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Baryon magnetic moments in quark-diquark model

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Abstract. The baryon magnetic moments in quark-diquark model are studied and it is found that the diquark (spin 1 and 0) mixing which may arise as a result of quark-gluon interaction inside a hadron, leads to a good agreement of theory with experiment.

Keywords. Baryon magnetic moments; quark.

1. Introduction

Spurred by recent improved measurements of hyperon magnetic moments, a number of new calculations (Kamal 1978, Bohm and Teese 1979; Tomozawa 1978; Franklin 1979, 1980; Geffen and Wilson 1980; Isgur and Karl 1980; Verma 1980; Lichtenberg 1981; Bohm *et al* 1982) of these moments have been made within the framework of the quark model. With the quark mass ratio determined from baryon masses, the simple quark model (De Rujula *et al* 1975) gave $\mu(\Lambda) = -0{\cdot}61$ in excellent agreement with experiment. But the measurements of other hyperon magnetic moments (Cox *et al* 1981) have clearly demonstrated that these cannot be fitted within the simple quark model. As a matter of fact the simple SU(6) approach is unable to explain some other properties like the neutron charge radius, g_A/g_V ratio as well. All these observations indicate SU(6) violation. Due to the lack of complete knowledge of hadron dynamics, such effects at low energy can at best be treated phenomenologically guided by quantum chromodynamics.

In the present investigation we study the effects of baryon wavefunction modification on the magnetic moments. We work in the framework of quark-diquark model for baryons. This model has been used to study some properties of the baryons. For example Zralek *et al* (1979) observed that SU(6) broken diquark model explains well the SU(6) violation in several static and dynamic properties like the total and differential cross-sections for a large body of quasi two-body reactions, neutron charge radius, neutron-proton structure functions etc. In the diquark model, a baryon is assumed to be made up of a diquark and a quark. The diquark can be in spin one or spin zero states. The SU(6) symmetry of the wavefunction can be broken by assigning different probabilities to these two types of diquarks which can be controlled through a mixing angle. We find that such a SU(6) breaking in the wavefunction leads to a good agreement between theory and experiment for the baryon magnetic moments.

2. Baryon wavefunction due to quark-quark interaction

The diquark model assumes that a baryon is made up of a diquark belonging to the symmetric 21 representation of SU(6) and a quark. A diquark can be in spin one or spin zero states. A diquark with two identical constituents exists only in spin one state while that made up of two different quarks can appear in both spin 1 and spin 0 multiplets. In the limit of exact SU(6) symmetry the probability amplitude for a diquark in a baryon to be in spin zero or spin one state will be governed by the CG coefficients involved. But due to SU(6) breaking caused by quark-gluon interaction, we may expect mixing between spin 1 and spin 0 states of a diquark in a baryon to become different from that given by the CG coefficient of SU(6). As we cannot calculate the coefficients in the wavefunction, we parametrize through a mixing angle in the following manner,

$$|ab\rangle_1 = |ab\rangle_1 \cos\phi_{ab} + |ab\rangle_0 \sin\phi_{ab},$$

$$|ab\rangle_0 = |ab\rangle_0 \cos\phi_{ab} - |ab\rangle_1 \sin\phi_{ab},$$

where the subscript denotes the spin of the diquark made up of two different quarks a and b and the parameter ϕ_{ab} contains the whole of dynamics. For the octet baryons, we have three types of diquarks containing ud, us and ds pairs and they would require three angles ϕ_{ud}, ϕ_{us} and ϕ_{ds} in the baryon wavefunction. Invoking isospin invariance we get

$$\phi_{us} = \phi_{ds} \, (= \phi)$$

and we fix $\phi_{ud}=0$ to preserve the successful results of SU(6) namely the ratio $\mu_p/\mu_n = -3/2$ in the nonstrange sector. This may be due to the fact that u and d have same mass and hence are identical with respect to strong interaction, so we demand the complete symmetry of the wavefunction while u and s being quarks with different masses, the symmetry may not be required.

In SU(6) symmetry, the wavefunction for the Σ^+ hyperon, for example, is given by

$$|\Sigma^+\uparrow\rangle = \frac{1}{\sqrt{6}} \sqrt{2}\left(\sqrt{\tfrac{2}{3}}\, s_1^+ s\downarrow - \frac{1}{\sqrt{3}} s_1^0 s\uparrow\right) - \left(\sqrt{\tfrac{2}{3}}\, s_4^+ u\downarrow - \frac{1}{\sqrt{3}} s_4^0 u\uparrow\right)$$

$$+ \frac{1}{\sqrt{2}} t_2 u\uparrow,$$

where

$$s_1^+ = u\uparrow u\uparrow,$$

$$s_1^0 = \frac{1}{\sqrt{2}}(u\uparrow u\downarrow + u\downarrow u\uparrow),$$

$$s_4^+ = \frac{1}{\sqrt{2}}(u\uparrow s\uparrow + s\uparrow u\uparrow),$$

$$s_4^0 = \tfrac{1}{2}\,(u\uparrow s\downarrow + u\downarrow s\uparrow + s\uparrow u\downarrow + s\downarrow u\uparrow),$$

$$t_2 = \tfrac{1}{2}\,(u\uparrow s\downarrow - u\uparrow s\downarrow - s\uparrow u\downarrow + s\downarrow u\uparrow).$$

As a result of spin 1 and spin 0 mixing due to hadron dynamics, the wavefunctions for Σ^+ now becomes

$$|\Sigma^+\uparrow\rangle = \frac{1}{\sqrt{6}}\Big[\sqrt{2}\left\{\sqrt{\frac{2}{3}}s_1^+ s\downarrow - \frac{1}{\sqrt{3}}s_1^0 s\uparrow\right\}$$

$$-(\cos\phi + \sqrt{3}\sin\phi)\times\left\{\sqrt{\frac{2}{3}}s_4^+ u\downarrow - \frac{1}{\sqrt{3}}s_4^0 u\uparrow\right\}\Big]$$

$$+\frac{1}{\sqrt{2}}\,t_2\,u\uparrow\left(\cos\phi - \frac{\sin\phi}{\sqrt{3}}\right).$$

The wavefunctions for all the baryons are given in the appendix.

3. Magnetic moments

Taking the expectation value of the magnetic moment operator

$$\frac{e_i}{2m_i}\bar{q}_i\,\vec{\sigma}\,q_i,$$

Table 1.

Moment	Present model* expression $xe/2m_u$	$\phi = 0°$ $y = 0{\cdot}63$	$\phi = 20°$, $y = 0{\cdot}79$	$\phi = 25°$, $y = 0{\cdot}88$	$\phi = 30°$, $y = 1$	exp ($n\,m$)
p	1	2·79*	2·79*	2·79*	2·79*	2·793
n	$-2/3$	− 1·86	− 1·86	− 1·86	1·86	− 1·913
Λ	$-y/3 + (y + 1/2)\mathrm{B}$	− 0·61*	− 0·61*	− 0·61*	− 0·61*	− 0·6138 ± 0·0047
Σ^+	$\frac{8+y}{9} - (y+2)\mathrm{B}$	2·69	2·46	2·33	2·16	2·33 ± 0·13
Σ^0	$\frac{2+y}{9} - (y+1/2)\mathrm{B}$	0·82	0·74	0·69	0·61	—
Σ^-	$\frac{y-4}{9} + (1-y)\mathrm{B}$	− 1·04	− 0·98	− 0·95	− 0·93	− 0·89 ± 0·14
Ξ^0	$\frac{-2-4y}{9} + (y+2)\mathrm{B}$	− 1·44	− 1·33	− 1·28	− 1·22	− 1·236 ± 0·014
Ξ^-	$\frac{1-4y}{9} + (y-1)\mathrm{B}$	− 0·50	− 0·70	− 0·79	− 0·93	− 0·75 ± 0·07
$\Sigma\Lambda$	$\frac{1}{\sqrt{3}}\left(1 - \frac{9}{2}\mathrm{B}\right)$	1·61	1·36	1·23	1·08	1·82 $^{+0{\cdot}25}_{-0{\cdot}18}$
Ω	$-y$	− 1·84	− 2·21	− 2·45	− 2·79	—

*$\mathrm{B} = 8\sin^2\phi/27$

in the baryon state expressed by the wavefunction given in the appendix, we obtain the corresponding value of the magnetic moment.

All the baryon magnetic moments are then expressed in terms of y *i.e.* m_u/m_s mass ratio and the diquark mixing angle as displayed in table 1. We compute the values of magnetic moment for different values of angle ϕ starting from the conventional SU(6) value of $\phi = 0$ to $\phi_{max} = 30°$ required by $\mu(\Lambda)$ in the limit of SU(3) symmetry ($y = 1$). The magnetic moments specially of Σ^+, Ξ^0 and Ξ^- are in good agreement for the value of ϕ lying between 20° and 30°. $\mu(\Sigma^-)$ is not affected much and stays around -1 in better agreement with a recent value $-0{\cdot}89 \pm 0{\cdot}14$ obtained by Ramieka (1981). The $\Sigma^0 - \Lambda^0$ transition moment is lowered as in any other model incorporating SU(3) breaking only. Ω^- is also calculated for different cases. It appears that the wavefunction modification due to presence of gluons inside the baryons, helps to bring the magnetic moment values closer to the experimental values.

Appendix

Diquark states SU(3) × SU(2) ⊂ SU(6)

(i) (6, 3) ⊂ 21.

$$s_1^+ = u{\uparrow}u{\uparrow}, \qquad s_1^0 = \frac{1}{\sqrt{2}}(u{\uparrow}u{\downarrow} + u{\downarrow}u{\uparrow})$$

$$s_2^+ = \frac{1}{\sqrt{2}}(u{\uparrow}d{\uparrow} + d{\uparrow}u{\uparrow}), \qquad s_2^0 = \tfrac{1}{2}(u{\uparrow}d{\downarrow} + u{\downarrow}d{\uparrow} + d{\uparrow}u{\downarrow} + d{\downarrow}u{\uparrow}),$$

$$s_3^+ = d{\uparrow}d{\uparrow}, \qquad s_3^0 = \frac{1}{\sqrt{2}}(d{\uparrow}d{\downarrow} + d{\downarrow}d{\uparrow}),$$

$$s_4^+ = \frac{1}{\sqrt{2}}(u{\uparrow}s{\uparrow} + s{\uparrow}u{\uparrow}), \qquad s_4^0 = \tfrac{1}{2}(u{\uparrow}s{\downarrow} + u{\downarrow}s{\uparrow} + s{\uparrow}u{\downarrow} + s{\downarrow}u{\uparrow}),$$

$$s_5^+ = \frac{1}{\sqrt{2}}(d{\uparrow}s{\uparrow} + s{\uparrow}d{\uparrow}), \qquad s_5^0 = \tfrac{1}{2}(d{\uparrow}s{\downarrow} + d{\downarrow}s{\uparrow} + s{\uparrow}d{\downarrow} + s{\downarrow}d{\uparrow}),$$

$$s_6^+ = s{\uparrow}s{\uparrow}, \qquad s_6^0 = \frac{1}{\sqrt{2}}(s{\uparrow}s{\downarrow} + s{\downarrow}s{\uparrow}).$$

(ii) (3*, 1) ⊂ 21

$$t_1 = \tfrac{1}{2}(u{\uparrow}d{\downarrow} - u{\downarrow}d{\uparrow} - d{\uparrow}u{\downarrow} + d{\downarrow}u{\uparrow}),$$

$$t_2 = \tfrac{1}{2}(u{\uparrow}s{\downarrow} - u{\downarrow}s{\uparrow} - s{\uparrow}u{\downarrow} + s{\downarrow}u{\uparrow}),$$

$$t_3 = \tfrac{1}{2}(d{\uparrow}s{\downarrow} - d{\downarrow}s{\uparrow} - s{\uparrow}d{\downarrow} + s{\downarrow}d{\uparrow}).$$

Baryon wave functions

$$|p\uparrow\rangle = \frac{1}{\sqrt{18}}\{2\, s_1^+ d\downarrow - \sqrt{2}\, s_1^0 d\uparrow - \sqrt{2}\, s_2^+ u\downarrow + s_2^0 u\uparrow + 3\, t_1 u\uparrow\},$$

$$|n\uparrow\rangle = \frac{1}{\sqrt{18}}\,[\sqrt{2} s_2^+ d\downarrow - s_2^0 d\uparrow - 2\, s_3^+ u_\downarrow + \sqrt{2}\, s_3^0 u\uparrow + 3\, t_1 d\uparrow],$$

$$|\Lambda\uparrow\rangle = \frac{1}{\sqrt{12}}\left[\left(\cos\phi - \frac{\sin\phi}{\sqrt{3}}\right)(\sqrt{2}\, s_4^+ d\downarrow - s_4^0 d\uparrow - \sqrt{2}\, s_5^+ u\downarrow + s_5^0 u\uparrow)\right.$$

$$\left. + (\cos\phi + \sqrt{3}\,\sin\phi)\,(t_2 d\uparrow - t_3 u\uparrow) + 2 t_1 s\uparrow\right],$$

$$|\Sigma^+\uparrow\rangle = \frac{1}{\sqrt{18}}\left[(2\, s_1^+ s\downarrow - \sqrt{2}\, s_1^0\, s\uparrow) - (\cos\phi + \sqrt{3}\,\sin\phi)\,(\sqrt{2}\, s_4^+\, u\downarrow\right.$$

$$\left. - s_4^0\, u\uparrow) + 3\left(\cos\phi - \frac{\sin\phi}{\sqrt{3}}\right) t_2\, u\uparrow\right],$$

$$|\Sigma^-\uparrow\rangle = \frac{1}{\sqrt{18}}\left[(2\, s_3^+ s\downarrow - \sqrt{2}\, s_3^0 s\uparrow) + (-\sqrt{2}\, s_5^+ d\downarrow + s_5^0 d\uparrow)\,(\cos\phi\right.$$

$$\left. + \sqrt{3}\,\sin\phi) + 3\, t_3 d\uparrow\left(\cos\phi - \frac{\sin\phi}{\sqrt{3}}\right)\right],$$

$$|\Sigma^0\uparrow\rangle = \tfrac{1}{3}\,(-\sqrt{2}\, s_2^+ s\downarrow + s_2^0 s\uparrow) + \tfrac{1}{6}\,(\cos\phi + \sqrt{3}\sin\phi)\,(\sqrt{2}\, s_5^+ u\downarrow - s_5^0 u\uparrow$$

$$+ \sqrt{2}\, s_4^+ d\downarrow - s_4^0 d\uparrow) - \tfrac{1}{2}\left(\cos\phi - \frac{\sin\phi}{\sqrt{3}}\right)(t_3 u\uparrow + t_2 d\uparrow),$$

$$|\Xi^0\uparrow\rangle = \frac{1}{\sqrt{18}}\left[(\sqrt{2}\, s_4^+ s\downarrow - s_4^0 s\uparrow)\,(\cos\phi - \sqrt{3}\,\sin\phi)\right.$$

$$\left. - 2\, s_6^+ u\downarrow + \sqrt{2}\, s_6^0 u\uparrow + 3\left(\cos\phi + \frac{\sin\phi}{\sqrt{3}}\right) t_2 s\uparrow\right],$$

$$|\Xi^-\uparrow\rangle = \frac{1}{\sqrt{18}}\left[(\sqrt{2}\, s_5^+\, s\downarrow - s_5^0 s\uparrow)\,(\cos\phi - \sqrt{3}\,\sin\phi)\right.$$

$$\left. - 2\, s_6^+ d\downarrow + \sqrt{2}\, s_6^0 d\uparrow + 3\left(\cos\phi + \frac{\sin\phi}{\sqrt{3}}\right) t_3 s\uparrow\right].$$

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