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# Dispersion theory relations and charmed baryon couplings

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**Abstract.** Dispersion theory sum rules proposed by Atkinson have been obtained for the invariant amplitudes of the elastic pion-charmed baryon scattering process. Saturating the sum rules with the known intermediate states, we obtain the pion-charmed baryon coupling constants.

**Keywords.** Dispersion relation; sum rules; charmed baryon couplings.

## 1. Introduction

One of the most important discoveries (Goldhaber *et al* 1976; Peruzzi *et al* 1976; Angelini *et al* 1979; Baltay *et al* 1979; Calicchio *et al* 1980) of the last decade has been the experimental detection of the charmed particles. The first evidence for charmed baryons came from neutrino reaction $\nu p \to \mu^- \Lambda \pi^+ \pi^+ \pi^+ \pi^-$, the hadrons arising from decay of a state of mass 2·43 GeV called $C_1^{++}$ (or $\Sigma_c^{++}$), the corresponding quark structure being *cuu*. The $C_1^{++}$ decays into $\pi^+ C_0$, where $C_0$ (or $\Lambda_c$) has 2·26 GeV mass and an isosinglet charmed baryon structure, $(1/\sqrt{2})c\,(ud - du)$. The above reaction thus arises from the sequence $\nu p \to \mu^- C_1^{++}$, $C_1^{++} \to \pi^+ C_0$, $C_0 \to \Lambda \pi^+ \pi^+ \pi^-$. The clearest evidence for $C_0$ comes presently from its $\Lambda \pi^+$ decay mode (Baltay *et al* 1979). $C_1$ state is found to be isospin-triplet and its higher state $C_1^*$ (or $\Sigma_c^*$) (mass 2·48 GeV) is the resonant state. Experiment indicates the decay of $C_1^*$ into $\pi C_0$. Coupling constants of the hadrons are more useful than decay widths, because the values of coupling constants throw light on hadronic reactions. So far only a few calculations (Bryan and Phillips 1968; Nagels *et al* 1975; Prasad and Singh 1980) have been performed to determine the hadronic coupling constants of charmed particles. However, these values of the coupling constants differ by considerable amounts in all these calculations. The present authors were therefore interested in calculating the meson–charmed baryons coupling constants which are not accessible to experimenters.

The purpose of this paper is to write the Atkinson type dispersion theory sum rules (Atkinson 1968; Seth *et al* 1970) for $\pi C_1$ elastic scattering process. These sum rules are saturated with low-lying intermediate states in the narrow width approximation. In the past, such an attitude (Atkinson 1968; Seth *et al* 1970) has resulted in useful algebraic relations among the masses and the coupling constants. Here these relations are used to determine coupling constants $g_{\pi C_1 C_1}$, $g_{\pi C_1 C_1^*}$, $g^V_{\rho C_1 C_1}$ and $g^T_{\rho C_1 C_1}$.

## 2. Sum rules and calculations

Dispersion theory sum rules can be obtained by equating two different unsubtracted dispersion relations for the same scattering amplitude with different variables held fixed. Briefly if a certain invariant amplitude $F(s,t,u)$ satisfies an unsubtracted dispersion relation with either the Mandelstam variable $t$ or $u$ held fixed, (in the present paper $s$ is the total centre of mass energy squared), one can write the following sum rule:

$$\int \frac{\operatorname{Im} F(s',t,\Sigma-s'-t)}{s'-s}\,\mathrm{d}s' = \int \frac{\operatorname{Im} F(s',\Sigma-s'-u,u)}{s'-s}\,\mathrm{d}s' \tag{1}$$

where

$$\sum = \sum_{i=1}^{4} m_i^2 = s+t+u \tag{2}$$

with $m_i$ as the mass of one of the two incident or two outgoing particles.

We write the dispersion theory sum rules for the invariant amplitudes $A$ and $B$ of $\pi C_1$ elastic scattering process. In the backward direction the Regge-pole description of the high-energy behaviour gives

$$A \sim s^{\alpha(u)-1/2},\quad B \sim s^{\alpha(u)-1/2}$$

as $s \to \infty$, where $\alpha(u)$ is the parameter of the leading Regge trajectory in the $u$-channel. For $\pi C_1$ elastic scattering process the $u$-channel Regge trajectories are $C_0$ and $C_1$. We take the slope of these trajectories equal to 0·33 (Barger and Phillips 1975). The trajectory parameters can be written as

$$\alpha_{C_0}(u) = 0{\cdot}5 + 0{\cdot}33\,(u - m_{C_0}^2)$$

$$\alpha_{C_1}(u) = 0{\cdot}5 + 0{\cdot}33\,(u - m_{C_1}^2)$$

These relations give $\alpha_{C_0}(u=0) = -1{\cdot}19$ and $\alpha_{C_1}(u) = -1{\cdot}45$. Thus both the amplitudes $A$ and $B$ are superconvergent at $u = 0$.

In the forward direction the asymptotic behaviour of the invariant amplitudes is

$$A \sim s^{\alpha(t)},\quad B \sim s^{\alpha(t)-1},$$

where $\alpha(t)$ is the leading Regge trajectory parameter in the $t$-channel. We take $\rho$-trajectory as the dominant one for isospin 1 amplitudes in the $t$-channel of the $\pi C_1$ elastic scattering process. For $\rho$ trajectory $\alpha(t=0) = 0{\cdot}57$, which shows that at $t = 0$, $B^{I_t=1}$ amplitude is convergent but the amplitude $A^{I_t=1}$ is not convergent or the dispersion relation without subtraction is valid only for $B^{I_t=1}$ amplitude whereas dispersion relations for $A^{I_t=1}$ amplitude need subtraction for its validity. In order to avoid subtraction one can use dispersion relations for $A/(s-u)$. Here we write the following four sum rules for $A/(s-u)$ and $B$ amplitudes at $t = u = 0$:

$$\int_{(m_\pi+m_{C_1})^2}^{\infty} \frac{\operatorname{Im} A^1(s',t=0)}{2(s'-m_{C_1}^2-m_\pi^2)(s'-2m_{C_1}^2-2m_\pi^2)}\,\mathrm{d}s' + \int_0^{\infty} \frac{\operatorname{Im} A^1(u',t=0)}{2u'(m_\pi^2+m_{C_1}^2-u')}\,\mathrm{d}u'$$

$$= \int_{(m_\pi+m_{C_1})^2}^{\infty} \frac{\operatorname{Im} A^1(s',u=0)}{s'(s'-2m_{C_1}^2-2m_\pi^2)}\,\mathrm{d}s' + \int_0^{\infty} \frac{\operatorname{Im} A^1(t',u=0)}{t'(2m_\pi^2+2m_{C_1}^2-t')}\,\mathrm{d}t' \tag{3}$$

$$\int_{(m_\pi+m_{C_1})^2}^{\infty} \frac{\operatorname{Im} A^2(s',t=0)}{2(s'-m_{C_1}^2-m_\pi^2)(s'-2m_{C_1}^2-2m_\pi^2)}\,\mathrm{d}s' + \int_0 \frac{\operatorname{Im} A^2(u',t=0)}{2u'(m_\pi^2+m_{C_1}^2-u')}\,\mathrm{d}u'$$

$$= \int_{(m_\pi+m_{C_1})^2}^{\infty} \frac{\mathrm{Im}\, A^2(s', u=0)}{s'(s'-2m_{C_1}^2-2m_\pi^2)} \mathrm{d}s' + \int_0^\infty \frac{\mathrm{Im}\, A^2(t', u=0)}{t'(2m_\pi^2+2m_{C_1}^2-t')} \mathrm{d}t' \tag{4}$$

$$\int_{(m_\pi+m_{C_1})^2}^{\infty} \frac{\mathrm{Im}\, B^1(s', t=0)}{(s'-2m_{C_1}^2-2m_\pi^2)} \mathrm{d}s' + \int_0^\infty \frac{\mathrm{Im}\, B^1(u', t=0)}{u'} \mathrm{d}u'$$

$$= \int_{(m_\pi+m_{C_1})^2}^{\infty} \frac{\mathrm{Im}\, B^1(s', u=0)}{(s'-2m_{C_1}^2-2m_\pi^2)} \mathrm{d}s' + \int_0^\infty \frac{\mathrm{Im}\, B^1(t', u=0)}{t'} \mathrm{d}t' \tag{5}$$

$$\int_{(m_\pi+m_{C_1})^2}^{\infty} \frac{\mathrm{Im}\, B^2(s', t=0)}{(s'-2m_{C_1}^2-2m_\pi^2)} \mathrm{d}s' + \int_0^\infty \frac{\mathrm{Im}\, B^2(u', t=0)}{u'} \mathrm{d}u'$$

$$= \int_{(m_\pi+m_{C_1})^2}^{\infty} \frac{\mathrm{Im}\, B^2(s', t=0)}{(s'-2m_{C_1}^2-2m_\pi^2)} \mathrm{d}s' + \int_0^\infty \frac{\mathrm{Im}\, B^2(t', u=0)}{t'} \mathrm{d}t', \tag{6}$$

where the superscripts 1 and 2 stand for the isospin of the $t$-channel Regge trajectories. Sum rules (3) and (5) are evaluated by taking the contributions from the known intermediate states $C_0$ (isospin = 0, $J^P = \frac{1}{2}^+$, mass = 2·26 GeV and charm = + 1), $C_1$ (isospin = 1, $J^P = \frac{1}{2}^+$, mass = 2·43 GeV and charm = + 1), $C_1^*$ (isospin = 1, $J^P = \frac{3}{2}$, mass = 2·48 GeV and charm = + 1) and $\rho$-meson in the sharp resonance approximation. The sum rules (4) and (6) get contribution from $C_0, C_1$ and $C_1^*$ only. For evaluating the contributions from different intermediate states we use the following effective Lagrangians:

$$L_{\pi bb'} = i g_{\pi bb'} \overline{u^{b'}} \gamma_5 u^b \phi^\pi$$

$$L_{\pi bb^*} = \frac{g_{\pi bb^*}}{m_\pi} \overline{\psi_\mu^{b^*}} u^b \partial_\mu \phi^\pi$$

$$L_{\rho bb} = g_{\rho bb}^V \overline{u^b} \gamma_\mu u^b V_\mu + g_{\rho bb}^T \overline{u^b} \frac{\sigma_{\mu\nu}}{2m_b} \partial_\nu u^b V_\mu$$

$$L_{\rho\pi\pi} = i g_{\rho\pi\pi} (\phi^\pi \partial_\mu \phi^\pi) V_\mu,$$

where $u(\psi_\mu)$ is the Dirac (Rarita-Schwinger) wave function for the 1/2 (3/2) spin particle and $\phi^\pi(V_\mu)$ is the wave function for pion (vector meson). Integrals in (3) to (6) are evaluated by using the relation

$$\frac{F}{s-s_r-i\varepsilon} = P\frac{F}{s-s_r} + i\pi F\,\delta(s-s_r). \tag{7}$$

The sum rules (3)–(6) are simplified in the following forms relating between masses of the particles and the coupling constants:

$$C_{10}^{tu} A_{C_0} g_{\pi C_1 C_0}^2 + c_{11}^{tu} A_{C_1^*} g_{\pi C_1 C_1^*}^2 = A_\rho g_{\rho C_1 C_1}^T g_{\rho\pi\pi} \tag{8}$$

$$C_{20}^{tu} A_{C_0} g_{\pi C_1 C_0}^2 + C_{21}^{tu} A_{C_1^*} g_{\pi C_1 C_1^*}^2 = 0 \tag{9}$$

$$C_{10}^{tu} B_{C_0} g_{\pi C_1 C_0}^2 + C_{11}^{tu} B_{C_1} g_{\pi C_1 C_1}^2 + C_{11}^{tu} B_{C_1^*} g_{\pi C_1 C_1^*}^2$$

$$= B_\rho (g_{\rho C_1 C_1}^V - g_{\rho C_1 C_1}^T) g_{\rho\pi\pi}, \tag{10}$$

$$C_{20}^{tu} B_{C_0} g_{\pi C_1 C_0}^2 + C_{21}^{tu} B_{C_1} g_{\pi C_1 C_1}^2 + C_{21}^{tu} B_{C_1^*} g_{\pi C_1 C_1^*}^2 = 0, \tag{11}$$

where

$$A_{C_0} = P_0^u (m_{C_1} - m_{C_0})/2(m_{C_1}^2 + m_\pi^2 - m_{C_0}^2)m_{C_0}^2,$$

$$A_{C_1^*} = P_1^u[\tfrac{2}{3}(m_{C_1^*} + m_{C_1})(E_1^2 - m_{C_1^*}^2) + \tfrac{1}{3}(E_1 + m_{C_1})(m_{C_1^*}^2 - m_{C_1}^2 - m_\pi^2)]/[2m_\pi^2(m_{C_1}^2 + m_\pi^2 - m_{C_1^*}^2)m_{C_1^*}^2],$$

$$A_\rho = -P_1^t/2m_{C_1}m_\rho^2,$$

$$B_{C_0} = P_0^u/m_{C_0}^2,$$

$$B_{C_1} = P_1^u/m_{C_1}^2,$$

$$B_{C_1^*} = -\tfrac{2}{3}P_1^u(2m_{C_1}^2 + m_{C_1}E_1 - E_1^2)/m_\pi^2,$$

$$B_\rho = -2P_1^t/m_\rho^2,$$

$$E_1 = (m_{C_1^*}^2 + m_{C_1}^2 - m_\pi^2)/2m_{C_1}^*,$$

$C_{ij}^{tu}$ is the crossing matrix element corresponding to $I_u = j$ to $I_t = i$ isospins and $P_j^u$ and $P_i^t$ are the isospin projection operators in the $u$-channel and $t$-channel, respectively. For $\pi C_1$ elastic scattering process, we find $P_0^u = 3$, $P_1^u = 2$, $P_1^t = 2$ (Golowich 1965; Seth 1969) and $C_{10}^{tu} = -\frac{1}{3}$, $C_{11}^{tu} = \frac{1}{2}$, $C_{20}^{tu} = \frac{1}{3}$, $C_{21}^{tu} = \frac{1}{2}$ (Seth 1969; Rebbi and Slanky 1970). Substituting the values of the mass of the particles, isospin projection operators and isospin crossing matrix elements, we get the following relations corresponding to sum rules (8)–(11):

$$0{\cdot}02\, g_{\pi C_1 C_0}^2 + 5{\cdot}72\, g_{\pi C_1 C_1^*}^2 = 0{\cdot}69\, g_{\rho C_1 C_1}^T g_{\rho\pi\pi}, \tag{12}$$

$$0{\cdot}02\, g_{\pi C_1 C_0}^2 - 5{\cdot}72\, g_{\pi C_1 C_1^*}^2 = 0 \tag{13}$$

$$-0{\cdot}20\, g_{\pi C_1 C_0}^2 + 0{\cdot}17\, g_{\pi C_1 C_1}^2 - 65{\cdot}36\, g_{\pi C_1 C_1^*}^2 = -6{\cdot}75\,(g_{\rho C_1 C_1}^V - g_{\rho C_1 C_1}^T) g_{\rho\pi\pi}, \tag{14}$$

$$0{\cdot}20\, g_{\pi C_1 C_0}^2 + 0{\cdot}17\, g_{\pi C_1 C_1}^2 - 65{\cdot}36\, g_{\pi C_1 C_1^*}^2 = 0. \tag{15}$$

Using $g_{\pi C_1 C_0}^2/4\pi = 51{\cdot}56$ (Prasad and Singh 1980) and solving (12)–(15) we find

$$g_{\pi C_1 C_1}/\sqrt{4\pi} = 3{\cdot}36, \tag{16}$$

$$g_{\pi C_1 C_1^*}/\sqrt{4\pi} = 0{\cdot}43, \tag{17}$$

$$g_{\rho C_1 C_1}^V g_{\rho\pi\pi}/4\pi = 6{\cdot}02, \tag{18}$$

$$g_{\rho C_1 C_1}^T g_{\rho\pi\pi}/4\pi = 3{\cdot}03. \tag{19}$$

Decay width of $\rho$ into $\pi\pi$ states equal to 154 MeV (Roos *et al* 1982) is used to obtain $g_{\rho\pi\pi}/\sqrt{4\pi} = 1{\cdot}72$. Using this value of $g_{\rho\pi\pi}$ in (18) and (19), we find

$$g_{\rho C_1 C_1}^V/\sqrt{4\pi} = 3{\cdot}50, \tag{20}$$

$$g_{\rho C_1 C_1}^T/\sqrt{4\pi} = 1{\cdot}76. \tag{21}$$

## 3. Discussion

We find that $g_{\pi C_1 C_1}/\sqrt{4\pi} = 3{\cdot}36$. This is in close agreement with the previous value $g_{\pi C_1 C_1}/\sqrt{4\pi} = 3{\cdot}98$ and 3·55 as determined by Dover *et al* (1977) from the calculations based on the models of Bryan and Phillips (1968) and Nagels *et al* (1975), respectively. Prasad and Singh (1980) have determined $g_{\pi C_1 C_1}/\sqrt{4\pi} = 4{\cdot}95$. Further, we get $g^V_{\rho C_1 C_1}/\sqrt{4\pi} = 3{\cdot}5$ and $g^T_{\rho C_1 C_1}/\sqrt{4\pi} = 1{\cdot}76$. These values are close to $g^V_{\rho C_1 C_1}/\sqrt{4\pi} = 2{\cdot}73$ and $1{\cdot}19$ and $g^T_{\rho C_1 C_1}/\sqrt{4\pi} = 1{\cdot}81$ and $3{\cdot}21$ as determined by Dover *et al* (1977). Prasad and Singh (1981) have determined $g^V_{\rho C_1 C_1}/\sqrt{4\pi} = 3{\cdot}1$ and $g^T_{\rho C_1 C_1}/\sqrt{4\pi} = 6{\cdot}1$ from the calculations based on superconvergence sum rules.

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