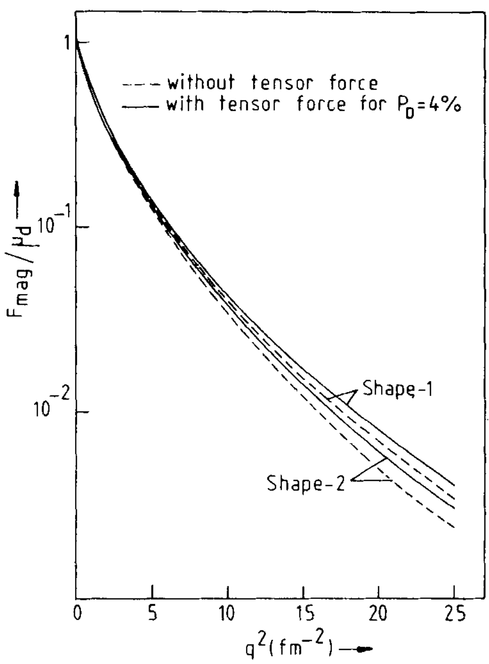

Figure 2. Magnetic form factor of deuteron for potential sets without and with tensor force ($P_D = 4\%$).

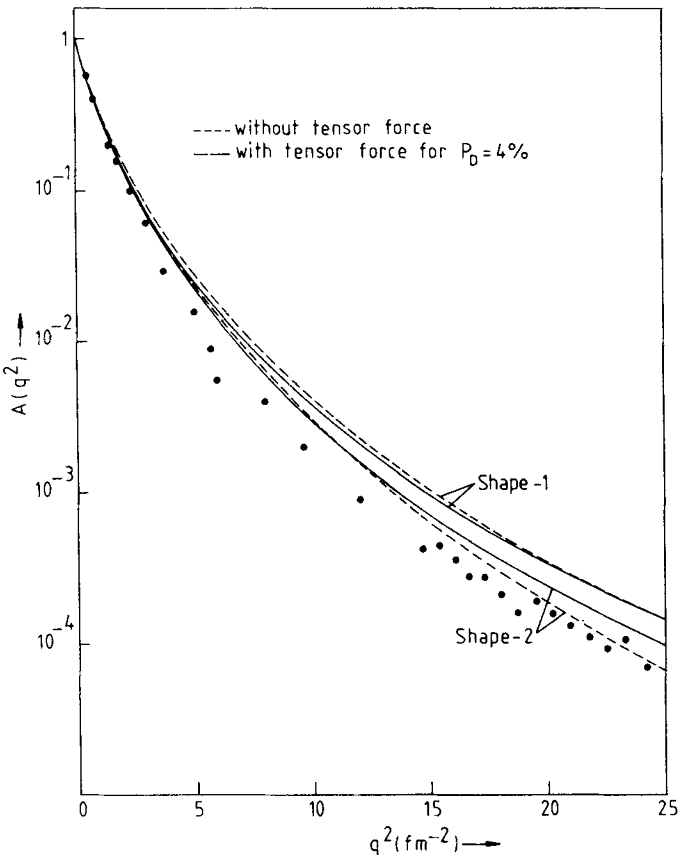

Figure 3. Deuteron form factor $A(q^2)$ for potential sets without and with tensor force (P_D = 4%). Experimental points are from Elias *et al* (1969).

the inclusion of a tensor term increases the form factors beyond certain q^2 values. Coupled with the fact that for small q^2 the form factors are reduced by the inclusion of tensor force (except for F_{mag}), there has to be a point at which the two curves—with and without tensor force—cross each other. This happens for $q^2 \sim$ 10–15 fm^{-2}. As a result throughout the relevant region of q^2 (up to 20 fm^{-2}) the effect of tensor force on the form factors remains small. Thus tensor force plays an even smaller role (due to first a decrease and then an increase) in the form factor calculations than could have otherwise been expected. To see that this (negligible role due to fluctuating effect) is not a mere coincidence for the particular set of parameters chosen, we have calculated the form factors for many other sets of parameters as well, which correspond to different P_D values ranging from 2 to 6% but all tuned to the same values of BE, scattering length and effective range, both for shape-1 and shape-2. Some of these results are shown in figures 4 and 5 for $A(q^2)$. The results clearly show that though there are qualitative differences between the various sets, the general conclusion that the inclusion of tensor force first reduces and then increases the form factors remains valid.

As for the comparison between the two shapes is concerned, the results with shape-2 are in better agreement with experimental data (Elias *et al* 1969) as is indicated in figure 3. (A similar conclusion was reached for the ^{3}He and ^{3}H form factors and the Coulomb

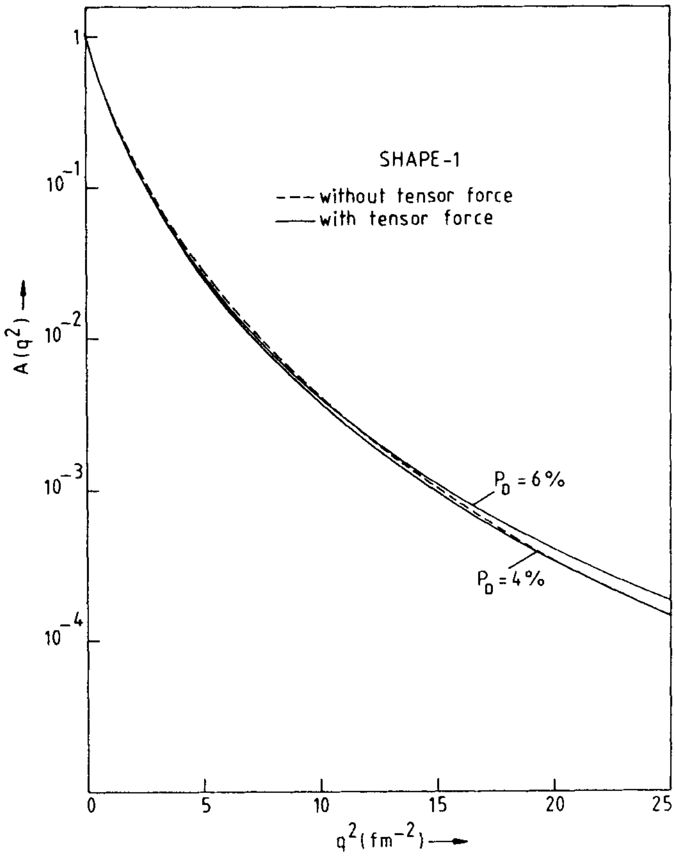

Figure 4. Comparison of deuteron form factor $A(q^2)$ for the potential sets of shape-1 characterized by different D-state probabilities.

energy of ^3He). In fact the results for shape-2 with purely central potential are embarassingly close to the experimental data. This, however, should not be taken too literally, since the inclusion of tensor term pushes up the form factors which will presumably be pushed down once again on the explicit inclusion of meson-exchange and other relativistic effects.

4. Conclusions

We thus conclude that the effect of tensor force on the deuteron form factor is very little up to $q^2 \sim 10$–$15\,\text{fm}^{-2}$. Beyond this q^2 value, the effect is in fact in the opposite direction to what was expected. Though the conclusion seems to be valid for all reasonable P_D values and seems to be model independent, atleast within the separable potential model, as results for two different potential shapes and many different P_D values indicate, it will be interesting to investigate whether the same is true of local potentials as well.

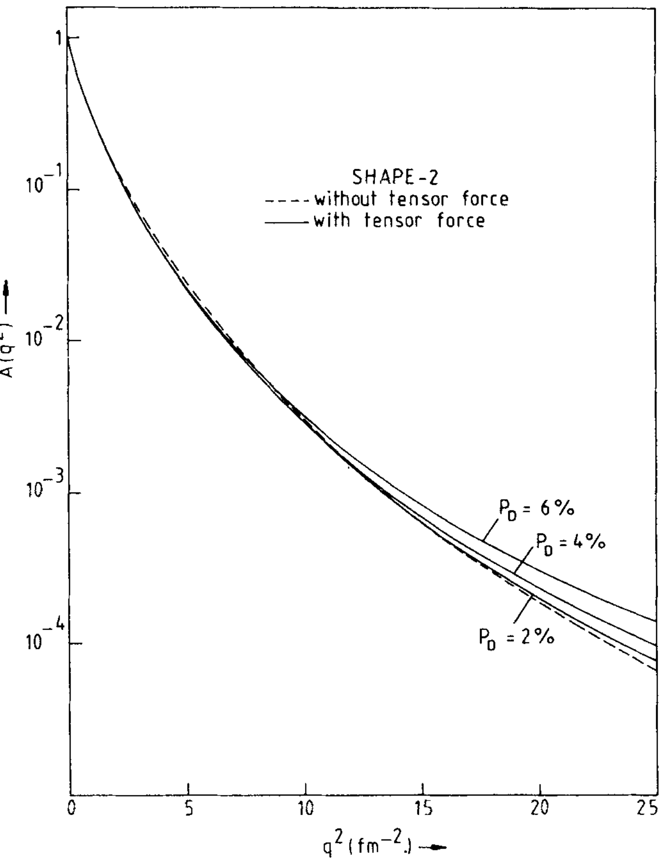

Figure 5. Comparison of deuteron form factor $A(q^2)$ for the potential sets of shape-2 characterized by different D-state probabilities.

Appendix

The normalization constant N fixed by the condition

$$\int [u^2(p) + w^2(p)]\, \mathrm{d}\mathbf{p} = 1, \tag{A1}$$

is given by

$$\frac{1}{N^2} = \int \frac{C^2(p) + T^2(p)}{(\alpha^2 + p^2)^2}\, \mathrm{d}\mathbf{p} \equiv I_1 + I_2. \tag{A2}$$

The relation between the strength λ of the triplet potential and the observed binding energy α^2/M of the deuteron is given by

$$\frac{1}{\lambda} = \int \frac{C^2(p) + T^2(p)}{\alpha^2 + p^2}\, \mathrm{d}\mathbf{p} \equiv I_3 + I_4. \tag{A3}$$

The integrals involved in the above expressions are given by

$$I_1 = \int_0^\infty \frac{C^2(p)\,d\mathbf{p}}{(\alpha^2+p^2)^2} = \frac{\pi^2}{\alpha\beta(\alpha+\beta)^3}, \text{ (for shape-1)}$$

$$I_1 = \int_0^\infty \frac{C^2(p)\,d\mathbf{p}}{(\alpha^2+p^2)^2} = \frac{\pi^2(\alpha^2+5\alpha\beta+8\beta^2)}{8\alpha\beta^5(\alpha+\beta)^5}, \text{ (for shape-2)}$$

$$I_2 = \int_0^\infty \frac{T^2(p)\,d\mathbf{p}}{(\alpha^2+p^2)^2} = \frac{\pi^2 t^2(5\alpha+\gamma)}{8\gamma(\alpha+\gamma)^5},$$

$$I_3 = \int_0^\infty \frac{C^2(p)\,d\mathbf{p}}{\alpha^2+p^2} = \frac{\pi^2}{\beta(\alpha+\beta)^2}, \text{ (for shape-1)}$$

$$I_3 = \int_0^\infty \frac{C^2(p)\,d\mathbf{p}}{\alpha^2+p^2} = \frac{\pi^2(\alpha^2+4\alpha\beta+5\beta^2)}{8\beta^5(\alpha+\beta)^4}, \text{ (for shape-2)}$$

$$I_4 = \int_0^\infty \frac{T^2(p)\,d\mathbf{p}}{\alpha^2+p^2} = \frac{\pi^2 t^2(5\alpha^2+4\alpha\gamma+\gamma^2)}{8\gamma(\alpha+\gamma)^4}.$$

The D-state probability P_D of the deuteron is given by

$$P_D = \int w^2(p)\,d\mathbf{p} = 4\pi N^2 \int_0^\infty \frac{T^2(p)\,p^2\,dp}{(\alpha^2+p^2)^2} = \frac{N^2\pi^2 t^2(5\alpha+\gamma)}{8\gamma(1+\gamma)^5}, \tag{A4}$$

and the quadrupole moment of the deuteron Q is given by

$$Q = -\frac{1}{4}\int d\mathbf{p}\left[\left\{u(p)+\frac{1}{\sqrt{8}}S(\mathbf{p})\,w(p)\right\}\chi_1'\right] \times\left\{3\frac{\partial^2}{\partial p_z^2}-\frac{\partial^2}{\partial \mathbf{p}^2}\right\}\left\{u(p)+\frac{1}{\sqrt{8}}S(\mathbf{p})\,w(p)\right\}\chi_1'$$
$$= \frac{\sqrt{8}\pi}{5}\int_0^\infty dp\left\{p\frac{\partial u}{\partial p}w - p^2\frac{\partial^2 u}{\partial p^2}w\right\} - \frac{\pi}{5}\int_0^\infty dp\left\{6w^2+p^2\left(\frac{\partial w}{\partial p}\right)^2\right\}. \tag{A5}$$

This expression for shape-1 *i.e.* for $C(p) = (\beta^2+p^2)^{-1}$, reduces to

$$Q = \frac{8\sqrt{8}\pi t N^2}{5}\left(I_{412}+I_{322}+I_{232}\right) - \frac{\pi^2 N^2 t^2}{160}\frac{(7\alpha^3+49\alpha^2\gamma+91\alpha\gamma^2+33\gamma^3)}{\gamma^3(\alpha+\gamma)^7}, \tag{A6}$$

and for shape-2, *i.e.* for $C(p) = (\beta^2+p^2)^{-2}$, reduces to

$$Q = \frac{8\sqrt{8}\pi t N^2}{5}\left(I_{422}+3I_{242}+2I_{332}\right)$$

$$-\frac{\pi^2 N^2 t^2}{160}\frac{(7\alpha^3+49\alpha^2\gamma+91\alpha\gamma^2+33\gamma^3)}{\gamma^3(\alpha+\gamma)^7}, \tag{A7}$$

where I_{mnr} stands for integral

$$I_{mnr}=\int_0^\infty \frac{p^6\,\mathrm{d}p}{(\alpha^2+p^2)^m(\beta^2+p^2)^n(\gamma^2+p^2)^r}, \tag{A8}$$

and accordingly

$$I_{412}=\frac{\pi}{32(\alpha+\beta)^4(\beta+\gamma)^2(\alpha+\gamma)^5}$$
$$\times\Big[\alpha\beta^2(5\alpha^2+4\alpha\beta+\beta^2)+\gamma^3(\alpha^2+4\alpha\beta+5\beta^2)$$
$$+\beta\gamma(10\alpha^3+33\alpha^2\beta+22\alpha\beta^2+5\beta^3)$$
$$+\gamma^2(5\alpha^3+22\alpha^2\beta+33\alpha\beta^2+10\beta^3)\Big],$$

$$I_{322}=\frac{\pi}{16(\alpha+\beta)^4(\beta+\gamma)^3(\alpha+\gamma)^4}$$
$$\times\Big[\alpha(\beta+\gamma)^3+4(\alpha^2+\beta\gamma)(\beta^2+3\beta\gamma+\gamma^2)+16\alpha\beta\gamma(\beta+\gamma)\Big],$$

$$I_{232}=\frac{\pi}{16(\alpha+\beta)^4(\beta+\gamma)^4(\alpha+\gamma)^3}$$
$$\times\Big[\beta(\alpha+\gamma)^3+4(\beta^2+\alpha\gamma)(\alpha^2+3\alpha\gamma+\gamma^2)+16\alpha\beta\gamma(\alpha+\gamma)\Big],$$

$$I_{422}=\frac{\pi}{32\alpha(\beta+\gamma)^3(\alpha+\beta)^5(\alpha+\gamma)^5}$$
$$\times\Big[5\alpha^2(\beta+\gamma)(\beta^2+\gamma^2+10\beta\gamma)+5\beta\gamma(\beta+\gamma)^3$$
$$+8\alpha(\alpha^2+\beta\gamma)(\beta^2+\gamma^2+3\beta\gamma)+\alpha(\beta+\gamma)^2(\beta^2+\gamma^2+18\beta\gamma)\Big],$$

$$I_{242}=\frac{\pi}{32\beta(\alpha+\gamma)^3(\alpha+\beta)^5(\beta+\gamma)^5}$$
$$\times\Big[5\beta^2(\alpha+\gamma)(\alpha^2+\gamma^2+10\alpha\gamma)+5\alpha\gamma(\alpha+\gamma)^3$$
$$+8\beta(\beta^2+\alpha\gamma)(\alpha^2+\gamma^2+3\alpha\gamma)+\beta(\alpha+\gamma)^2(\alpha^2+\gamma^2+18\alpha\gamma)\Big],$$

$$I_{332}=\frac{\pi}{16(\alpha+\gamma)^4(\beta+\gamma)^4(\alpha+\beta)^5}$$

$$\times\Big[\alpha^3(\beta+4\gamma)+\alpha^2(5\beta^2+16\gamma^2+24\beta\gamma)$$
$$+\alpha(\beta^3+\gamma^3+15\beta\gamma^2+23\beta^2\gamma)$$
$$+\alpha\gamma(\beta^2+11\gamma^2+23\beta\gamma)$$
$$+\gamma(4\beta^3+3\gamma^3+12\beta\gamma^2+16\beta^2\gamma)\Big].$$

The expressions for the scattering length a_t and effective range r_{0t} for the two shapes of $C(p)$ *i.e.* $C(p)=(\beta^2+p^2)^{-n}$, $n=1, 2$, can be written as follows:

$$\frac{1}{a_t}=-\frac{\beta^4}{2\pi^2\lambda}+\frac{\beta}{2}+\frac{t^2\beta^4}{16\gamma^3},\quad \text{(for shape-1)} \tag{A9}$$

$$\frac{1}{a_t}=-\frac{\beta^8}{2\pi^2\lambda}+\frac{5\beta}{16}+\frac{t^2\beta^8}{16\gamma^3},\quad \text{(for shape-2)} \tag{A10}$$

$$r_{0t}=\frac{2\beta^2}{\pi^2\lambda}+\frac{1}{\beta}-\frac{t^2\beta^2}{8}\left(\frac{\beta^2}{\gamma^5}+\frac{2}{\gamma^3}\right),\quad \text{(for shape-1)} \tag{A11}$$

$$r_{0t}=\frac{4\beta^6}{\pi^2\lambda}+\frac{15}{8\beta}-\frac{t^2\beta^6}{8}\left(\frac{\beta^2}{\gamma^5}+\frac{4}{\gamma^3}\right).\quad \text{(for shape-2)} \tag{A12}$$

The expressions for $u(r)$ and $w(r)$, the radial S- and D-state wavefunctions of the deuteron in configuration space are:

$$u(r)=A(\exp(-\alpha r)-\exp(-\beta r))+Br\exp(-\beta r), \tag{A13}$$

where $$A=\frac{\sqrt{2}\,\pi N}{\beta^2-\alpha^2};\ B=0 \qquad \text{(for shape-1)},$$

and $$A=\frac{\sqrt{2}\pi N}{(\beta^2-\alpha^2)^2};\ \mathrm{B}=-\frac{\pi N}{\sqrt{2}\beta(\beta^2-\alpha^2)}\ \text{(for shape-2)},$$

and

$$w(r)=C\Big[\frac{\alpha^2}{3}(\exp(-\alpha r)-\exp(\gamma r))-\frac{\gamma(\gamma^2-\alpha^2)}{6}\cdot re^{-\gamma r}$$
$$+\left(\frac{1}{r^2}+\frac{\alpha}{r}\right)\exp(-\alpha r)-\left(\frac{1}{r^2}+\frac{\gamma}{r}+\frac{\gamma^2-\alpha^2}{2}\right)\exp(-\gamma r)\Big], \tag{A14}$$

where $$C=\frac{3\sqrt{2}\pi Nt}{(\gamma^2-\alpha^2)^2}.$$

Finally, the expressions for the structure functions C_E, C_Q and C_S can be written as:

$$C_E(q^2)=\int_0^\infty\left(u^2+w^2\right)j_0\left(\frac{1}{2}qr\right)\mathrm{d}r\equiv UU+WW, \tag{A15}$$

$$C_Q(q^2) = \frac{3}{\eta\sqrt{2}} \int_0^\infty \left(uw - \frac{w^2}{2\sqrt{2}} \right) j_2\left(\frac{1}{2}qr\right) \mathrm{d}r \equiv \frac{3}{\eta\sqrt{2}} \left(UW_2 - \frac{WW_2}{2\sqrt{2}} \right), \tag{A16}$$

$$C_S(q^2) = \int_0^\infty \left(u^2 - \frac{w^2}{2} \right) j_0\left(\frac{1}{2}qr\right) \mathrm{d}r + \frac{1}{\sqrt{2}} \int_0^\infty \left(uw + \frac{w^2}{\sqrt{2}} \right) j_2\left(\frac{1}{2}qr\right) \mathrm{d}r$$

$$\equiv \left(UU - \frac{WW}{2} \right) + \frac{1}{\sqrt{2}} \left(UW_2 + \frac{WW_2}{\sqrt{2}} \right), \tag{A17}$$

where writing $b = \frac{1}{2}q$

$$UU = \int_0^\infty u^2(r) j_0(br)\, \mathrm{d}r$$

$$= \frac{A^2}{b} \tan^{-1} \left\{ \frac{2b(\beta+\alpha)(\beta-\alpha)^2}{4\alpha\beta(\beta+\alpha)^2 + b^2(3\beta^2 + 2\beta\alpha + 3\alpha^2) + b^4} \right\}$$

$$+ B^2 \cdot \frac{4\beta}{(4\beta^2 + b^2)^2} + 2AB \cdot \frac{(\beta-\alpha)(3\beta+\alpha)}{(4\beta^2+b^2)\{(\beta+\alpha)^2 + b^2\}},$$

$$WW = \int_0^\infty w^2(r) j_0(br)\, \mathrm{d}r$$

$$= \frac{C^2}{72} \left[\frac{8\alpha^4}{b} \tan^{-1} \left\{ \frac{2b(\gamma+\alpha)(\gamma-\alpha)^2}{4\gamma\alpha(\gamma+\alpha)^2 + (3\gamma^2 + 2\gamma\alpha + 3\alpha^2) b^2 + b^4} \right\} \right.$$

$$+ 3b(4\alpha^2 + b^2) \tan^{-1}\left(\frac{b}{2\alpha}\right) + \frac{6}{b}$$

$$\times \left\{ (\gamma^2 - \alpha^2)^2 - 4\alpha^2 b^2 - b^4 \right\} \tan^{-1}\left(\frac{b}{\alpha+\gamma}\right)$$

$$+ \frac{8\gamma^3(\gamma^2-\alpha^2)^2}{(4\gamma^2+b^2)^2} - \frac{8\alpha^2\gamma(\gamma-\alpha)^2(\gamma+\alpha)(3\gamma+\alpha)}{(4\gamma^2+b^2)\{(\gamma+\alpha)^2+b^2\}}$$

$$\left. - \frac{6(\gamma+\alpha)(\gamma-\alpha)^2(2\gamma^2 - 2\alpha\gamma + b^2)}{(4\gamma^2+b^2)} \right],$$

$$UW_2 = \int_0^\infty u(r) w(r) j_2(br)\, \mathrm{d}r$$

$$= \frac{A \cdot C}{24b^3} \left[b^2(3b^2 + 4\alpha^2) \tan^{-1}\left(\frac{b}{2\alpha}\right) \right.$$

$$- \left\{ 3(\gamma^2 - \beta^2)(\gamma^2 + \beta^2 - 2\alpha^2) \right.$$

$$\left. - b^2(6\beta^2 - 2\alpha^2 + 3b^2) \right\} \tan^{-1}\left(\frac{b}{\beta+\gamma}\right)$$

$$+\left\{3\left(\gamma^2-\alpha^2\right)^2-b^2\left(4\alpha^2+3b^2\right)\right\}\tan^{-1}\left(\frac{b}{\alpha+\gamma}\right)$$

$$-\left\{3\,(\beta^2-\alpha^2)^2+b^2\,(6\beta^2-2\alpha^2+3b^2)\right\}\tan^{-1}\left(\frac{b}{\alpha+\beta}\right)$$

$$-3b\,(\alpha-\beta)\,(\beta-\gamma)\,(\gamma-\alpha)-4\gamma\,(\gamma^2-\alpha^2)\,b^3$$

$$\times\left\{\frac{1}{(\beta+\gamma)^2+b^2}-\frac{1}{(\alpha+\gamma)^2+b^2}\right\}\Bigg]$$

$$+\frac{B\cdot C}{6b^3}\Bigg[3\beta(\beta^2-\alpha^2+b^2)\tan^{-1}\left\{\frac{b(\alpha-\gamma)}{(\beta+\gamma)(\alpha+\beta)+b^2}\right\}$$

$$-3b(\gamma-\alpha)(\gamma+\alpha-\beta)+\frac{b^3(3\gamma^2-\alpha^2)+3b\gamma\,(\beta+\gamma)\,(\gamma^2-\alpha^2)}{(\beta+\gamma)^2+b^2}$$

$$+\frac{2b^3\gamma(\beta+\gamma)(\gamma^2-\alpha^2)}{\{(\beta+\gamma)^2+b^2\}^2}-\frac{2\alpha^2b^3}{(\alpha+\beta)^2+b^2}\Bigg],$$

$$WW_2=\int_0^\infty w^2\,(r)\,j_2\,(br)\,\mathrm{d}r$$

$$=\frac{C^2}{144b^3}\Bigg[b^2\,(8\alpha^4-6\alpha^2b^2-3b^4)\tan^{-1}\left(\frac{b}{2\alpha}\right)$$

$$-\Big\{12\gamma^6+6\,(b^2-4\alpha^2)\,\gamma^4+12\,(\alpha^2-b^2)\,\alpha^2\gamma^2+2\,(3b^2-\alpha^2)$$

$$\times\,\alpha^2b^2+3b^6\Big\}\tan^{-1}\left(\frac{b}{2\gamma}\right)$$

$$+2\Big\{6\gamma^2\,(\gamma^2-\alpha^2)^2+b^2\,(3\gamma^4-5\alpha^4-6\alpha^2\gamma^2)+6\alpha^2b^4$$

$$+3b^6\Big\}\tan^{-1}\left(\frac{b}{\alpha+\gamma}\right)$$

$$+6b\,(\alpha+\gamma)\,(\gamma-\alpha)^2\,(b^2+2\alpha\gamma)+\frac{16\alpha^2\gamma b^3\,(\gamma^2-\alpha^2)}{(\alpha+\gamma)^2+b^2}$$

$$-\frac{8b\gamma\,(\gamma^2-\alpha^2)}{4\gamma^2+b^2}\left\{b^2\,(3\gamma^2-\alpha^2)+3\gamma^2\,(\gamma^2-\alpha^2)+\frac{2\gamma^2b^2\,(\gamma^2-\alpha^2)}{4\gamma^2+b^2}\right\}\Bigg].$$

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