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Some puzzles in the results of ultrasonic attenuation in superconductors

B K BASU

Tata Institute of Fundamental Research, Homi Bhabha Road, Bombay 400005, India

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Abstract. Two aspects of ultrasonic attenuation in superconductors are examined: (i) electron drag on dislocations and (ii) anomalous results in energy gap measurements. None of these features is physically understandable at present.

Keywords. Ultrasonic attenuation; superconductors; electron drag; energy gap measurements; dislocations.

1. Introduction

It is well-known that the ultrasonic attenuation in superconductors is given by the BCS (Bardeen *et al* 1957) relation,

$$\frac{\alpha_s}{\alpha_n} = \frac{2}{\exp[\Delta(T)/kT]+1} \tag{1}$$

where α_n and α_s are the electronic attenuations in the normal and superconducting states respectively, $2\Delta(T)$ the energy gap at temperature T, and k the Boltzmann constant. The BCS theory also predicts that $\Delta(T)/\Delta(0)$ is a universal curve irrespective of the $\Delta(0)$ value. Many ultrasonic measurements have been done to evaluate the energy gap value and the situation before 1970 has been reviewed by Rayne and Jones (1970). The situation now is not very different. In most metals the energy gap deduced from ultrasonic results agree with the BCS energy gap. However, in some materials there are anomalous results. In this paper we discuss the anomalous results with particular reference to Pb and where these results lead us to.

2. The anomalous behaviour

A typical measurement giving α_n and α_s is shown in figure 1. This is a recent measurement (Sathish *et al* 1983) in a high purity single crystal of lead (starting material 99·999% purity) grown in our laboratory. The frequency of the measurement was 36 MHz. α_n in the figure was taken with a transverse magnetic field of 1 kgauss. Both α_s and α_n were measured at a low ultrasonic amplitude to avoid amplitude-dependent effect, particularly in the superconducting state. The data in terms of (1) were analysed by assuming that the α_s at the lowest attained temperature was the zero of electronic attenuation. In other words the background attenuation was assumed to be in-

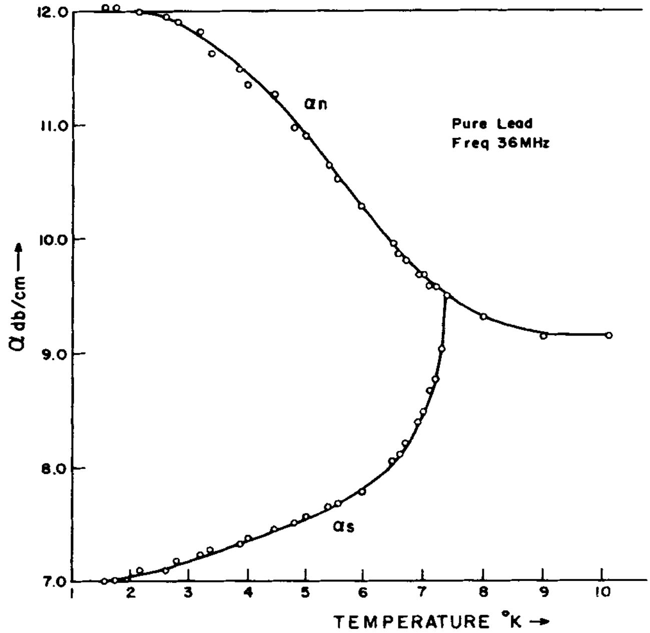

Figure 1. A typical ultrasonic attenuation curve of a superconductor.

dependent of temperature. This is the standard method of analysis in all ultrasonic measurements.

Equation (1) is independent of frequency. But in many experiments in high purity materials particularly in Pb (Deaton 1966; Fate *et al* 1968b; Randorff and Marshall 1970; Sathish *et al* 1983). In (Binnie *et al* 1974; Conley and Reed 1981) and Hg (Newcomb and Shaw 1968) α_s/α_n was observed to be strongly frequency-dependent. In doped specimens α_s/α_n was nearly independent of frequency, but it strongly deviated from the BCS energy gap (Fate *et al* 1968b; Newcomb and Shaw 1968; Binnie *et al* 1974; Sathish *et al* 1983).

3. Background attenuation

Love and Shaw (1964) first showed that the attenuation in Pb above a certain stress level is amplitude-dependent in the superconducting state. Amplitude-dependent effect was also reported in the normal state with stress amplitude much above than that in the superconducting state. This effect has been observed in several materials and investigated in great detail by many researchers. It is now believed that this behaviour is due to the change in the background dislocation attenuation at high stress amplitudes (Tittman and Bömmel 1966). In most materials dislocations cause substantial attenuation of the ultrasonic wave and the physical picture of the attenuation is as follows. In a lattice dislocations are held in position by defects called pinning points. These points may be the intersections of dislocations or the presence of foreign atoms near dislocations. When an ultrasonic wave passes through the lattice it causes dislocations to vibrate like free strings held firmly by the pinning points against the viscous forces in the lattice. The viscous forces are due to phonons at high temperatures. At low temperatures phonon population decreases and therefore in metals the viscous force is primarily due to electrons. The theory of ultrasonic attenuation due to dislocations has been worked out by Granato and Lücke (1956). This theory has been verified in great detail (Stern and Granato 1962) in situations where the dislocation attenuation can be unmistakably separated out from other sources of attenuation.

When the amplitude of the ultrasonic wave is large the dislocations can overcome the forces of the pinning points and can cause amplitude-dependent attenuation. In this case it is not possible to quantitatively predict the attenuation because of the difficulty of ascertaining the details of dislocation vibration. However, the amplitude-dependent effect is a signature of the dislocations breaking away from the pinning points. Thus the observation of the amplitude-dependent effect occurring at lower amplitudes in the superconducting state compared to that in the normal state implies reduction of electron drag on dislocation motion in the superconducting state. That electrons provide less drag on moving dislocations has also been shown in a different set of experiments (Suenaga and Galligan 1972) on plastic deformation in the normal and superconducting states. This feature can be reconciled if one assumes that only the normal electrons provide drag on dislocations. Since the number of normal electrons reduces with the decrease of temperature in the superconducting state one would expect the electron drag to reduce with the decrease of temperature. This has been observed in many experiments.

Since high amplitude ultrasonic experiments indicate a difference in the electron drag on dislocations between the normal and superconducting states there is no reason to believe that the same should not be true in low amplitude experiments. It then follows that the background attenuation due to dislocations in the superconducting state should be dependent on temperature. In the normal state several experiments (Hikata and Elbaum 1967; Hikata *et al* 1970) show that the dislocation drag and thereby attenuation is independent of temperature.

Mason (1966a and b) first pointed out that the low amplitude ultrasonic attenuation in the superconducting state should be dependent on temperature. He showed that if the temperature dependence of the background attenuation due to dislocations is taken into account properly, then the frequency dependence of α_s/α_n as well as the deviation from the BCS curve can be explained in some cases. He used the electron drag on dislocations to be given by,

$$\frac{B_{es}}{B_{en}} = \frac{2}{\exp[\Delta(T)/kT]+1} = \frac{\alpha_s}{\alpha_n} \tag{2}$$

which is according to the assumption that only the normal electrons provide the drag. Above B_{en} and B_{es} give the electron drag on dislocations in the normal and the superconducting states respectively.

Although some of the calculations of Mason on electron drag have been found incorrect the basic physical idea advanced by him should be valid. In some cases it has been found that suitable corrections of the background attenuation seem to explain some of the deviation from the BCS result. We (Basu 1970) had earlier analysed some ultrasonic data on lead by Fate *et al* (1968b) (high purity specimen Pb 12) which showed strong frequency dependence. The nature of the correction is shown in figure 2. The line curves are the experimentally observed data for a frequency of 51 MHz. The dashed lines are the corrected data. The dotted curve is the curve for α_s for the BCS energy gap

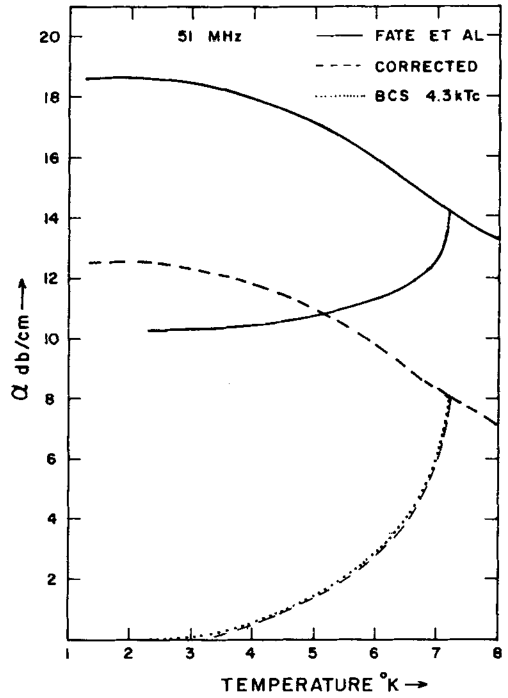

Figure 2. A possible explanation of the anomalous behaviour in Pb (Pb 12, Fate *et al* 1968b) in terms of temperature dependent background attenuation.

$2\Delta(0) = 4{\cdot}3\,kT_c$. The above correction makes use of (2) and contains an assumed parameter which is the drag coefficient in the superconducting state at the lowest temperature. The result gives the normal state dislocation drag coefficient close to the experimentally measured value (Hikata and Elbaum 1967) and also accounts for the frequency dependence of α_s/α_n in the sample.

4. The puzzles

Although the above explanation looks good it need not be correct. Unfortunately, there is no direct method to separate the background attenuation from the measured attenuation data without some assumptions. In his experiment Fate (1968a) compared the normal state attenuation data of pure Pb 12 which was deformed 4% to obtain amplitude independent data with many other undeformed specimens doped with impurities and found excellent fit with the Pippard's (1955) relation for the longitudinal ultrasonic wave. He, therefore, concluded that the dislocation background attenuation was independent of temperature and the same in the normal and superconducting states. Fate's conclusion completely contradicts Mason's viewpoint and needs to be checked further. We felt that we should look for changes in α_s and α_n in the same specimen before and after deformation (Sathish *et al* 1983). The idea behind this experiment is that the electronic attenuations will remain the same with or without deformation, but the background attenuation will change causing changes in the measured α_s and α_n curves. We did this experiment with two specimens, one of pure lead (99·999 % purity) (specimen A) and the other pure lead doped with 0·1 at. % gold (specimen B). Specimen A showed strong frequency dependence; α_s/α_n ratio, near the transition temperature increasing with decrease of frequency in the range of frequencies 12 MHz to 108 MHz. The doped specimen B showed very little spread of α_s/α_n with frequency, with α_s/α_n smaller than that expected from the BCS gap $2\Delta(0) = 4{\cdot}3\, kT_c$, near the transition temperature.

Because of gold the background attenuation in specimen B before deformation was very small, which increased three times as measured from the attenuation α_s at the lowest temperature for the frequency 36 MHz after deformation. The curves α_s and α_n in this case remained the same before and after deformation within the error of our experimental investigation. This is shown in figure 3. For specimen A deformation did not alter the background attenuation very much and the results are shown in figure 4. The α_s/α_n ratio remained the same before and after deformation. This brings to the first puzzle. The above results show without much doubt that the conclusion of Fate is correct and one cannot associate the anomalous behaviour of ultrasonic attenuation to background dislocation attenuation. What happens then to the dislocation drag by electrons? Unmistakably, the electron drag on dislocations is different in the normal and superconducting states at high stress amplitudes. But these seem to be the same at low stress amplitudes in the two states, contrary to all expectations.

The second puzzle is how to explain the anomalous ultrasonic results. In doped specimens there is no frequency dependence of α_s/α_n, but no single BCS energy gap curve fit the data throughout the temperature range. In this case the mean free path of electrons l is less than the ultrasonic wavelength, and the ultrasonic wave sees the

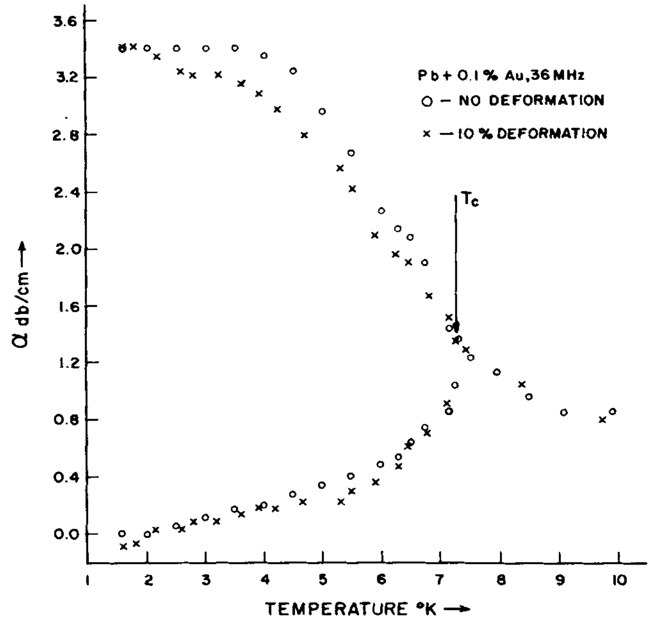

Figure 3. Ultrasonic attenuation curves of a lead specimen doped with 0·1 at. % gold before and after plastic deformation.

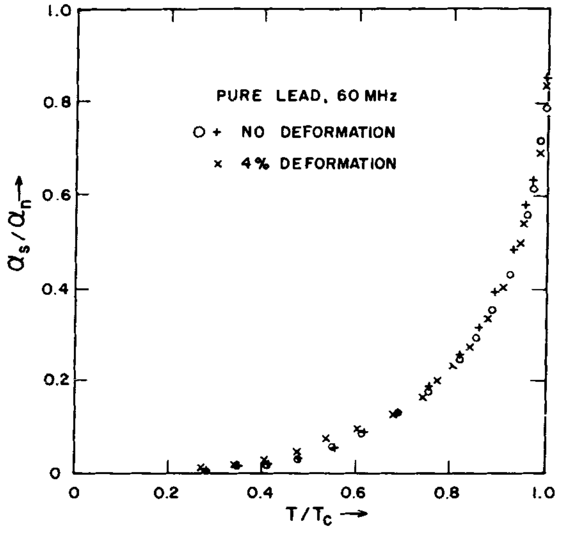

Figure 4. α_s/α_n ratio of a pure lead specimen before and after plastic deformation.

smeared-out Fermi surface. Several authors have analysed the results in terms of two BCS energy gaps with reasonable agreements. However, these agreements have problems. First of all one has to reconcile with unusual gap values. Secondly, tunnelling experiments, which are equivalent to these measurements, seem to give single BCS energy gap.

In high purity specimens the main problem is the frequency dependence of α_s/α_n. The electronic attenuation of the ultrasonic wave in the normal state depends on the product ql where q is the wavevector of the ultrasonic wave and l the mean free path of electrons. Since q is the same in the normal and superconducting states one may examine the concept that l is different in the two states. This assumption, *i.e.*, the phonon limited mean free paths are different in the two states was examined by Fate *et al* (1968b). These authors were partially successful in explaining their data. Unfortunately, this line of arguments cannot be sustained as there are wide divergences of the observed results, even on the nature of frequency dependences. For example Fate *et al* (1968b) showed that in Pb α_s/α_n decreases with increase of frequency whereas two other results (Randorff and Marshall 1970; Sathish *et al* 1983) showed just the opposite dependence in approximately the same purity specimens. This puts us back to square one, *i.e.*, the effect must be a material dependent effect. The question is how to look for it and the answer, if any, is not clear at all.

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