

Mass spectrum of the dimuons produced in relativistic heavy-ion collisions

D SYAM

Saha Institute of Nuclear Physics, 92, A.P.C. Road, Calcutta 700 009, India

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Abstract. The mass spectrum of dimuons produced from the matter in the central region of rapidity in ultra-relativistic heavy-ion collisions is calculated in accordance with Bjorken's recently proposed model for relativistic heavy-ion collisions. The matter in this central region is assumed to consist of a deconfined quark-gluon plasma phase and a pionized phase. Distinct enhancements of the dimuon mass spectrum below 500 MeV, due to the quark-gluon phase, are predicted for a deconfinement phase-transition temperature $T_c < 200$ MeV.

Keywords. Heavy ion collisions; relativistic hydrodynamics; quark-gluon phase; dimuon mass spectrum.

1. Introduction

It has been suggested (Shuryak 1980; Chin 1982; Domokos 1983) that the mass spectrum of dileptons (*i.e.* lepton pairs) produced in relativistic heavy ion collisions should serve as a good diagnostic tool for deciding whether a quark-gluon plasma may have been formed in the said collision. Since these dileptons may come from quark-antiquark annihilation, or, *e.g.* $\pi^+ \pi^-$ annihilation, the shape of the mass spectrum is sensitive to the critical temperature T_c —currently estimated to be $\lesssim 200$ MeV (Kuti *et al* 1980, 1981; Kajantie *et al* 1981)—for phase-transition between the quark-gluon plasma and the hadronic phase (composed mainly of pions) and also to the manner in which the original system of colliding nucleons evolve into the final set of observed particles, *i.e.* to the equations of motion of the hadronic and quark-gluon matter. Chin has presented a scenario in which a central collision between two heavy ions produces a spherical blob of hot, compressed matter; which subsequently expands and cools in a manner governed by the equations of relativistic hydrodynamics, undergoing a phase-transition from the quark-gluon phase (provided the initial temperature of the fireball is above T_c) to the hadronic phase at T_c . Local thermal equilibrium is supposed to prevail at each point of the evolving mass of fluid. Chin convoluted the resulting temperature distribution with the rate of emission ($dN(T)/dM d^4x$) of lepton pairs of given invariant mass (M) from a unit volume of the fluid to obtain the required mass spectrum. In the present paper we adopt the hydrodynamic scenario developed by Bjorken (1983) and Baym *et al* (1983). We consider ultra-relativistic collisions between identical heavy ions for which $E_{\text{beam}}/A > 50$ GeV in the c.m. frame. Following the studies of Bjorken, Baym and their co-workers, we assume that subsequent to the collision between the two heavy ions, which is supposed to take place at $t = z = 0$ (we shall be using the cylindrical polar co-ordinate system), two leading clusters, moving

with nearly the velocity of light in opposite directions, carry away the baryon numbers of the two heavy ions. Between the leading clusters, spanning the central region of rapidity, lies some fluid, composed, immediately after the collision, of a quasi-freely streaming set of quarks, antiquarks and gluons. In this paper we focus our attention solely on the evolution of this fluid body, which is expected to emit dileptons in the central region of rapidity. Note that the initial configuration of the fluid, in this model, has cylindrical symmetry which is preserved during the subsequent evolution. Bjorken's second assumption is that the initial conditions for hydrodynamic flow are Lorentz-invariant with respect to longitudinal motion of the fluid elements. These initial conditions are imposed at a proper time $(\tau) = \tau_0 \sim 0$ (1 fm), where $\tau = (t^2 - z^2)^{1/2}$. τ_0 , however, is *a priori* unknown. Between $\tau = 0$ and $\tau = \tau_0$, the evolution of the matter, which essentially consists of quasi-freely streaming quarks and gluons, cannot be described by hydrodynamics and consequently thermodynamics. We shall have to invoke some hypothesis to describe the temperature-distribution in this pre-equilibrium state or phase. More about this phase in § 3.2. We emphasize at this point that, in our view, at $\tau \approx \tau_0$, the interactions between the streaming particles become strong enough (through 'infrared slavery' of QCD) to not only thermalise the fluid, but also cause hadronization if the temperature T_0 at $\tau = \tau_0$ is less than T_c . Further, T_c refers to the critical temperature for phase transition between (locally) thermally equilibrated plasma and the hadronic phase. After finding out the temperature distribution according to this scenario, we follow the footsteps of Chin.

A different model for the evolution of the (assumed spherical) fireball produced in heavy ion collisions is given by Hasegawa and Tanaka (1983). They solve a differential equation for diffusion of temperature through space, to obtain the required temperature distribution (as a function of r and t). Hasegawa (1983) has worked out the dilepton mass spectrum with this $T(r, t)$. With this introduction we turn to describe (very briefly) the hydrodynamical aspects in § 2, the dilepton production rates from the quark-gluon phase and the hadronic phase in § 3, and the results are summarized in § 4.

2. Hydrodynamic aspects (Bjorken's model)

The expansion of the quark-gluon plasma/hadronic phase, for $\tau \geq \tau_0$, is governed by the relativistic hydrodynamic equation (Landau 1965):

$$\partial_\mu T_{\mu\nu} = 0, \quad (1)$$

where the energy momentum tensor $T_{\mu\nu}$ is given by

$$T_{\mu\nu} = (\varepsilon + p)u_\mu u_\nu - pg_{\mu\nu}, \quad (2)$$

where ε is the energy density, p is the pressure, $u_\mu = (\gamma, \gamma\mathbf{v})$, \mathbf{v} being the velocity of the fluid (with $c \equiv 1$), $\gamma = (1 - \mathbf{v}^2)^{-1/2}$ and $g_{\mu\nu} = (1, -1, -1, -1)$. Now Bjorken's hypothesis that the initial conditions of hydrodynamic flow are Lorentz-invariant with respect to longitudinal motion, together with the covariant nature of (1) implies that this initial Lorentz symmetry is preserved by the dynamics. Thus any local thermodynamic variable, say the temperature $T(r, z, t)$, is related to the corresponding quantity on the $z = 0$ plane by a simple transformation, *e.g.* $T(r, z, t) = T(r, 0, (t^2 - z^2)^{1/2})$. Consequently it suffices to confine our attention to the $z = 0$ plane. Baym *et al* (1983)

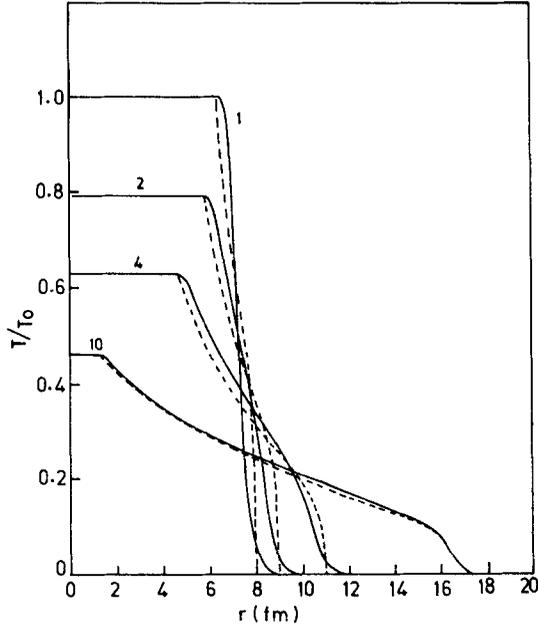


Figure 1. Temperature distributions as functions of r for cylindrical expansion of the fluid *la Baym et al* (1983). Solid lines correspond to numerical solution of the coupled differential equations of Baym *et al*; dashed lines correspond to approximate solutions as explained in the text. Each pair of curves is labelled by the time (in fm) that has elapsed since the collision. The hydrodynamic flow is assumed to commence at $t = \tau_0 = 1$ fm. Here $R = 7$ fm; $c_s = 1/\sqrt{3}$.

have solved the hydrodynamical equations on this ($z = 0$) plane for a massless quark-gluon plasma or hadronic phase (note that for $T > m_\pi$, the mass of the pion plays an unimportant role in the equation of state), with baryon number density $n_B = 0$. The resulting temperature distribution as a function of r and t is shown in figure 1 for $\tau_0 = 1$ fm, $R =$ radius of the colliding ions $= 7$ fm (for uranium-uranium collision) and $c_s =$ velocity of sound $= 1/\sqrt{3}$ (for an ideal fluid). The use of the same equation of state ($p = c_s^2 \varepsilon$), and hence the same value for c_s , for both phases, implies that the underlying phase transition is second order (no entropy change). At present it is not clear (from QCD) whether the phase transition in question is of first order or second order. However, we believe that our conclusions will not be drastically affected by the nature of the phase-transition. In the figure, T_0 is the temperature of the fluid at $\tau = \tau_0$, or on the plane $z = 0$ at $t = \tau_0$. According to the estimates of Baym *et al* (1983) $T_0 \approx 0.2$ GeV. Unfortunately the numerical solution of the coupled differential equations of Baym *et al* (equations (3.8) and (3.9) of their paper) take a large amount of time. To cut down the computation time for the calculation of the dilepton mass-spectrum we have adopted the following strategy. Instead of solving the differential equations numerically we have used the following analytic approximation to T given by Baym *et al*:

$$T(r, 0, t) \equiv T(r, t) = T_R(r, t) (\tau_0/t) c_s^2 [1 + (1 - V_R(r, t) c_s)^{-1}]^{1/2} \quad (3)$$

where V_R and T_R are the Riemann solutions to V_r (transverse velocity of the fluid) and T , with

$$V_R = \tanh \alpha_R,$$

$$T_R = T_0 \exp(-c_s \alpha_R)$$

and

$$\alpha_R = \frac{1}{2} \ln [(1 + c_s)(t + r - R)/(1 - c_s)(t + R - r)].$$

This approximation does not reproduce the part of T/T_0 vs r distribution that is flat. In fact the analytic approximation gives unphysical results when r is less than the right hand end of the flat region. We compute the height of the flat portion at any given time by interpolation from the known (from the numerical solution of the differential equations referred to above) values of T/T_0 over the flat portion as a function of time. The results of analytic approximation plus interpolation are compared with the numerical solution in figure 1.

3. Dimuon mass spectrum

3.1 Production rates

Following the computation of the temperature we compute $dN(T)/dM d^4x$, dN being the number of lepton pairs produced within the four-volume d^4x , in the mass interval $M, M + dM$, where M is the invariant mass of the lepton pair. The dilepton production rate at any temperature T due to the annihilation of massless u and d quarks at zero baryon density is given by (Nikishov 1959; Chin 1982; Domokos 1983):

$$dN/dM d^4x|_q = \frac{5}{9} \cdot (\alpha^2/\pi^3) \left(1 + \frac{2m_l^2}{M^2}\right) \left(1 - \frac{4m_l^2}{M^2}\right)^{1/2} MT^2 H(M^2/T^2). \quad (4)$$

The contribution of $s\bar{s}$ annihilation is small compared to $u\bar{u}$ or $d\bar{d}$ annihilation for $M < 1$ GeV (the mass range of interest to us), because of the mass of the s -quark (~ 300 MeV) and hence is overlooked (However, see Sinha 1983). The effect of $s\bar{s}$ annihilation (at large M) is merely to replace $5/9$ in (4) by $6/9$. In (4), m_l is the mass of a lepton (assumed to be μ^\pm in the present investigation) and

$$H(\xi^2) = \int_0^\infty dx (e^x + 1)^{-1} \ln [1 + \exp(-\xi^2/4x)]. \quad (5)$$

The production rate from the hadronic phase, which is mainly due to pion annihilation ($\pi^+\pi^- \rightarrow \rho \rightarrow \gamma^* \rightarrow \mu^+\mu^-$) is given by (Nikishov 1959; Chin 1983):

$$dN/dM d^4x|_{\text{pion}} = \frac{1}{12} \cdot (\alpha^2/\pi^3) \cdot |F_\pi(M^2)|^2 \cdot \left(1 - \frac{4m_\pi^2}{M^2}\right) \left(1 + \frac{2m_l^2}{M^2}\right) \cdot \left(1 - \frac{4m_l^2}{M^2}\right)^{1/2} \cdot MT^2 \cdot G(W, \lambda), \quad (6)$$

with $W = \frac{M^2}{2m_\pi^2} - 1, \quad \lambda = \frac{m_\pi}{T},$

and $G(W, \lambda) = \int_\lambda^\infty dx (e^x - 1)^{-1} \cdot \ln \left[\frac{1 - \exp(-Wx - P(x^2 - \lambda^2)^{1/2})}{1 - \exp(-Wx + P(x^2 - \lambda^2)^{1/2})} \right], \quad (7)$

where $P = (W^2 - 1)^{1/2}$. (8)

The pion form factor $|F_\pi(M^2)|^2$ is given by a Breit-Wigner form (Gounaris and Sakurai 1968; Benaksas *et al* 1972):

$$|F_\pi(M^2)|^2 = c_1 / [(m_\rho^2 - M^2)^2 + c_2 (M^2 - 4m_\pi^2)^3 / M^2], \quad (9)$$

where m_ρ (the ρ -meson mass) = 0.7723 GeV, $c_1 = 0.3894$ and $c_2 = 0.0469$.

The resulting production rates at three selected temperatures (same as those chosen by Chin) for each phase are shown in figures 2a and 2b.

We have in the same vein, as in §2, decided not to use expressions (4) and (6) directly in the computations for the dilepton mass spectrum but replace these by an analytic approximation, in the case of (4) and an interpolation-based approximation in the case of (6) to cut down the computation time. Thus we find that if we replace $H(\xi^2)$ by $A(\pi\xi/2)^{1/2} \exp(-B\xi)$ and find A and B by least-square-fitting to $H(\xi^2)$ for a given T ($\xi = M/T$), then excellent fits are obtained between $H(\xi^2)$ and $A(\pi\xi/2)^{1/2} \exp(-B\xi)$ for a wide range of values of T . This is shown in figure 2a, where $A = 0.675$ and $B = 0.9317$, and the least square fit has been made to the curve corresponding to $T = 0.2$ GeV.

The choice of interpolation as a suitable scheme in the case of (6) stems from the observation that the pion spectrum is dominated by the form factor $|F_\pi(M^2)|^2$, and its shape is rather insensitive to temperature variations over the temperature range shown in figure 2b. The result ensuing from this scheme may be examined by looking at the curves for $T = 0.175, 0.225$ GeV, which are calculated in two ways: by numerically integrating $G(W, \lambda)$ and by interpolation, the inputs in the latter approach being the values of $dN/dM d^4x|_{\text{pion}}$ for 20 values of M ($0.2 \text{ GeV} < M < 1 \text{ GeV}$) at $T = 0.2$ GeV and the values of $dN/dM d^4x|_{\text{pion}}$ at $M = 0.715$ GeV for three values of temperature: $T = 0.15, 0.2$ and 0.25 GeV.

3.2 The convolution integral

The convolution part is rather straightforward. However we are yet to specify our assumption regarding the temperature distribution before the onset of hydrodynamic flow, which is supposed to commence at $\tau = \tau_0$ fm (We shall use this temperature distribution in conjunction with (4) to compute the contribution to the mass spectrum from the prehydrodynamic stage). Recall, however, that the quarks and gluons are in a non-thermal equilibrium state during this period ($0 < \tau < \tau_0$). We have to bear with this procedure, until a suitable theoretical framework for particle production is developed for this domain of τ . It may be suggested that the momentum distribution functions for the sea quarks, as revealed by deep inelastic scattering (on the nucleus in question), be used for the momentum distributions of the quarks (instead of using thermodynamic momentum distributions) in the central region, over the period $0 < \tau < \tau_0$. However, we have considerable doubts about whether the same functions may remain applicable, firstly because of the creation of additional (pairs of) quarks and gluons and secondly because the temperature of the system would be different from that inside a nucleus or a nucleon ($\lesssim 50$ MeV, following some estimates, *e.g.* Angelini and Pazzi 1982).

In the present paper we have principally used the following assumption:

$$T(r, t)|_{t < \tau_0} = T_0 \cdot (t/\tau_0), \quad r < R \\ = 0, \quad r \geq R \quad (10a)$$

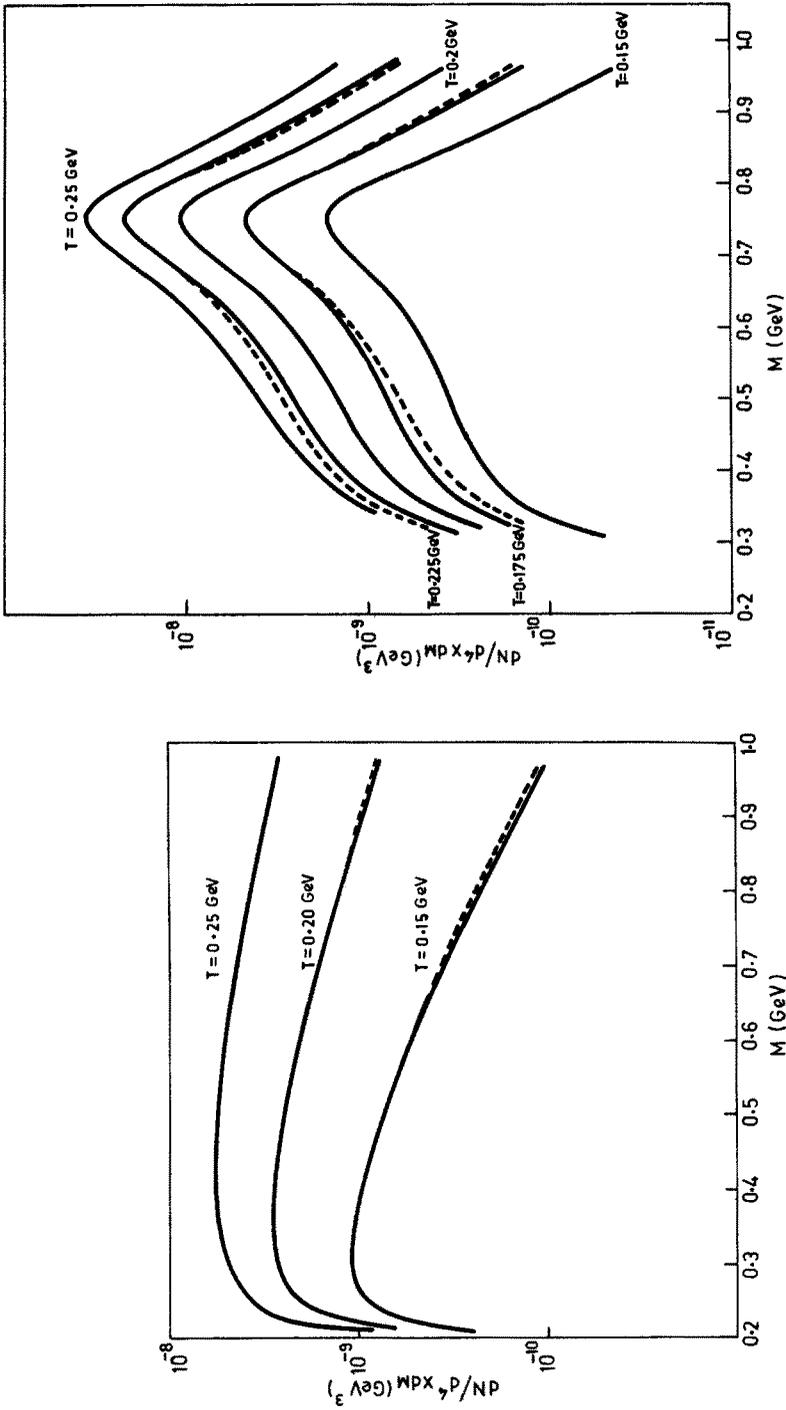


Figure 2. The dimuon production rates at three different temperatures as a function of the dimuon invariant mass. In (a), From quark-gluon phase; the solid lines present the result of numerical integration while the dashed lines give the results of a least square-fit algorithm as explained in the text. In (b), From pion phase; the solid lines represent the result of numerical integration while the dashed lines correspond to an interpolation-based approximation as explained in the text.

where T_0 is the temperature at $t = \tau_0$ and R is the radius of the colliding nucleus. In addition, for a typical situation ($\tau_0 = 1$ fm; $R = 7$ fm; $T_0 = 0.18$ GeV; $T_c = 0.16$ GeV) we have examined the effects of the following three forms:

$$(i) \quad T(r, t)|_{t < \tau_0} = T_0 \cdot (t/\tau_0)^2, \quad r < R \\ = 0, \quad r \geq R, \quad (10b)$$

$$(ii) \quad T(r, t)|_{t < \tau_0} = T_0 \cdot (t/\tau_0)^{1/2}, \quad r < R \\ = 0, \quad r \geq R, \quad (10c)$$

and also the extreme situation

$$(iii) \quad T(r, t)|_{t < \tau_0} = T_0, \quad r < R \\ = 0, \quad r \geq R. \quad (10d)$$

We now write down the formula for dN/dM :

$$dN/dM = 2\pi \int_0^t dt \int_0^r r dr \int_{-t}^t dz [dN(T(r, z, t))/dM d^4x] \quad (11)$$

with

$$dN(T)/dM d^4x = dN(T)/dM d^4x|_Q, \quad \text{if } T > T_c \text{ for } \tau \geq \tau_0 \\ \text{and for all values of } T \text{ if } \tau < \tau_0 \\ = dN(T)/dM d^4x|_{\text{pion}}, \quad \text{if } T_F < T \leq T_c, \tau \geq \tau_0 \\ \text{where } T_F = \text{Freeze-out temperature} = m_\pi \\ = 0, \text{ if } T_F \geq T, \tau \geq \tau_0.$$

Observe that the use of $dN(T)/dM d^4x|_Q$ for $\tau_0 > \tau > 0$ stems from the assumption that the two nuclei 'collide' at $t = z = 0$, triggering a 'phase-transition' from nucleons (hadrons) to quasi-freely moving quarks and gluons at that instant of time. This is a reasonable assumption only for ultra-relativistic energies ($E_{\text{beam}}/A > 50$ GeV). The situation, in reality, is quite complicated (Kajantie *et al* 1983); because of the non-zero thickness of the Lorentz-contracted nuclei, the 'collision process' consumes an amount of time $t \approx 2RA/E_{\text{beam}}$ which is about 1 fm for $R \approx 7$ fm and $E_{\text{beam}}/A \approx 14$ GeV. Thus nucleons (presumably along with pions) and streams of quarks and gluons co-exist in this $0 < t < 2RA/E_{\text{beam}}$ period.

4. Results and conclusion

Our results for the dimuon mass spectrum are presented in figures 3 and 4. Use has been made of (10a) in arriving at these results. In both figures $\tau_0 = 1$ fm and the t -integration is truncated at $t = 15$ fm. Since for $\tau_0 = 1$ fm freeze-out occurs around $\tau \leq 4$ fm (for $T_0 \leq 0.23$ GeV), the value of $|z|$ at which freeze-out occurs at $t = 15$ fm is $|z_F| \gtrsim 14$ fm. The fluid elements having such a $|z|$ may be considered to belong to the leading clusters and are therefore left out of consideration here.

The critical temperature T_c is 0.16 GeV for figure 3. The least-square-fit parameters A, B are evaluated at $T = 0.17$ GeV for this figure. The curves have the same general characteristics as those reported by Chin, except for the following two features:

(i) At the lower value of T_0 ($= 0.15$ GeV) the low mass ($M \lesssim 0.35$ GeV) contribution is significant (practically absent in Chin's model). The reason for this is our assumptions

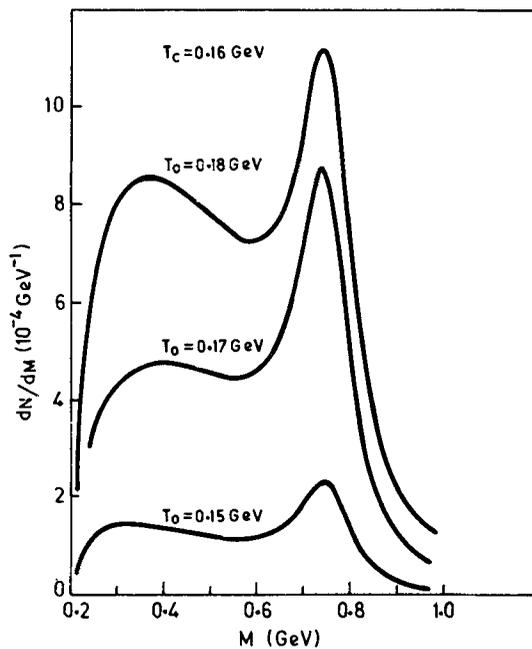


Figure 3. The dimuon mass-spectrum for three initial temperatures $T_0 = 0.15, 0.17$ and 0.18 GeV . The assumed deconfinement temperature is $T_c = 0.16 \text{ GeV}$. $\tau_0 = 1 \text{ fm}$ and $R = 7 \text{ fm}$.

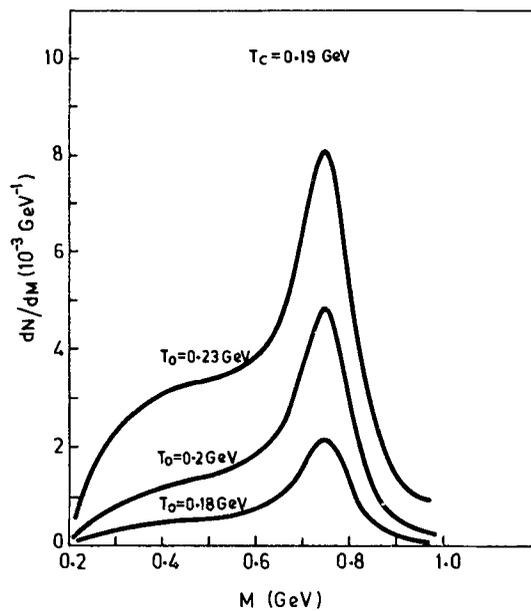


Figure 4. The dimuon mass-spectrum for three initial temperatures $T_0 = 0.18, 0.2$ and 0.23 GeV . $T_c = 0.19 \text{ GeV}$; $\tau_0 = 1 \text{ fm}$; $R = 7 \text{ fm}$.

embodied in (10a) and (11), which imply that between $\tau = 0$ and $\tau = \tau_0$ only contributions from the quark-gluon (QG) phase occur.

(ii) However at the upper value of T_0 ($= 0.18$ GeV) the relative contribution of the quark-gluon phase is less than that reported by Chin. The reason for this is that the cylindrical plasma cools faster than the spherical plasma (Baym *et al* 1983), so that in the former case, the QG phase exists for a shorter period of time (A minor cause for the relatively smaller contribution is the underestimation (by a few percent) of the temperature in the region where the analytic approximation is supposed to work—see figure 1).

The critical temperature T_c is 0.19 GeV for figure 4. The least-square-fit parameters are evaluated at $T = 0.21$ GeV. No new comments need be added over and above Chin's remarks and our preceding comments.

In figures 3 and 4 τ_0 was set equal to 1 fm and the calculations were performed for $R = 7$ fm (uranium-uranium collision). In figure 5 we present a comparison of the mass-spectrum for $\tau_0 = 1$ fm and $\tau_0 = 2$ fm (both for $R = 7$ fm) and also add in figure 5 the mass-spectrum for $^{12}\text{C}-^{12}\text{C}$ collision ($R = 2.75$ fm) with $\tau_0 = 1$ fm. All the curves are for $T_0 = 0.18$ GeV; $T_c = 0.16$ GeV. It may be noted that the shapes of the three curves are rather similar. The effect of a smaller radius is to cut down the production four-volume ($\sim R^2$). On the other hand the effect of a larger τ_0 is to increase this four-volume, since, as shown by Bjorken (1983) T/T_0 behaves as $\sim (\tau_0/\tau)^{1/3}$ so that it takes a corresponding longer time for the fluid to cool down below T_c : because of this reason we have truncated the t -integration at 30 fm for $\tau_0 = 2$ fm.

Finally, in figure 6, we compare the predicted mass-spectra for the various choices for

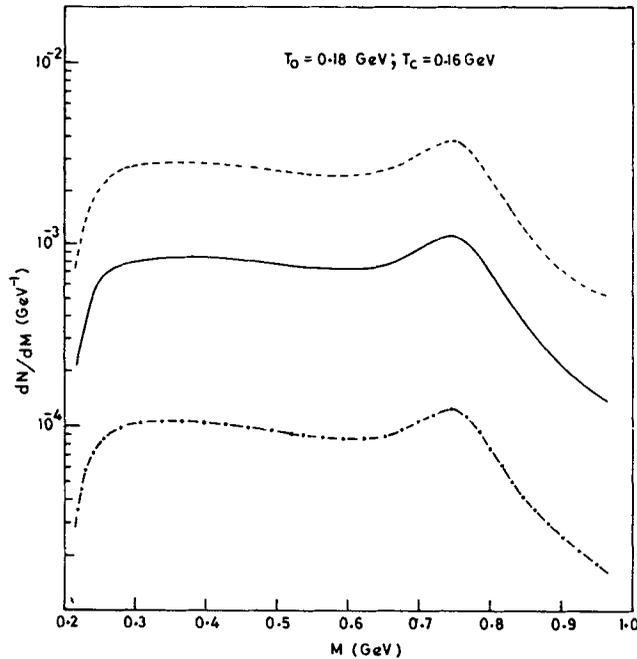


Figure 5. The dimuon mass spectra, corresponding to $T_c = 0.16$ GeV, for (i) $R = 7$ fm, $\tau_0 = 2$ fm—dashed curve; (ii) $R = 7$ fm, $\tau_0 = 1$ fm—solid curve; (iii) $R = 2.75$ fm, $\tau_0 = 1$ fm—dash-dotted curve. See text for details.

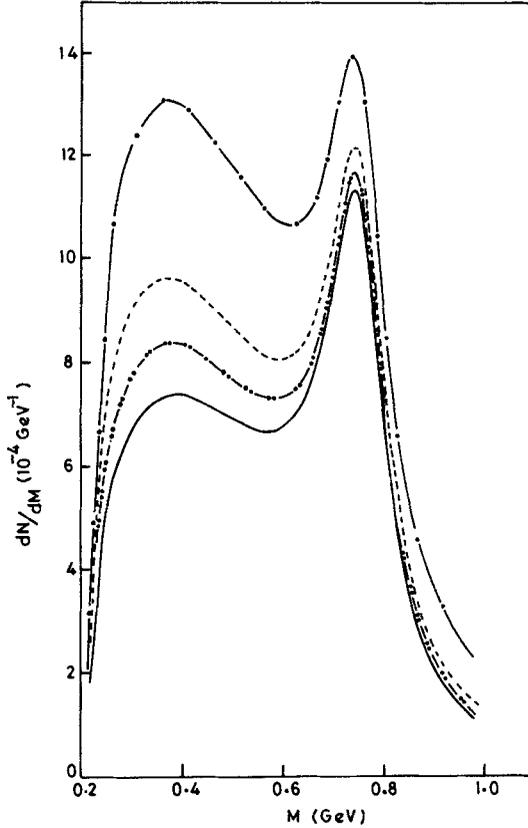


Figure 6. Dimuon mass spectra corresponding to the various possibilities in equations (10a) to (10d). Here $T_c = 0.16$ GeV, $T_0 = 0.18$ GeV, $R = 7$ fm and $\tau_0 = 1$ fm. Double dotted curve—equation (10a); solid curve—equation (10b); dashed curve—equation (10c); dash-dotted curve—equation (10d).

the temperature-distribution in $0 < \tau < \tau_0$. As expected, the shape of the mass-spectrum below $M \approx 0.6$ GeV is rather sensitive to the nature of the distribution. However the double-humped character is not lost even for the choice (10b), which tends to suppress the contribution from quark-antiquark annihilation occurring over the period $0 < \tau < \tau_0$. On the other hand (10d) gives the most pronounced contribution as anticipated.

Experimental data on heavy-ion-collision induced dilepton production are not yet available. However there exist some data on dimuon production in π -nucleus and p -nucleus collisions (Anderson *et al* 1976; Grannan *et al* 1978; Bunnell *et al* 1978; Haber *et al* 1980). These data suggest a double-hump (or at least a plateau plus hump) structure, with the humps coming at $M \approx 0.35$ GeV and $M \approx 0.75$ GeV. Chin has given a competent description of these and how they compare with his results. Because of the general similarity between Chin's results and our results, a fresh comparison of these experimental data with our theoretical calculations is somewhat redundant. We would however like to point out the following:

According to Haber *et al* (1980) the number of μ -pairs produced/collision (in π - p interactions) is $\sim 3-4 \times 10^{-6}$. Since in uranium-uranium collisions the production

four-volume is 10^2 – 10^3 times larger, we expect that the number of μ -pairs/collision would be $O(10^{-3}$ – $10^{-2})$, in accord with our theoretical calculations (for the higher values of T_0). This raises hopes that a confrontation of the theoretical results with experimental data, when available, would turn out to favour the assumed model.

In conclusion, then, a double-humped mass-distribution would, in general, testify to the production of a (locally) thermally equilibrated quark-gluon plasma; while the absence of even a low plateau in the dimuon mass-range $0.3 < M < 0.6$ GeV would, in our opinion, cast serious doubts on the validity of the Bjorken model of relativistic heavy-ion collisions. We also mention that there exist other signatures of a quark-gluon plasma; *e.g.* the transverse momentum distribution of dileptons and the ratio of the number of hard photon to dileptons emitted from the plasma are quite characteristic (Sinha 1983).

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References

- Anderson K J *et al* 1976 *Phys. Rev. Lett.* **37** 799
 Angelini C and Pazzi R 1982 *Phys. Lett.* **B113** 343
 Baym G, Friman B L, Blaizot J P, Soyeur M and Czyz W 1983 *Nucl. Phys.* **A407** 541
 Benaksas D *et al* 1972 *Phys. Lett.* **B39** 289
 Bjorken J 1983 *Phys. Rev.* **D27** 140
 Bunnell K *et al* 1978 *Phys. Rev. Lett.* **40** 136
 Chin S A 1982 *Phys. Lett.* **B119** 51
 Domokos G 1983 *Phys. Rev.* **D28** 123
 Gounaris G J and Sakurai J J 1968 *Phys. Rev. Lett.* **21** 244
 Grannan D M *et al* 1978 *Phys. Rev.* **D18** 3150
 Haber B *et al* 1980 *Phys. Rev.* **D22** 2107
 Hasegawa A 1983 *Progr. Theor. Phys.* **69** 689
 Hasegawa A and Tanaka H 1983 *Progr. Theor. Phys.* **69** 685
 Kajantie K, Montonen C and Pietarinen E 1981 *Z. Phys.* **C9** 253
 Kajantie K, Raitio R and Ruuskanen P V 1983 *Nucl. Phys.* **B222** 152
 Kuti J, Lukacs B, Polonyi J and Szlachanyi K 1980 *Phys. Lett.* **B95** 75
 Kuti J, Polonyi J and Szlachanyi K 1981 *Phys. Lett.* **B98** 199
 Landau L D 1965 *Collected papers of L D Landau*, (ed.) D Ter Haar (Oxford: Pergamon Press) p. 665
 Nikishov A I 1959 *JETP* **9** 937
 Shuryak E V 1980 *Phys. Rep.* **C61** 72
 Sinha B 1983 *Phys. Lett.* **B128** 91

Note added in proof:

The value of T over the flat portion in figure 1 is directly given in terms of Bjorken's scaling solutions viz. $T/T_0 = (\tau_0/t)^{1/3}$. Interpolation is thus avoidable.