Pion absorption by $^{16}\text{O}$ using realistic interactions—II: Free pion absorption followed by emission of proton

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Abstract. Matrix element of the Galilean invariant nonrelativistic reduction of the pseudoscalar-pseudovector interaction has been calculated for free pion absorption by a single nucleon inside the nucleus of $^{16}\text{O}$. The Hartree-Fock wavefunctions obtained with the unitary-model-operator approach starting with the hard-core nucleon-nucleon interaction have been used for the $\pi$-capturing nucleon in the initial state. The initial pion distortion in the presence of nuclear field of the absorbing nucleus prior to its absorption together with the Coulomb interaction with the finite nuclear size has been taken into account. The distortion of the emitted proton in the field of the residual nucleus has also been considered. The differential cross-sections have been obtained and calculated results are compared with the previous experimental and theoretical work.

Keywords. Pion absorption; UMOA wavefunctions; bound neutron; pion-nucleon interaction; Klein-Gordon equation; initial state wavefunctions; final state wavefunctions.

1. Introduction

The possibility of using pionic interactions in understanding nuclear structure has become increasingly attractive in recent years. In such interaction mechanism the $\pi$-meson may be viewed merely as a probe of nuclear structure, and information may be obtained as to how pions interact with nucleons bound in a nucleus vis-à-vis hole states. Since pion absorption takes place only within a nucleus, any study of the process involves or presupposes an understanding of pion propagation within the range of strong forces. This may be studied by means of pi-nucleus reactions or pi-mesic atoms.

The ($\pi^+, p$) reaction has the advantage that only one particle, a proton is required to be detected in the final state. Experimental measurements are comparatively simple since the capture leading to low-lying states of the residual nucleus results in two-body final states with distinct signatures. This reaction provides access to single-neutron hole-states.

With the advent of meson factories, this reaction has been studied by different authors both experimentally and theoretically. However, there is discrepancy between theory and experiment. This could possibly be explained if one takes into account correlations between nucleons in the $\pi$-absorbing nucleus, besides distortion of the pion and proton waves. This is because, in this process there is large transfer of momentum (about 400 MeV/c in the forward direction) where the nuclear correlations become important.
In this work, we have used the Galilean-invariant form of the \( ps-pv \) interaction and performed calculations with the \( 1N \)-mechanism for \( \pi \)-absorption. We take into account various effects such as the initial pion distortion in the presence of nuclear fields of the absorbing nucleus prior to its absorption, the correlated wavefunctions for the nucleus and the distortion of the emitted proton in the field of residual nucleus.

2. Theory of pion absorption

In the first order time-dependent perturbation theory, the differential cross-section for the \((\pi^+, p)\) process is given by

\[
\frac{d\sigma}{d\Omega} = \frac{M}{(2\pi)^2} \frac{k}{q} E_x \sum_f |\langle f | H | i \rangle|^2,
\]

where \( M \) is the mass of the nucleon, \( k \) and \( q \) are momenta of the proton and pion respectively and \( E_x \) is the total pion energy. \( |i\rangle \) and \( |f\rangle \) represent the initial and final states of the system respectively whereas \( H \) is the Galilean invariant non-relativistic pseudoscalar-pseudovector interaction.

\[
H = \sum_{\mu} \sum_{\pi} \left[ \sigma \cdot \nabla_{\pi} \tau \cdot \phi - \frac{\mu}{M} \tau \cdot \phi \sigma \cdot \nabla_N \right],
\]

where \( f \) is the pion nucleon coupling constant, and \( \sigma \) and \( \tau \) are nucleon spin and isospin Pauli matrices. In this expression, \( \nabla_{\pi} \) acts only on \( \phi \), the pionic field and \( \nabla_N \) only on the nuclear wavefunction, the sum extending over all nucleons. The initial state consists of the positive pion and the absorbing nucleus while the final state is made up of vacuum state of pionic field, the emitted proton and the residual nucleus.

2.1 The nuclear wavefunction

In the pion absorption process there is high momentum transfer. Hence one has to consider the nuclear wavefunction which takes into account the short range correlations. For this we have employed wavefunction generated by Hartree-Fock type variation calculation (Kakkar 1969) in the framework of unitary model operator approach (Shakin et al 1967a, b). The wavefunction in this scheme is given by

\[
|^{16}\text{O}\rangle = \sum_m (jj; m - m o)\psi_{\text{neutron}}^{\text{HF}} (jm)\psi_{\text{c}}^{\text{HF}} (j - m),
\]

\[
= \sum_m (-)^{j-m} \sum_{n} C_{lm}^l |n l j m \rangle_{\text{HO}} \psi_{\text{c}}^{\text{HF}} (j - m),
\]

\[
= \sum_m (-)^{j-m} \sum_{n,\lambda} C_{lm}^l |l^\frac{1}{2} j; \lambda \Lambda m \rangle |n l \lambda \rangle \chi_{l^\frac{1}{2} \Lambda} \psi_{\text{c}}^{\text{HF}} (j - m),
\]

where \((l^\frac{1}{2} j; \lambda \Lambda m)\) are the Clebsch-Gordan coefficients. \( \psi_{\text{neutron}}^{\text{HF}} \) and \( \psi_{\text{c}}^{\text{HF}} \) are wavefunctions of the \( \pi \)-capturing neutron and the rest of the nucleus respectively, \( C_{lm}^l \) are the mixing coefficients occurring in the expansion of Hartree-Fock (HF) wavefunctions in the harmonic-oscillator (HO) basis. For spherical HF the summation is restricted only over radial quantum number \( n \) and \( \chi_{l^\frac{1}{2} \Lambda} \) is the spin-part of the neutron wavefunction.
2.2 Pion wavefunction

In the absence of any distortion, the pion wave is represented by a plane wave. But essentially the incident pion wave experiences the effect of coulomb and nuclear fields prior to its absorption by the nucleon. Considering the kinetic energy of the incident pion as 70 MeV, a relativistic treatment is required to take into account the pion distortion. Hence one has to solve the appropriate Klein-Gordon equation to generate the pion-waves. Since the pion-nucleus optical potential is not completely understood, it has been commonly assumed that it can be introduced (Baker et al 1958a, b) into the Klein-Gordon equation as a fourth component of a four-vector with the energy $E_\pi$ and the coulomb potential $V_c$. Hence

$$E_\pi \rightarrow E - V_c - V_N.$$  \hspace{1cm} (6)

Then

$$E^2 \approx (E - V_c)^2 - 2E_\pi V_N.$$  

The additional terms $2V_c V_N$ and $V_N^2$ are arbitrarily dropped from the expression for $E^2$. The Klein-Gordon equation now reads in natural units ($\hbar = c = 1$)

$$(-\nabla^2 + \mu^2) \psi = (E^2 - 2V_c E_\pi + V_c^2 - 2E_\pi V_N) \psi.$$  \hspace{1cm} (7)

Expanding $\psi$ in partial waves as

$$\psi = \sum_i \frac{u_i(r)}{r} l^i p_i (\cos \theta)$$  \hspace{1cm} (8)

and substituting it in (7), we get the following equation for each partial wave

$$\left[ -\frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial}{\partial r} \right) + \frac{l(l+1)}{r^2} \right] \frac{u_i}{r} p_i (\cos \theta) = \left[ q^2 + V_c^2 - 2E_\pi (V_c + V_N) \right] \frac{u_i}{r} p_i (\cos \theta).$$  \hspace{1cm} (9)

The pion-nucleus optical potential employed here is the standard Kisslinger potential

$$V_N = -Aq^2 b_0 \rho(r) + A b_1 \nabla \cdot \rho \nabla,$$  \hspace{1cm} (10)

where $A$ is the nuclear mass in A.M.U., $\rho$ is the nuclear matter density which is assumed to be spherically symmetric, i.e. $\rho = \rho(r)$. $b_0$ and $b_1$ are potential parameters which are related to the pion-nucleon phase shifts.

We can write (10) as

$$2E_\pi V_N \psi = \left[ A_1 \rho + A_2 \nabla \cdot \rho \nabla \right] \psi.$$  \hspace{1cm} (11)

With this substitution, (9) reduces to the following radial equation for each partial wave

$$(A_2 \rho - 1)u''_i + A_2 \rho' u'_i = \left\{ \frac{A_2 \rho'}{r} + q^2 - 2V_c E_\pi + V_c^2 $$
$$ - A_1 \rho + \frac{l(l+1)}{r^2} (A_2 \rho - 1) \right\} u_i.$$  \hspace{1cm} (12)

where the primes denote derivatives with respect to $r$. This equation has been solved to generate the pion partial waves.
2.3 The final state

In the absence of any interaction with the residual nucleus, the outgoing proton will be a plane wave. The proton wavefunction is then given by

$$\psi_p = \exp(ik \cdot r) \chi_{1/2A'}.$$  \hspace{1cm} (13)

Essentially the emitted proton wave experiences distortion due to the residual nucleus. The interaction potential between the outgoing proton and the residual nucleus is represented by a square well optical potential of radius $R_0$ and strength

$$V = -(U_0 + iW_0).$$  \hspace{1cm} (14)

Owing to its high kinetic energy, the distortion of the proton wave is treated according to high energy Glauber prescription (Jackson 1970). Accordingly, the wavefunction of the proton can be written as

$$\psi_p = \exp(ik \cdot r) \exp \left[ -\frac{i}{v} \int_0^\infty V(r + b\hat{k}) \, db \right],$$

where $v$ is the proton velocity and $\hat{k}$ is a unit vector in the direction $k$. For a square well potential, it has been shown (Yu 1966) that, in this approximation, the momentum of the outgoing proton gets modified as

$$k \to k' = k \left( 1 + U_0 M \frac{M}{k^2} - iW_0 \frac{M}{k^2} \right)$$

and the cross-section is reduced by a factor of $\exp(-2W_0 M\overline{R}/k)$ where

$$\overline{R} = \langle (R_0^2 - r^2 \sin^2 \theta)^{1/2} \rangle = 3R_0/4,$$

$\theta$, being the angle between the vectors $k$ and $r$. Thus the final state will be represented by

$$|f\rangle = \exp(ik' \cdot r) \chi_{1/2A} D\psi_e(j - m'),$$

where $D$ is the damping factor given by $\exp(-W_0 M\overline{R}/k)$.

2.4 Matrix element

The matrix element is given by

$$\langle f|H|i\rangle = \sum_{n\Lambda} \frac{j}{\mu} \frac{(-)^{j-m'}}{(2j+1)^{1/2}} C_{\Lambda n}^l(l^{1/2} j; \lambda \Lambda m')DF$$

where

$$F = \exp(ik' \cdot r)\chi_{1/2A'} 0|\sigma \cdot \nabla_\pi \tau \cdot \phi - \frac{\mu}{M} \tau \cdot \phi \sigma \cdot \nabla_N |n l\lambda \chi_{1/2A} + q\rangle,$$

where $|+q\rangle$ represents a positive pion of incident momentum $q$ while $|0\rangle$ is the vacuum state of pion.

The quantity $\langle 0|\tau \cdot \phi |+q\rangle$ is given by

$$\langle 0|\tau \cdot \phi |+q\rangle = \frac{-1}{(2E_\pi)^{1/2}} \phi(q, r) \tau_+,$$

where the pion wavefunction $\phi(q, r)$ is given by (8), $\tau_+$ is the isospin-raising operator.
and where
\[ \tau_{+} | \text{neutron} \rangle = | \text{proton} \rangle. \]  
(20)

Using the gradient formula as given by (Edmonds 1960)
\[ \nabla_{\mu} (\Phi(r) Y_{\ell}^{m} (\theta, \phi)) = - \left[ \frac{(l+1)}{(2l+1)} \right]^{1/2} (l+1 \ell; m-\mu m) \]
\[ \frac{d}{dr} \left( \frac{l}{r} \right) \Phi_{\ell} (r) Y_{\ell+1}^{-m} (\theta, \phi) \]
\[ + \left[ \frac{l}{(l+1)} \right]^{1/2} (l-1 \ell; m-\mu m) \]
\[ \frac{d}{dr} \left( \frac{l+1}{r} \right) \Phi_{\ell} (r) Y_{\ell+1}^{m} (\theta, \phi). \]
(21)

Substituting the above relations in the matrix element and further solving it reduces to
\[ \langle f| H | i \rangle = \sum_{\ell L n^{'}} 8 \pi^{2} \frac{3 \ell L + L}{2E_{\pi}} \left[ \left( \frac{f}{\mu} \right) \left( \frac{(-)^{j-m'}}{\ell+1} \right)^{1/2} C_{\ell n}^{l} (l \frac{1}{2} j; \lambda \Lambda m') A_{l+1} A_{L n'} \right] \]
\[ \times D_{L n'} (\Omega_{\pi}) \left( I_{1} - \frac{\mu \pi}{M I_{2}} \right), \]
(22)

where \( L n' \) are quantum numbers for proton wavefunctions and \( I_{1} \) and \( I_{2} \) are complicated series of terms involving radial and angular integrals. They are too complex to be reproduced here.

3. Calculational details

The calculations have been done to study the rate of absorption of (\( \pi^{+}, p \)) reaction in \( ^{16}O \) nucleus. Absorption phenomena have been studied by considering the pion absorbing bound neutron in \( 0s_{\frac{1}{2}}, 0p_{\frac{3}{2}} \) and \( 0p_{\frac{1}{2}} \) states. The incident pion kinetic energy has been taken to be 70 MeV. This energy has been considered because it is well below the (3, 3) resonance region so that resonance complexities do not have any effect. Further the energy is optimum to use Glauber’s approximation for the emitted proton. Finally experimental results are available (Bachelier et al 1977, 1980) for this energy. As explained earlier, the pion wavefunctions have been generated by solving Klein-Gordon equation.

It has been observed (Koch and Sternheim 1972) that for low energy pion nucleus elastic scattering around 80 MeV, the Kisslinger model pion optical potential gives better fit to the experimental value. We have used the standard Kisslinger form (Kisslinger 1955) of pion-nucleus optical potential. At incident pion kinetic energy of 70 MeV, there is no best fit values available for the potential parameters \( b_{0} \) and \( b_{1} \). However \( b_{0} \) and \( b_{1} \) are related to \( a_{0} \) and \( a_{1} \) through energy and momentum of the incident pion (Auerbach et al 1967)
\[ b_{0} = -2 (2\pi)^{3} E_{\pi} a_{0}/q^{2}, \]
\[ b_{1} = -2 (2\pi)^{3} E_{\pi} a_{1}. \]
(23)
Now \( a_0 \) and \( a_1 \) are approximately constants for the energy region 50–125 MeV. Recently measurements (Malbrough et al 1978; Blecher et al 1979; Dytman et al 1979) have been done for low energy \( \pi^+ \) scattering from light nuclei in the energy range 30–50 MeV and the results have been used to calculate the best fit values for the Kisslinger-potential parameters \( b_0 \) and \( b_1 \) in that energy range. Using the best fit values of \( b_0 \) and \( b_1 \) at 50 MeV (Malbrough et al 1978), we have calculated \( a_0 \) and \( a_1 \). With this set of \( a_0 \) and \( a_1 \) calculation has been done to find out \( b_0 \) and \( b_1 \) at 70 MeV. Further, looking at the best fit values of \( b_0 \) and \( b_1 \) as given by the above authors it is clear that there is uncertainty with \( \text{Re } b_0 \) and \( \text{Im } b_0 \). The largest theoretical uncertainty with \( \text{Re } b_0 \) is also apparent in the calculation done by Auerbach et al (1967). In view of the theoretical uncertainty in \( \text{Re } b_0 \) and also in \( \text{Im } b_0 \) as evident from the best fit values mentioned above, we have varied these parameter to get the right order of cross-section in the framework of standard Kisslinger optical model potential. The \( b_0 \) and \( b_1 \) values used in the calculation are given in table 1.

In order to solve the Klein-Gordon equation the PIRK (Eisenstein and Miller 1974) code has been employed. The density function has been generated from the radial HF wavefunctions of (5). Assuming the nucleus to be spherically symmetric uniform charge distribution, the coulomb potential used in the calculation is given by

\[
V_c(r) = \frac{Ze^2}{2R_c} \left[ (3 - (r/R_c)^2) \right] \quad r \leq R_c, \\
= \frac{Ze^2}{r} \quad r \geq R_c,
\]

where \( Ze \) is the nuclear charge and \( R_c \) is the charge radius of the nucleus. The \( R_c \) value has been obtained from the electron scattering data.

By using the principle of conservation of energy and momentum, the proton momentum \( k \) can be shown to be

\[
k = \sqrt{\frac{(q \cos \theta_L/M_{A-1}) + \left[ (q^2 \cos^2 \theta_L/M_{A-1}^2) + 2(1/M + 1/M_{A-1})(E_n - SE - E_Y - q^2/2M_{A-1}) \right]^{1/2}}{1/M + 1/M_{A-1}}}
\]

where \( M_{A-1} \) is the mass of the recoiling nucleus, \( SE \) the neutron separation energy in the target nucleus, \( E_Y \) the excitation energy of the residual nucleus and dynamical quantities refer to the laboratory coordinate system. The oscillator well parameter \( \alpha = (Mo/h)^{1/2} \) is taken to be 125 MeV, a value obtained from electron and proton scattering experiments. The coefficients \( C_{lm} \) used in the nuclear wavefunction are listed in table 2. The proton optical well parameters have been taken from a recent compilation (Passatore 1975). The justification of using the square well has been discussed in §4.

| Table 1. | Pion optical potential parameters for \( ^{16}\text{O} \). |
|----------|-----------------|-----------------|-----------------|-----------------|
| \( \text{Re } b_0 \) |
| \( \text{Im } b_0 \) |
| \( \text{Re } b_1 \) |
| \( \text{Im } b_1 \) |
| Remarks |
| \( -2.31 \) |
| \( 0.007 \) |
| \( 6.87 \) |
| \( 0.92 \) |
| Theoretically evaluated parameters (Malbrough et al 1978) at 70 MeV |
| \( -4.6 \) |
| \( -0.2 \) |
| \( 6.87 \) |
| \( 0.92 \) |
| Values employed in present calculation |
Table 2. Expansion coefficients $C^{(1)}_{lm}$ for $^{16}\text{O}$ obtained with Sussex interaction.

<table>
<thead>
<tr>
<th>Occupied HF orbit</th>
<th>$C_{00}$</th>
<th>$C_{01}$</th>
<th>$C_{02}$</th>
</tr>
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<tbody>
<tr>
<td>$0s_{1/2}$</td>
<td>0.9433</td>
<td>0.3140</td>
<td>0.1079</td>
</tr>
<tr>
<td>$0p_{3/2}$</td>
<td>0.9236</td>
<td>0.3447</td>
<td>0.1677</td>
</tr>
<tr>
<td>$0p_{1/2}$</td>
<td>0.9445</td>
<td>0.2882</td>
<td>0.1577</td>
</tr>
</tbody>
</table>

4. Results and discussion

The differential cross-sections for positive pion absorption by $^{16}\text{O}$ have been obtained separately for $0s_{1/2}$ neutron, $0p_{1/2}$ neutron and $0p_{3/2}$ neutron. The initial cross-section for the reaction which employed plane waves for both the pion and the proton with neutron represented by the harmonic oscillator wavefunction exhibited a high degree of sensitivity with variation in $\alpha$, the oscillator well parameter. Within allowable range the cross-section fluctuated by orders of magnitude, depending upon the proton emission angle. By taking into account the pion and proton distortion and employing the correlated wavefunction for the nucleus, we find that the cross-section is very much stabilised with a variation of a mere 15%. Figure 1 compares the differential cross-sections for pion absorption by $0s_{1/2}$ neutron, $0p_{1/2}$ neutron and $0p_{3/2}$ neutron.

![Figure 1](image_url)

**Figure 1.** Proton emission cross-section for 70 MeV pion absorption by $0s_{1/2}$, $0p_{1/2}$ and $0p_{3/2}$ neutron for the reaction $^{16}\text{O}(\pi^+, p)^{15}\text{O}$. Pion waves distorted by the standard Kisslinger potential.
is seen that the cross-section increases in magnitude by nearly 50\% at 0° proton angle consecutively from absorption by 0s_{1/2} through 0p_{1/2} to 0p_{3/2} neutron. The position of minima shifts slightly towards lower angles as absorption is considered by 0s_{1/2}, 0p_{1/2} and 0p_{3/2} neutron, in accordance with the experimental results. Finally we have compared the calculated cross-section for pion absorption by 0p_{1/2} neutron and 0p_{3/2} neutron with the experimental results (Bachelier et al 1977). In figure 2 the comparison has been done for absorption by 0p_{1/2} neutron. In this case the calculated cross-section has been scaled down by a factor of four. The calculated cross-section compares favourably with the experimental result. Here the experimental minima is beautifully reproduced. Figure 3 compares the calculated cross-section with the experimental

![Figure 2](image1.png)

**Figure 2.** Proton emission cross-section for 70 MeV pion absorption by 0p_{1/2} neutron for the reaction \(^{16}\)O (\(\pi^+, p\)) \(^{12}\)O; K: Present work with standard Kisslinger potential (scaled by factor of four); M: Calculation by Miller and Phatak (1974); Experimental data: Bachelier et al (1977).

![Figure 3](image2.png)

**Figure 3.** Proton emission cross-section for 70 MeV pion absorption by 0p_{3/2} neutron for the reaction \(^{16}\)O (\(\pi^+, p\)) \(^{12}\)O; K: Present work with standard Kisslinger potential; M: Calculation by Miller and Phatak (1974); Experimental data: Bachelier et al (1977).
results for absorption by $0p_{3/2}$ neutron. It is seen that the experimental data is reproduced reasonably well. The experimental minima is around $70^\circ$, whereas ours is at $85^\circ$. In both the figures the curve $M$ represents the calculation done by Miller and Phatak. It is evident that the calculation done by Miller and Phatak does not reproduce the experimental behaviour well which shows a rise in cross-section at backward angles.

The proton distortion has been taken into account according to the high-energy approximation. A complex square well potential has been employed to treat the proton-nucleus scattering. Strictly speaking, the potential should not plunge to zero abruptly. But the proton energy being large, the absorption is expected to be more or less uniform throughout the nuclear volume rather than enhanced at the nuclear surface. Again, the details of the nuclear surface, however, are expected to play an important role for nuclei with large $Z$ where coulomb repulsion is most effective. Moreover, the spin orbit term is just about 2–3 MeV at the proton energy concerned. Thus we believe that the error introduced due to square well is likely to be small.

5. Conclusions

The present analysis of $(\pi^+, p)$ reaction with oxygen leads to the following conclusions. (i) By taking into account the pion distortion and HF wavefunctions for the nucleus, the undesirable sensitivity of the cross-section with parameters like the oscillator well depth parameter $\alpha$ disappears. (ii) All the three effects like pion distortion, nuclear correlations and final state proton distortion are important in general, with pion distortion being dominant. (iii) In the present framework, the calculated cross-section reproduces the experimental behaviour reasonably well.

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References

Bhalerao R S and Waghmare Y R 1975 *Pramana* 5 154
Blecher M et al 1979 *Phys. Rev.* C20 1884
Eisenstein R A and Miller G A 1974 *Phys. Commun.* 8 130
Jackson D F 1970 *Nuclear reactions* (London: Methuen and Co) p. 64
Kakkar I 1969 PhD thesis, IIT Kanpur
Kisslinger L S 1955 *Phys. Rev.* 98 761
Koch J H and Sternheim M M 1972 *Phys. Rev.* C6 1118
Miller G A and Phatak S C 1974 *Phys. Lett.* B51 129
Passotore G 1975 *Nucl. Phys.* A248 509
Shakin C M, Waghmare Y R and Hull M H 1967a *Phys. Rev.* 161 1006
Yu D U L 1966 *Ann. Phys.* 38 392