

A new approach to particle confinement

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Abstract. A new approach to permanent confinement of non-relativistic and relativistic particles inside microscopic regions of space is presented. Motion in suitably chosen energy-dependent potentials turns out to be such that the size of orbits of particles bound in such potentials decreases when energy is supplied to them from external sources and there exists a maximum size of these orbits. The energy spectrum is purely discrete without any continuum. The dynamics of such particles requires the introduction of a space-dependent metric in the Hilbert space of states to ensure conservation of probability.

Keywords. Permanent confinement; energy-dependent potentials; quarks; elliptic orbits.

1. Introduction

It is now widely believed that fractionally-charged, spin $\frac{1}{2}$ quarks constitute the hadrons. This was motivated by the hadron symmetries and is consistent with the deep inelastic lepton-hadron scattering data. But, on the other hand, the non-observation of free quarks has led to the belief that quarks are permanently bound inside hadrons by some mechanism. A full understanding of this mechanism is yet to evolve. Attempts to explain confinement by field-theoretic methods and through phenomenological models are in progress. Quantum chromodynamics (QCD), a gauge theory based on exact $SU_c(3)$ colour symmetry of quarks had emerged as a favourable candidate for strong interaction dynamics. The concept of "asymptotic freedom" (Politzer 1973; Gross and Wilczek 1973) resulting from $SU_c(3)$ justified the quark-parton model (Feynman 1972) which successfully describes hard scattering processes. On the other hand the construction of bound states of quarks to form hadrons on the basis of QCD has not been possible due to ignorance of quark-quark interaction in the low energy region.

Various phenomenological models have been proposed to explain the riddle of quark confinement and explain some properties of hadrons. The bag model for example presumes the qualitative features of confinement and has been applied to hadron phenomenology with considerable success (Chodos *et al* 1974; Hasenfratz and Kuti 1978). The potential models have used non-relativistic Schrödinger equation with power law (linear, harmonic oscillator) potential to explain heavy quarkonia spectrum (Quigg and Rosner 1979).

In this paper we present a new approach to permanent confinement of particles. In §2, we discuss the criteria for confinement in general. In §3, the confinement of a non-relativistic particle is considered by using Schrödinger equation in energy-dependent potential. The confinement of a relativistic particle and its various consequences are discussed in §4.

2. Confinement criteria

Usually, in a bound system (like the atom) the constituents can be excited from the ground state to the continuum and become free when we supply adequate amount of energy from an external source. If the constituents are to be permanently confined, the energy spectrum should be discrete without a continuum. Supply of energy from outside, in that case, would excite the constituents to one of the higher discrete states. This happens to be the case for a particle moving in a three-dimensional harmonic oscillator potential. Here the size of the orbit increases when energy is supplied to the system. The size of the system can therefore be increased indefinitely by supplying sufficient energy. Thus although the particle can never be made free, its confinement is not limited to a small domain. It would be interesting to look for potentials where in addition to having discrete energy spectrum one has orbits which shrink in size when energy is supplied from outside. For this to happen, the wave function should behave as

$$\sim \exp[-f(E)r],$$

where $f(E)$ is an increasing function of energy and r is the distance from a fixed centre. In that case, as energy increases, the particle has a greater probability of being closer to the centre than anywhere else. Thus increase of energy brings the particle closer to the centre rather than taking it away as it happens to be the case for conventional bound systems. However, when energy is taken away from such systems, the size of the orbits of the constituents would increase. Therefore, to ensure confinement in a small domain it would be necessary to limit the orbit size to a maximum finite value at the lowest allowed energy. It is the aim of the present investigations to look for potentials which are tailored for this kind of confinement of particles. The above confinement criteria are seen to be satisfied by potentials which in addition to having space dependence are also energy-dependent.

3. Non-relativistic particle

We consider a non-relativistic particle in a potential which depends linearly on energy and is spherically symmetric in space:

$$V = V(E, r) = -\left(\frac{a}{r} - b^2\right)E, \quad (1)$$

where E is the energy of the particle and a and b are constants. The particle is described by the Schrödinger equation

$$\left[-\frac{\nabla^2}{2m} + V(E, r)\right]\psi = E\psi, \quad (2)$$

with $\hbar = 1$.

3.1 Analytic solution

As the potential is spherically symmetric we can separate the radial and angular parts:

$$\psi_{nlm} = R_{nl}(r)Y_{lm}(\theta, \varphi).$$

The radial equation takes the form:

$$\frac{1}{r^2} \frac{\partial}{\partial r} \left(r^2 \frac{\partial}{\partial r} \right) R - \frac{l(l+1)}{r^2} R + 2mE \left(\frac{a}{r} - b^2 + 1 \right) R = 0. \quad (3)$$

Now, choosing $R(r) \sim r^l \exp(-\lambda r) \xi(r)$, where λ is a constant to be determined, we get

$$r \xi''(r) + 2(l+1-\lambda r) \xi'(r) + [\{\lambda^2 - 2m(b^2-1)E\}r + 2maE - 2\lambda(l+1)] \xi(r) = 0. \quad (4)$$

If we change to the dimensionless variable $\rho = 2\lambda r$, and set

$$\lambda^2 = 2m(b^2-1)E, \quad (5)$$

equation (4) reduces to the confluent hypergeometric equation

$$\rho \xi''(\rho) + (2l+2+\rho) \xi'(\rho) - \left\{ (l+1) - \frac{maE}{\lambda} \right\} \xi(\rho) = 0, \quad (6)$$

whose solution is

$$\xi(r) = CF \left(l+1 - \frac{maE}{\lambda}, 2l+2; 2\lambda r \right).$$

As we seek solution for confined states, the radial function must vanish asymptotically. So the above confluent hypergeometric series must reduce to a polynomial, the condition for which is

$$l+1 - \frac{maE}{\lambda} = -n_r, \quad (7)$$

where n_r is a non-negative integer. Equations (5) and (7) give the energy spectrum

$$E_n = 2(b^2-1)n^2/ma^2, \quad (8)$$

where $n = (n_r + l + 1)$. The energy levels are discrete without continuum. From (5) and (8) we get

$$\lambda = 2(b^2-1)n/a, \quad (9)$$

and the radial wavefunction is given by

$$R_{n_l}(r) \sim r^l \exp \{ -2(b^2-1)nr/a \} F(l+1-n, 2l+2; 4(b^2-1)nr/a). \quad (10)$$

It will be noticed from (8) and (10) that the exponential part of the wavefunction is an increasing function of energy (in this case $\sim E^{1/2}$). This feature is quite different from that of other conventional bound systems where it is a decreasing function of energy.

3.2 Orbits

Permanent confinement is best demonstrated in terms of orbits. To obtain this we use a semiclassical approximation which is valid in the limit $E \rightarrow \infty$. Putting

$$\psi = \psi_0 \exp \{ i(2mE)^{1/2} \Sigma \},$$

in (2), we get in the limit $E \rightarrow \infty$,

$$(\nabla \Sigma)^2 = \left(\frac{a}{r} - b^2 + 1 \right). \quad (11)$$

Here Σ is an energy-independent eikonal function and $(\nabla\Sigma)$ gives the direction of motion of the particle. Since the potential is spherically symmetric, the motion will take place in a plane. We write (11) in plane polar co-ordinates:

$$\left(\frac{d\Sigma_r}{dr}\right)^2 + \frac{1}{r^2}\left(\frac{d\Sigma_\theta}{d\theta}\right)^2 = \frac{a}{r} - b^2 + 1, \quad (12)$$

where $\Sigma(\mathbf{r}) \equiv \Sigma(r, \theta) = \Sigma_r(r) + \Sigma_\theta(\theta)$.

From (12), we have

$$d\Sigma_\theta/d\theta = \text{constant} = \alpha_\theta \text{ (say),}$$

$$\text{and hence, } d\Sigma_r/dr = \left(\frac{a}{r} - b^2 + 1 - \frac{\alpha_\theta^2}{r^2}\right)^{1/2}. \quad (13)$$

For convenience we introduce two quantities J_r and J_θ :

$$J_\theta = \oint d\theta \frac{d\Sigma_\theta}{d\theta} = 2\pi\alpha_\theta, \quad (14)$$

$$\text{and } J_r = \oint dr \frac{d\Sigma_r}{dr} = \oint dr \left(\frac{a}{r} - b^2 + 1 - \frac{\alpha_\theta^2}{r^2}\right)^{1/2}. \quad (15)$$

Integrating (15) by standard methods and using (14) we get

$$J_r + J_\theta = \pi a/(b^2 - 1)^{1/2}. \quad (16)$$

This looks very much like a relation which one comes across in the solution of Hamilton-Jacobi equation for motion of a charged particle in Coulomb potential. So it is natural to impose the "Böhr-Sommerfeld conditions":

$$(2mE)^{1/2} J_r = 2\pi n_r, \quad (17)$$

with $n_r = 0, 1, 2, \dots$ etc., and

$$(2mE)^{1/2} J_\theta = 2\pi(l + 1), \quad (18)$$

with $l = 0, 1, 2, \dots$ etc. These conditions give us

$$(2mE)^{1/2} (J_r + J_\theta) = 2\pi(n_r + l + 1) \equiv 2\pi n, \quad (19)$$

where $n = (n_r + l + 1) = 1, 2, 3, \dots$ etc.

From (19) and (16) we get

$$E_n = 2(b^2 - 1)n^2/ma^2, \quad (20)$$

which is same as (8).

Now, using (13), (14), (15), (18) and following the method of Jena and Pradhan (1981) we obtain the orbit equation for the bound constituents:

$$\frac{1}{r} = \frac{amE}{(l+1)^2} \left[1 + \left\{ 1 - \frac{2(b^2 - 1)(l+1)^2}{a^2 mE} \right\} \cos(\theta - \theta_0) \right]. \quad (21)$$

This represents an ellipse with the origin as one of the foci and having eccentricity

$$e = \left\{ 1 - \frac{2(b^2 - 1)(l+1)^2}{a^2 mE} \right\}^{1/2}. \quad (22)$$

Substituting the allowed value of E from (20) in (21) and (22) we get,

$$\frac{1}{r} = \left\{ \frac{2(b^2 - 1)n^2}{a(l+1)^2} \right\} \left[1 + \left\{ 1 - \frac{(l+1)^2}{n^2} \right\}^{1/2} \cos(\theta - \theta_0) \right], \quad (23)$$

and
$$e = \{1 - (l+1)^2/n^2\}^{1/2}. \quad (24)$$

Thus the bound constituents have closed elliptic orbits with semi-major axis A and semi-minor axis B given by

$$A = a/2(b^2 - 1), \quad (25)$$

and
$$B = a(l+1)/2(b^2 - 1)n \quad (26)$$

The semi-major axis is thus independent of energy and for a given value of l , the semi-minor axis decreases with increase of energy. It is clear from (25) and (26) that no orbit can have size greater than the semi-major axis $a/2(b^2 - 1)$.

3.3 Probability density and current density

With the energy-dependent potential

$$V(E, r) = -\left(\frac{a}{r} - b^2\right)E,$$

the Schrödinger equation

$$\left[-\frac{\nabla^2}{2m} + V(E, r) \right] \psi = E\psi,$$

becomes
$$-\frac{\nabla^2}{2m}\psi = i\left(\frac{a}{r} - b^2 + 1\right)\frac{\partial\psi}{\partial t}, \quad (27)$$

for which the equation of continuity takes the form

$$\frac{\partial}{\partial t} \left\{ \psi^* \left(\frac{a}{r} - b^2 + 1 \right) \psi \right\} + \nabla \cdot \left\{ \frac{-i}{2m} (\psi^* \overleftrightarrow{\nabla} \psi) \right\} = 0. \quad (28)$$

The probability density and probability current density are thus given by

$$\rho = \psi^* \left(\frac{a}{r} - b^2 + 1 \right) \psi,$$

and

$$\mathbf{j} = -\frac{i}{2m} \psi^* \overleftrightarrow{\nabla} \psi.$$

The additional multiplicative factor in the probability density is a space-dependent metric which is to be used in normalisation of wavefunction and calculation of expectation values:

$$\int d^3x \psi^* \left(\frac{a}{r} - b^2 + 1 \right) \psi = 1,$$

and

$$\langle Q \rangle = \int d^3x \psi^* Q \left(\frac{a}{r} - b^2 + 1 \right) \psi.$$

Such metrics have been used in the context of superheavy atoms with electrons moving in strong fields where the effective potential becomes energy-dependent (Zeldovich and Popov 1972; Reinhardt and Greiner 1977).

4. Relativistic particle

We consider a relativistic spin $\frac{1}{2}$, particle described by Dirac equation in energy-dependent potentials:

$$[\gamma_\mu p_\mu - m + \gamma_\mu V_\mu(E, r) + \gamma_5 \gamma_\mu A_\mu(E, r) + S(E, r) + \gamma_5 P(E, r) + \sigma_{\mu\nu} T_{\mu\nu}(E, r)]\psi = 0, \quad (29)$$

where V_μ , A_μ , S , P and $T_{\mu\nu}$ are vector, axial vector, scalar, pseudoscalar and tensor respectively.

Now, writing $T_{0i} = F_i$, $T_{ij} = \varepsilon_{ijk} G_k$ and $\psi = \begin{pmatrix} \varphi \\ \chi \end{pmatrix}$, we get the coupled equations

$$\begin{aligned} \{\boldsymbol{\sigma} \cdot (\mathbf{p} + \mathbf{V} - \mathbf{F}) + A_0 - P\} \chi - (E - m + V_0 + S - \boldsymbol{\sigma} \cdot \mathbf{G} + \boldsymbol{\sigma} \cdot \mathbf{A}) \varphi &= 0, \\ \{\boldsymbol{\sigma} \cdot (\mathbf{p} + \mathbf{V} + \mathbf{F}) + A_0 + P\} \varphi - (E + m + V_0 - S + \boldsymbol{\sigma} \cdot \mathbf{G} + \boldsymbol{\sigma} \cdot \mathbf{A}) \chi &= 0. \end{aligned} \quad (30)$$

With suitable choice of the strengths of the potentials, these equations take convenient forms and lead to interesting consequences:

4.1 Dirac equation analogous to Maxwell equation in a medium

If we choose $\mathbf{F} = -\mathbf{V}$, $\mathbf{A} = -\mathbf{G}$, $A_0 = -P$ and $V_0 = S - m$, then equation (30) reduce to

$$\begin{aligned} \boldsymbol{\sigma} \cdot \mathbf{p} \varphi &= E \chi, \\ \{\boldsymbol{\sigma} \cdot (\mathbf{p} + 2\mathbf{V}) - 2P\} \chi &= (E + 2V_0 - 2\boldsymbol{\sigma} \cdot \mathbf{G}) \varphi. \end{aligned} \quad (31)$$

Further, choosing the energy dependence of the potentials as

$$(V_0 - \boldsymbol{\sigma} \cdot \mathbf{G}) = E(\varepsilon - 1)/2,$$

and $(\boldsymbol{\sigma} \cdot \mathbf{V} - P) = \frac{1}{2} [(\varepsilon - 1)\boldsymbol{\sigma} \cdot \mathbf{p} + \boldsymbol{\sigma} \cdot (\varepsilon \mathbf{p})]$,

where ε is an energy-independent function, we get

$$\begin{aligned} \boldsymbol{\sigma} \cdot \mathbf{p} \varphi &= E \chi, \\ \boldsymbol{\sigma} \cdot \mathbf{p} (\varepsilon \chi) &= E (\varepsilon \varphi). \end{aligned} \quad (32)$$

These are analogous to the Maxwell equations,

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad \text{and} \quad \nabla \times \mathbf{H} = \frac{\partial \mathbf{D}}{\partial t}, \quad (33)$$

in a medium characterized by the dielectric constant ε and magnetic permeability $\mu = 1/\varepsilon$. Here $\mathbf{D} = \varepsilon \mathbf{E}$ and $\mathbf{H} = \mathbf{B}/\mu = \varepsilon \mathbf{B}$. The analogy becomes clear if we take the time-dependence of \mathbf{E} and \mathbf{B} as

$$(\mathbf{E}, \mathbf{B}) \sim \exp(-i\omega t)$$

and write Maxwell equations (33) in the form (Akhiezer and Berestetskii 1965)

$$\begin{aligned} (\mathbf{S} \cdot \mathbf{p})_{jk} E_k &= \omega (iB_j), \\ (\mathbf{S} \cdot \mathbf{p})_{jk} (\varepsilon B_k) &= -\omega (i\varepsilon E_j), \end{aligned} \quad (34)$$

where $p_k = -i\partial_k$, $(S_i)_{jk} = -i\varepsilon_{ijk}$ and make the identifications

$$\varphi \equiv \mathbf{E}, \chi \equiv i\mathbf{B} \quad \text{and} \quad \boldsymbol{\sigma} \equiv \mathbf{S}.$$

It may be worth noting here that (33) with $\varepsilon = 1/\mu = \exp(-\alpha r^2)$ leads to confinement of photons. This has been demonstrated in a recent work by Khare and Pradhan (1983).

Since the equations of motion of a relativistic spin $\frac{1}{2}$ particle in energy-dependent potential can be cast into a form analogous to that of Maxwell equations in a medium where the function ε in (32) plays the role of dielectric constant of the medium, it is natural to expect that (32) will lead to confinement of massless Dirac particle.

4.2 Dirac equation with only scalar and vector potentials with $V_\mu = (V_0, 0, 0, 0)$.

In this case equations (30) reduce to

$$\begin{aligned} \boldsymbol{\sigma} \cdot \mathbf{p}\varphi &= (E + V_0 + m - S)\chi, \\ \text{and} \quad \boldsymbol{\sigma} \cdot \mathbf{p}\chi &= (E + V_0 - m + S)\varphi \end{aligned} \quad (35)$$

If the space-dependence of V_0 and S are taken to be spherically symmetric, the solution can be written in the form (Berestetskii *et al* 1971)

$$\begin{pmatrix} \varphi \\ \chi \end{pmatrix} = \begin{pmatrix} f(r)\Omega_{jm}(\theta, \varphi) \\ ig(r)\Omega_{j'm}(\theta, \varphi) \end{pmatrix}, \quad (36)$$

where $f(r)$ and $g(r)$ are radial function, $\Omega_{jm}(\theta, \varphi)$ are spinor spherical harmonics, $l = j \pm \frac{1}{2}$ and $l' = 2j - l$. Equations (35) now reduce to the radial equations,

$$\begin{aligned} f' + \frac{1+k}{r}f - (E + m - S + V_0)g &= 0, \\ g' + \frac{1-k}{r}g + (E - m + S + V_0)f &= 0, \end{aligned} \quad (37)$$

where

$$\begin{aligned} K &= (j + \frac{1}{2}) = l, \quad \text{for } j = l - \frac{1}{2}, \\ &= -(j + \frac{1}{2}) = -(l + 1), \quad \text{for } j = l + \frac{1}{2}, \end{aligned}$$

is a positive or negative integer and can never take the value zero.

4.2a Massless case: For this case ($m = 0$) we choose the potentials as

$$V_0(E, r) = S(E, r) = \frac{E}{2} \left(\frac{a_0}{r} - b_0 \right), \quad (38)$$

where a_0 and b_0 are energy- and space-independent parameters. Equations (37) now become

$$\begin{aligned} f' + \frac{1+k}{r}f - Eg &= 0, \\ g' + \frac{1-k}{r}g + E \left(\frac{a_0}{r} - b_0 + 1 \right) f &= 0. \end{aligned} \quad (39)$$

This is identical with the equation of massless coloured quarks in a colour-dielectric medium considered by us earlier (Jena and Pradhan 1984). Solving (39) by the method followed in this reference we get equispaced energy-spectrum

$$E_n = 2(b_0 - 1)^{1/2} n/a_0, \quad (40)$$

with $n = 1, 2, 3 \dots$ etc and the solution

$$\{f(r), g(r)\} \sim \exp[-2(b_0 - 1)nr/a_0],$$

so that higher energy states are closer to the centre than the lower energy ones. Thus the state of massless Dirac particles in energy-dependent scalar and vector potentials given by (38) is identical to that of massless coloured quarks in a colour-dielectric medium.

By following the analysis of § 3.2, it is easy to see that the massless Dirac particles in the potentials of (38) also form closed elliptic orbits having identical features with those of the non-relativistic case.

4.2b Massive case: Here equations (37) take the form

$$\begin{aligned} f' + \frac{1+k}{r} f - (E+m)g &= 0, \\ g' + \frac{1-k}{r} g + E\left(\frac{a_0}{r} - b_0 + 1 - m/E\right) f &= 0. \end{aligned} \quad (41)$$

Now, changing the radial co-ordinate from r to $\rho = \left(1 + \frac{m}{E}\right)r$ we get

$$\begin{aligned} f' + \frac{1+k}{\rho} f - Eg &= 0, \\ g' + \frac{1-k}{\rho} g + E\left(a_0/\rho - \frac{b_0 - 1 + m/E}{1 + m/E}\right) f &= 0, \end{aligned} \quad (42)$$

which have the same form as those of (39). In this case, energy is given by the cubic equation

$$E^3 + mE^2 - \frac{4(b_0 - 1)}{a_0^2} n^2 E - 4mn^2/a_0^2 = 0, \quad (43)$$

which has one imaginary and two real roots. The physically acceptable real solution is

$$\begin{aligned} E_n = \frac{i(3)^{1/2}}{2} \left[\left\{ \frac{2mn^2(4-b_0)}{3a_0^2} - \frac{m^3}{27} \right. \right. \\ \left. \left. + \left(\frac{64(1-b_0)^3 n^6}{27 a_0^6} + \frac{4m^2 n^4 (134 - 106b_0 - b_0^2)}{27 a_0^4} - \frac{4m^4 n^2}{27 a_0^2} \right)^{1/2} \right\}^{1/3} \right. \\ \left. - \left\{ \frac{2mn^2(4-b_0)}{3a_0^2} - \frac{m^3}{27} - \left(\frac{64(1-b_0)^3 n^6}{27 a_0^6} \right. \right. \right. \\ \left. \left. \left. + \frac{4m^2 n^4 (134 - 106b_0 - b_0^2)}{27 a_0^4} - \frac{4m^4 n^2}{27 a_0^2} \right)^{1/2} \right\}^{1/3} \right], \end{aligned} \quad (44)$$

which in the massless limit reduces to equation (40) i.e.

$$E_n \xrightarrow{m=0} 2(b_0 - 1)^{1/2} n/a_0.$$

Further, the solution is

$$\{f(r), g(r)\} \sim \exp[-\{(m + E)(m + E(b_0 - 1))\}^{1/2} r],$$

so that the higher energy solutions are closer to the origin than the lower energy ones.

4.3 Probability density and probability current

With the energy-dependent scalar (S) and vector $\{V_\mu = (V_0, 0, 0, 0)\}$ potentials given by (38), the Dirac equation is

$$\left[\gamma_0 E - \boldsymbol{\gamma} \cdot \mathbf{p} - m + \frac{1}{2} E(\gamma_0 + 1) \left(\frac{a_0}{r} - b_0 \right) \right] \psi = 0. \quad (45)$$

The equation of continuity, in this case, gives the probability density

$$\rho = \psi^\dagger \left\{ 1 + \frac{1}{2} (1 + \gamma_0) \left(\frac{a_0}{r} - b_0 \right) \right\} \psi,$$

and the probability current density

$$\mathbf{j} = \psi^\dagger \boldsymbol{\alpha} \psi.$$

The space-dependent multiplicative factor $\left\{ 1 + \frac{1}{2} (1 + \gamma_0) \left(\frac{a_0}{r} - b_0 \right) \right\}$ occurring in the probability density is a metric and is to be used in the determination of normalization constant and calculation of expectation values.

5. Discussion

We have presented a new approach to the problem of permanent confinement of particles. The potentials in this approach are energy-dependent. The expectation values have to be calculated using a space-dependent metric as a consequence of resulting non-hermiticity of the Hamiltonian. We have also shown that the massless Dirac particles in suitably chosen potentials can be equivalently described by a space-dependent generalized dielectric function. This analysis may throw some light on the choice of potentials for confining quarks in hadrons.

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