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Longitudinal plasma waves in the presence of monopoles of electron mass and electron charge

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Abstract. Dispersion relation for plasma waves in the presence of magnetic monopoles of electron mass and electron charge, is derived. It is shown that magnetic monopole charges, in general, dominate the dispersion. When monopoles form a fraction of the main body of plasma, there are two dominant oscillations. It is suggested that there can be electromagnetic emissions at these frequencies by nonlinear conversions. Possible application to the pulsar neighbourhood is envisaged.

Keywords. Magnetic monopoles; longitudinal plasma waves; pulsar; dispersion relation.

1. Introduction

The existence of magnetic monopoles was theoretically inferred by Dirac (Jackson 1975). The angular momentum and quantum conditions also lead to the existence of magnetic monopoles (Saha 1936, 1949). Maxwell's equations become symmetric and the charge quantisation leads to units of magnetic monopole values for every unit of electric charge (Schwinger 1969). However, the experimental verifications for the existence of magnetic monopoles have so far not been very successful. But the recent observation (Bartlett *et al* 1981; Cabrera 1982) shows that magnetic monopole charge particles are very small in number as compared to the electron-charged particles. In a plasma with magnetic monopoles of electron mass, the electromagnetic waves remain transverse in nature (Wilson and Swamy 1979). In a magnetised plasma, the magnetic monopole charges dominate the dispersion for the circularly polarised waves (Kulkarni 1983). In the absence of ambient magnetic field, magnetic monopole charges completely take over the dispersion.

In the following, we study the propagation characteristics of high frequency longitudinal electrostatic type of waves and their dispersion. The heavy ions with opposite charges provide the neutralising background for charges in the charge distribution. Wave propagation in a typical plasma is discussed and the possible wave frequency regions are identified. Finally the situations under which such waves can be observed are suggested.

2. Dispersion relation for longitudinal waves

The plasma is assumed to be uniform and homogeneous, with no external electric field E_0 and magnetic field B_0. There are N_1 electrons having no magnetic monopole charges at temperature T_1, N_2 magnetic monopole charges of monopole charge g, of electron mass m, and electron charge e, at temperature T_2. 0 as a superscript or as subscript represent equilibrium values. The CGS system of units is used. Equal numbers of positive ions and opposite massive monopoles provide the neutralising background.

The equation of motion for N_1^0 electrons having charge e, mass m is given by

$$m\frac{d\mathbf{v}}{dt} = -e\mathbf{E} - \nabla P_1 \tag{1}$$

and for N_2^0 is given by

$$m\frac{d\mathbf{v}}{dt} = -e\mathbf{E} + g\mathbf{B} - \nabla P_2, \tag{2}$$

where P_1 and P_2 are the pressure energies equivalent to $N_1^0 \cdot T_1$ and $N_2^0 \cdot T_2$ respectively. Temperature is in the units of energy. **E** and **B** are perturbed wave electric and magnetic fields respectively. The equation of state for the electrons is $P_0 = A\rho_0^{\gamma}$ [A, constant and γ is adiabaticity, ρ_0 = density = N_m^0]. For the self-consistent solution for E and B fields, (1) and (2) are solved along with

$$\nabla \cdot \mathbf{E} = -4\pi e \int n_1^1 \, d^3x, \tag{3}$$

$$\nabla \cdot \mathbf{B} = +4\pi g \int n_2^1 \, d^3x. \tag{4}$$

We linearise these equations with standard perturbations method such that

$$N_1 = N_1^0 + n_1^1 \text{ etc.}$$

where $n_1^1/N_1^0 \ll 1$.

Retaining only first order perturbations and using the harmonic variations of the type $f(x,t) \sim \exp(i(wt - kx))$, we solve (1) and (3), (2) and (4), in 1-dimension, to obtain:

$$\left(1 - \frac{\omega_{p_1}^2}{\omega^2 - k^2C_{s_1}^2} - \frac{\omega_{p_2}^2}{\omega^2 - k^2C_{s_2}^2}\right)E + \frac{\omega_{p_2}\omega_{g_2}}{(\omega^2 - k^2C_{s_2}^2)}B = 0, \tag{5}$$

$$\frac{\omega_{g_2}\omega_{p_2}}{(\omega^2 - k^2C_{s_2}^2)}E + \left(1 - \frac{\omega_{g_2}^2}{\omega^2 - k^2C_{s_2}^2}\right)B = 0, \tag{6}$$

where $\omega_{p_1}^2$ is the square of electron plasma frequency $= 4\pi N_1^0 e^2/m$ and $\omega_{g_2}^2$ is the square of monopole charge plasma frequency $= 4\pi N_2^0 g^2/m$; k is the wavenumber of the wave of frequency ω: $C_s = (\gamma P_0/\rho_0)^{1/2} = (\gamma T/m)^{1/2}$ is the electron sound velocity. Equations (5) and (6) when solved gives

$$1 - \frac{\omega_{p_1}^2}{\omega^2 - k^2C_{s_1}^2} - \frac{\omega_{p_2}^2}{\omega^2 - k^2C_{s_2}^2} - \frac{\omega_{g_2}^2}{\omega^2 - k^2C_{s_2}^2}\left(1 - \frac{\omega_{p_1}^2}{\omega^2 - k^2C_{s_1}^2}\right) = 0 \tag{7}$$

which describes the dispersion relation for the longitudinal waves.

In the case when

$$\omega^2 = \omega_{p_1}^2 + k^2C_{s_1}^2, \tag{8}$$

the nontrivial solution is possible for $\omega_{p_2} = 0$, saying that the monopole changes are absent. Equation (8) is dispersion relation for electrostatic waves discussed in the text books (e.g. Krall and Trivelpiece).

3. Wave characteristics

In the case, monopole charges are at the same temperature as the main body, $(C_{s_1} = C_{s_2})$ the solution of (7) is

$$\omega^2 = k^2 C_s^2 + \tfrac{1}{2}(\omega_{p_1}^2 + \omega_{p_2}^2 + \omega_{g_2}^2) \pm \tfrac{1}{2}(\omega_{p_1}^2 - \omega_{g_2}^2), \tag{9}$$

$(\omega_{g_2}^2 \gg \omega_{p_2}^2$, therefore it is neglected in the fourth term).

We get two roots for (9). One relation is

$$\omega^2 = k^2 C_s^2 + (\omega_{p_1}^2 + \omega_{p_2}^2). \tag{10}$$

This is nothing but (8), which shows that the electrostatic waves propagate in two-component plasma as if it were only one. The second relation

$$\omega^2 = k^2 C_s^2 + \omega_{g_2}^2, \tag{11}$$

shows that there exists a new region where the longitudinal wave propagates.

In the presence of monopole charges, the second frequency range exists near (11) where electrostatic waves can be excited. The natural frequency component is that of monopole charge plasma frequency. Since $\omega_{g_2}^2 \gg \omega_{p_2}^2$, we expect even for moderate densities of monopole charges, ω_g^2 is in a very high frequency range, thereby heavy ions do not contribute to the wave propagation.

4. Discussion

When a small number of monopoles are dispersed through a plasma containing a large number of electrons without magnetic charges, we see that the longitudinal static type perturbation can be excited. There exist two regions of natural frequencies (a) ω_p and (b) ω_g and in the neighbourhood of these, waves could be strongly excited. These two-frequency regions can be well separated.

Assuming the pulsars to be uniformly distributed over a spherical shell at a distance R, the observed flux can be that due to a single pulsar. For fluxes $F_M \approx 10^{-9}\,\mathrm{cm}^{-2}\,\mathrm{sec}^{-1}$ by Cabrera (1982) and $F_M \approx 10^{-15}\,\mathrm{cm}^{-2}\,\mathrm{sec}^{-1}$ by Salpeter *et al* (1982), the monopole density N_0 at the pulsar surface will be

$$N_0 = (F_M R^2 / C\gamma_p^2)\,\mathrm{cm}^{-3}$$

where γ_p is the pulsar radius.

The distances are typically in kilo par sec ($1\,\mathrm{kps} \approx 10^{15} \times 3\,\mathrm{km}$) and radius γ_p is $\approx 10\,\mathrm{km}$. The density N_0 will be in the range 10^{11} to $10^5\,\mathrm{cm}^{-3}$. This is very large and comparable to electron densities. At distances away from surfaces there could be a medium where the effects discussed in this paper, are possible. Moreover, monopoles with electron masses dominate the dispersion for $N_0 > N_e\,(e^2/g^2)$ (N_e is the electron densities) which is very low as compared to N_e. The waves will suffer dispersion. The high densities of magnetic monopoles put severe restriction on the existence of magnetic field on pulsars similar to the restrictions on galactic magnetic fields (Salpeter *et al* 1982; Arons and Blandford 1983), which need careful analyses.

The charge quantisation discussed in Jackson (1975), Schwinger (1969) and also by Saha (1939, 1949) does not put any restriction for the masses of monopoles. However, the elegant calculation regarding minimum masses of such particles, termed as electromagnetic masses by Saha and also the grand unified theories disputes the existence of low mass particles. Here in this paper, we have assumed that the charge quantisation is applicable to particle of electron mass also as they carry fundamental charge e. Therefore, the analysis is valid for such a medium.

5. Conclusion

The behaviour of electrostatic type waves shows that there are two dominant modes of oscillations. The monopole charges have behaviour similar to electrostatic waves. The effects of such waves in wave-wave conversions need further study.

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