

A mean-field, effective medium theory of random binary alloys III. The Ising model with competing interactions

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Abstract. The Ising model with competing interactions is studied in a mean field effective medium approach. The phase diagram of such model alloys is studied. We conclude that for all ratios of the competing interaction moments, a spin glass phase always exists at low temperatures for certain concentration regimes.

Keywords. Random magnetic alloys; spin glasses; Ising model, competing interactions.

1. Introduction

The phases of the quenched random alloy systems one of whose components is magnetic (the so-called spin glass alloys) have been studied in great detail (Edwards and Anderson 1975; Plefka 1976; Kaneyoshi 1976; Morita and Horiguchi 1976; Mookerjee 1978, 1979). At the origin of the spin glass phase, which all of these alloys show at low temperatures and low concentrations of the magnetic constituent, is the fluctuating RKKY interaction. Focus on binary and ternary alloys all of whose constituents are magnetic began about the same time (Matsubara 1974; Katsura and Matsubara 1974; Aharony 1975, 1978; Luttinger 1976; Eggarter and Eggarter 1977; Oguchi and Ueno 1979; Katsura *et al* 1979; Medvedev and Zaborov 1982a, b). Here the dominant interaction is the short-ranged direct exchange. The sign of this interaction is governed by the type of atoms occupying the two neighbouring sites. These interactions could thus compete with one another, having random signs, and a spin glass phase may occur in a concentration range where there is a transition from a ferromagnetic low temperature phase to an antiferromagnetic one. There are ample experimental evidences of the spin glass phase in these alloys. Menshikov *et al* (1981) have shown, in the experimental phase diagram spin glass and mixed phases in γ -FeNiCr alloys. γ -FeNiMn alloys with a high iron concentration have a state very similar to that of amorphous magnets, characterised by a developed short-ranged ferro- and antiferromagnetic ordering which changes to a long-ranged antiferromagnetic at a higher and a long ranged ferromagnetic at a lower iron concentrations (Menshikov and Teplykh 1977). In a FeNi binary alloy with 70.3% Fe, a drop in the magnetisation and asymmetrically placed hysteresis loop about the origin was observed by Rode *et al* (1975) at low temperatures. This behaviour is peculiar to spin glass phases. In CrFe (Shull and Beck 1974) between 15 to 20 at % Fe 'micro-

magnetic' behaviour has been observed. Aitken *et al* (1982) have found spin glass behaviour with a susceptibility cusp at 8°K in a NiMn alloy with 26% Mn. They also report displaced hysteresis loops at 4°K in NiMn (22% Mn) in field cooling experiments. Low temperature critical neutron scattering in NiMn suggests that there may be spin glass type behaviour in certain concentration regions (Kuzmin and Menshikov 1979). But the spin glass type behaviour revealed in the alloy systems mentioned above may not be exactly identical to the canonical spin glasses (AuFe). In fact, Menshikov *et al* (1981) in explaining their experimental phase diagrams of γ -FeNiMn mentioned about the existence of non-collinear ferromagnets or more generally a mixed phase. In an earlier work (Mookerjee 1979) on canonical spin glasses, the probable existence of such non-collinear or randomly-canted phases was also indicated.

In this paper we report the phases of such binary systems with mixed interactions. We shall adopt the mean-field, effective medium approach of Mookerjee (1978, 1979). We do not consider a homogeneous, but a species-dependent local magnetisation and local field distribution. Some of the complicated interpenetrating sublattice ideas introduced by Katsura *et al* (1979) and Medvedev and Zaborov (1982a) are seen not to be essential to the physical understanding of the phases. The phase diagram obtained is qualitative similar to the studied mentioned above, but there are minor but not unimportant differences which we shall point out subsequently. It is also argued that our results are more in keeping with physical interpretation of the known experimental data.

2. The local internal field

We first consider a random Ising model where

$$H = -\frac{1}{2} \sum_k \sum_{j \neq k} J(|\mathbf{r}_k - \mathbf{r}_j|) S_k S_j;$$

here J is the direct exchange interaction, decaying fast with $|\mathbf{r}_k - \mathbf{r}_j|$. It is unnecessary to assume that J is nearest neighbour alone, that can be incorporated as a special case. However, J is random and depends upon the atoms at the sites \mathbf{r}_k and \mathbf{r}_j . In the binary alloy AB , it can take three values J_{AA} , J_{AB} and J_{BB} .

The free energy is

$$\begin{aligned} F = \frac{1}{2} & \left[\sum_{ij \in AA} \sum J_{ij}^{AA} \sigma_i^A \sigma_j^A + \sum_{ij \in AB} \sum J_{ij}^{AB} (\sigma_i^A \sigma_j^B + \sigma_i^B \sigma_j^A) \right. \\ & + \sum_{ij \in BB} \sum J_{ij}^{BB} \sigma_i^B \sigma_j^B \left. \right] - k_B T \left(\sum_{i \in A} \ln \cosh (\beta h_i^A) \right. \\ & \left. + \sum_{i \in B} \ln \cosh (\beta h_i^B) \right) \end{aligned} \quad (1)$$

here σ_i are the random local magnetisations $\langle S_i \rangle$ taking the values σ_i^A or σ_i^B depending whether the site r_i has an A or B atom. While the stable phases are found by minimising the free energy with respect to the local magnetisations. This yields a set of equations

$$\left. \begin{aligned} \sum J_{ik}^{AA} [\sigma_i^A - \tanh(\beta h_i^A)] + \sum J_{ik}^{AB} [\sigma_i^B - \tanh(\beta h_i^B)] &= 0, \\ \sum J_{ik}^{AB} [\sigma_i^A - \tanh(\beta h_i^A)] + \sum J_{ik}^{BB} [\sigma_i^B - \tanh(\beta h_i^B)] &= 0, \end{aligned} \right\} \quad (2)$$

since the J 's are independent, this leads to

$$\sigma_i^A = \tanh(\beta h_i^A) \text{ and } \sigma_i^B = \tanh(\beta h_i^B). \quad (3)$$

While

$$h_i^A = \sum_j J_{ij}^{AA} \sigma_j^A + \sum_j J_{ij}^{AB} \sigma_j^B \text{ and } h_i^B = \sum_j J_{ij}^{BA} \sigma_j^A + \sum_j J_{ij}^{BB} \sigma_j^B, \quad (4)$$

are the random 'local' internal mean fields.

Unlike the canonical spin glass alloys (Mookerjee 1978, 1979), the probability density of the local internal field depends upon whether the site is occupied by a A or a B atom. The correspondence is $h_i^B = \sigma_i^B = 0$, if the atom B is nonmagnetic.

We shall now proceed to evaluate the conditional probability density $P_A(h_i^A)$ of the local internal field at r_i , provided the site is occupied by an A atom. $P_B(h_i^B)$ follows in an exactly similar manner

$$P_A(h_i^A) = \sum_{\mathbf{r}_1 \dots \mathbf{r}_{N-1}} \Pr(\mathbf{r}_1 \dots \mathbf{r}_{N-1}) \delta[h_i^A - \sum_j J(|\mathbf{r}_i - \mathbf{r}_j|) \sigma_j].$$

We assume first that there is no clustering or chemical correlations so that any atom can occupy any site with equal probability. The site r_i is occupied by an A atom, while the remaining $N - 1$ atoms of which $N_1 - 1$ are A and $N_2 = N - N_1$ are B are distributed among the remaining $N - 1$ sites. In the thermodynamic limit $N_1 - 1/N \rightarrow c_A$ and $N_2/N \rightarrow c_B$ the respective concentrations of the A and B constituents.

$$\begin{aligned} P_A(h_i^A) &= (\frac{1}{2}\pi) \int dk \exp[ikh_i^A] \sum_{\mathbf{r}_1 \dots \mathbf{r}_{N-1}} \sum_{\mathbf{r}_1 \dots \mathbf{r}_{N-1}} (1/N-1)^{N_1+N_2-1} \\ &\int \dots \int \prod_{j \in A} dh_j^A \delta[h_j^A - \sum J_{ij}^{AA} \sigma_j^A - \sum J_{ij}^{AB} \sigma_j^B] \prod_{j \in B} dh_j^B \\ &\delta[h_j^B - \sum J_{ij}^{AB} \sigma_j^A - \sum J_{ij}^{BB} \sigma_j^B] \times \\ &\exp[-ik(\sum J_{ij}^{AA} f(h_j^A) + \sum J_{ij}^{AB} f(h_j^B))], \end{aligned} \quad (5)$$

where $f(x) = \tanh(\beta x)$. We now replace the δ functions by their configuration averages (Klein 1968; Mookerjee 1978) so that the product term in (5) reduces to

$$\prod_{j \in \alpha, \alpha = A, B} P_{\alpha}(h_j^{\alpha}).$$

A little algebra then yields

$$P_A(h_i^A) = (\frac{1}{2}\pi)^{-1} \int dk \exp(ikh_i^A) [1 - F_1(k)/(N-1)]^{N_1-1} [1 - F_2(k)/(N-1)]^{N_2}$$

where
$$F_1(k) = \sum_{\mathbf{r}} \int dh_j^A P_A(h_j^A) [1 - \exp\{-ikJ_{ij}^{AA} f(h_j^A)\}],$$

$$F_2(k) = \sum_{\mathbf{r}} \int dh_j^B P_B(h_j^B) [1 - \exp\{-ikJ_{ij}^{AB} f(h_j^B)\}]. \quad (6)$$

In the thermodynamic limit,

$$P_A(h_i^A) = (1/2\pi) \int dk \exp [ikh_i^A - (c_A F_1 + c_B F_2)].$$

This is a complicated nonlinear integral equation. However, in case the spatial moments $I_{\alpha\alpha}^{(n)} = \sum_R [J^{\alpha\alpha}(R)]^n$ decrease rapidly with n , we may expand the exponentials in the F 's and neglect all I_n $n \geq 3$. The integral then can be exactly carried through. Mookerjee (1978) showed the validity of this procedure for the RKKY interaction, and a very similar argument can also be carried out for any interaction falling as R^{-m} , $m > 3$. In any situation, if I_3, I_4 etc are not negligible then we may estimate all the moments of the distribution. Instead of having the simple Gaussian form, as shown subsequently, we shall have a distribution with skewness and kurtosis.

This yields

$$P_A(h_i^A) = (2\pi J_1^{A^2})^{-1/2} \exp [-(h_i^A - J_0^A)^2 / (2J_1^{A^2})] \quad (7)$$

where
$$J_0^A = c_A m_A \sum_R J^{AA}(R) + c_B m_B \sum_R J^{AB}(R),$$

$$J_1^{A^2} = c_A q_A \sum_R J^{AA^2}(R) + c_B q_B \sum_R J^{AB^2}(R),$$

where $m_{\alpha} = [\sigma_{\alpha}^2]$ and $q_{\alpha} = [\sigma_{\alpha}^4]$ $\alpha = A$ or B , are the configuration-averaged Edwards-Anderson type order parameters for the A and B type of atoms. Similarly we may obtain:

$$P_B(h_i^B) = (2\pi J_1^{B^2})^{-1/2} \exp [-(h_i^B - J_0^B)^2 / (2J_1^{B^2})]. \quad (8)$$

The J_0^B and $J_i^{B^2}$ are defined as before replacing A by B at r_i . From (4), (7) and (8) we obtain the equations for the order parameters

$$\begin{aligned}
 m_A &= (2\pi)^{-1/2} \int dz \exp(-1/2 z^2) \tanh \beta [(T_{11}m_A + T_{12}m_B) \\
 &\quad + (T_{\sigma 1}^2 q_A + T_{\sigma 2}^2 q_B)^{1/2} z], \\
 m_B &= (2\pi)^{-1/2} \int dz \exp(-1/2 z^2) \tanh \beta [(T_{21}m_A) + (T_{22}m_B) \\
 &\quad + (T_{\sigma 3}^2 q_A + T_{\sigma 4}^2 q_B)^{1/2} z], \\
 q_A &= (2\pi)^{-1/2} \int dz \exp(-1/2 z^2) \tanh^2 \beta [(T_{11}m_A + T_{12}m_B) \\
 &\quad + (T_{\sigma 1}^2 q_A + T_{\sigma 2}^2 q_B)^{1/2} z], \\
 q_B &= (2\pi)^{-1/2} \int dz \exp(-1/2 z^2) \tanh^2 \beta [(T_{21}m_A + T_{22}m_B) \\
 &\quad + (T_{\sigma 3}^2 q_A + T_{\sigma 4}^2 q_B)^{1/2} z],
 \end{aligned} \tag{9}$$

where $k_B T_{\alpha\alpha'} = c_{\alpha'} \sum_R J^{\alpha\alpha'}(R)$ with $\alpha\alpha' = 1$ for A , and 2 for B

and $k_B^2 T_{\sigma n}^2 = c_{\alpha} \sum_R J^{\alpha'\alpha''}(R)^2$ with $n = 1 = \alpha \in A, \alpha'\alpha'' \in AA$,

$$n = 2 = \alpha \in B, \alpha'\alpha'' \in AB,$$

$$n = 3 = \alpha \in A, \alpha'\alpha'' \in BA,$$

$$n = 4 = \alpha \in B, \alpha'\alpha'' \in BB.$$

3. The phase diagram

Equations (9) form the basis of our analysis of the phases. To start with we study the paramagnetic phase boundary. Near the vicinity of this all the four quantities m_A, m_B, q_A and q_B are small, so that we may expand the $\tanh(x)$ and $\tanh^2(x)$ functions in the integrand carry out the Gaussian integrals. This converts the complicated integral equations to tractable algebraic ones.

$$\begin{aligned}
 m_A &= \frac{1}{T} (T_{11} m_A + T_{12} m_B) - (1/3T^3) (T_{11} m_A + T_{12} m_B)^3 \\
 &\quad - \frac{1}{T^3} (T_{11} m_A + T_{12} m_B) (T_{\sigma 1}^2 q_A + T_{\sigma 2}^2 q_B),
 \end{aligned}$$

$$\begin{aligned}
m_B &= \frac{1}{T} (T_{21} m_A + T_{22} m_B) - (1/3T^3) (T_{21} m_A + T_{22} m_B)^3 \\
&\quad - \frac{1}{T^3} (T_{21} m_A + T_{22} m_B) (T_{g3}^2 q_A + T_{g4}^2 q_B), \\
q_A &= \frac{1}{T^2} (T_{g1}^2 q_A + T_{g2}^2 q_B) + \frac{1}{T^2} (T_{11} m_A + T_{12} m_B)^2, \\
q_B &= \frac{1}{T^2} (T_{g3}^2 q_A + T_{g4}^2 q_B) + \frac{1}{T^2} (T_{21} m_A + T_{22} m_B)^2. \tag{10}
\end{aligned}$$

The above equations contain, as their possible solutions, an inhomogeneous situation where the A and B sublattices order at different temperatures. Such situations were also inherent in the earlier work of Katsura *et al* (1979). In such cases complex situations where the A and B sublattices order differently: *e.g.* the A atoms order ferromagnetically while the B atoms antiferromagnetically. In ordered crystalline alloys this is a possibility. However, in the random alloys under study all experimental evidence points to unique Curie, Neel or spin glass temperatures, where the entire system orders in a particular way. The experimentally measured magnetisation is a bulk average $m = c_A m_A + c_B m_B$ and it is this which is the relevant order parameter. Moreover as this magnetisation vanishes at a unique Curie temperature T_c , we should seek solutions of the type

$$m_B = \eta(T, c) m_A \text{ and } q_B = \mu(T, c) q_A.$$

With this substitution, the set of equations (10) provide four equations for the four unknowns m_A , q_A , η and μ . A possible solution is $m_A = q_A = 0$ ($= m_B = q_B = 0$). This corresponds to the paramagnetic phase. The non-zero solution for m_A is given by

$$\begin{aligned}
m_A^2 &= [3T^3(T^2 - T_{g1}^2 - \mu T_{g2}^2)/(3T^2 - 2T_{g1}^2 - 2\mu T_{g2}^2)] [(T - T_{11} - \eta T_{12}) / \\
&\quad T(T_{11} + \eta T_{12})^3] \\
&= [3T^3(\mu T^2 - \mu T_{g4}^2 - T_{g3}^2)/(3\mu T^2 - 2T_{g3}^2 - 2\mu T_{g4}^2)] \\
&\quad [(\eta T - T_{21} - \eta T_{22})/T(T_{21} + \eta T_{22})^3]
\end{aligned}$$

Suppose that at T_c , $\eta = \eta_c$ and $\mu = \mu_c$, then T_c is that temperature at which this other solution vanishes. The above expressions may vanish if either of the two terms in the square brackets vanish. We shall later show that the vanishing of the first term occurs in those concentration ranges where we have a paramagnetic to a spin glass transition. In those regimes where $T_c > T_g$, these terms remain positive. Thus the equation for T_c is obtained from

$$T_c - T_{11} - \eta_c T_{12} = \eta_c T_c - T_{21} - \eta_c T_{22} = 0$$

Since $q_A = \beta^2(T_{11} + \eta T_{12})^2 m_A^2 / (1 - \beta^2(T_{g1}^2 + \mu T_{g2}^2))$ q_B also vanishes at the paramagnetic boundary. The above equation yields

$$T_c = \frac{1}{2}(T_{11} + T_{22}) \pm \frac{1}{2} [(T_{11} - T_{22})^2 + 4T_{12}T_{21}]^{1/2}.$$

If $T_{12}T_{21} - T_{11}T_{22} > 0$, then one of the above roots is positive and the other negative. The positive root gives the physical Curie temperature. For the nearest neighbour interaction we can show that the negative root is the negative of the Neel temperature. This can be easily seen if we rewrite the Hamiltonian in terms of the staggered spins $H = \frac{1}{2}\Sigma - J(R_i)\hat{S}_i \hat{S}_j$. For nearest neighbour interaction each $J_0 = ZJ$ is replaced by $-J_0 = -ZJ$ and the expression for T_N is $T_N = -\frac{1}{2}(T_{22} + T_{11}) \mp \frac{1}{2} [(T_{22} - T_{11})^2 + 4T_{21}T_{12}]^{1/2}$. The positive root of this has exactly the same magnitude as the negative root of T_c . For longer ranged forces, the T_N obtained from the negative root of T_c is not the Neel temperature and T_N has to be calculated from the Hamiltonian expressed in terms of the staggered magnetisation in a similar way.

In case $T_{12}T_{21} - T_{11}T_{22} < 0$, there is no negative root. The lower positive root has no physical significance and there is no antiferromagnetic regime.

For the spin glass boundary we must have $m_A = 0$, but $q_A \neq 0$. Such solutions are possible provided

$$T_g^2 - (T_{g1}^2 + \mu_g T_{g2}^2) = T_g^2 \mu_g - T_{g3}^2 - \mu_g T_{g4}^2 = 0,$$

$$\text{so that } T_g^2 = \frac{1}{2}(T_{g1}^2 + T_{g4}^2) \pm \frac{1}{2} [(T_{g1}^2 - T_{g4}^2)^2 + 4T_{g2}^2 T_{g3}^2]^{1/2}.$$

For the spin glass transition only the greater positive root has any significance. A spin glass transition from the paramagnetic phase will be observed only in those regions where the T_g lies above T_c or T_N . In the regions where this is not so the spin glass temperature T_g has no physical significance, as the small q expansion itself becomes invalid.

In another regime, *i.e.* $T=0$, equations (9) also reduce to soluble algebraic equation. $\tan h(x) \rightarrow 2H(x) - 1$ where $H(x)$ is the Heaviside step function, so that

$$m_A = \phi [(T_{11} + \eta T_{12}) m_A / (\sqrt{2}(T_{g1}^2 + T_{g2}^2)^{1/2})], q_A = 1, \quad (13)$$

$$\eta m_A = \phi [(T_{21} + \eta T_{22}) m_A / \sqrt{2}(T_{g3}^2 + T_{g4}^2)^{1/2}], \mu = 1,$$

where $\phi(x)$ is the error integral function. Equation (13) assume even simpler form in the neighbourhood of the spin-glass-ferromagnetic transition where m_A is small and $\phi(x) = (\sqrt{2/\pi}) x \exp(-x^2)$. Using this expansion for small m_A we obtain one root $m_A = 0$, this is relevant to the spin glass phase. Another non-zero solution

$$m_A^2 = [(a_1 + \eta a_2) - \sqrt{\frac{1}{2}\pi}] / [\sqrt{\frac{1}{2}\pi} (a_1 + \eta a_2)^2] = [(a_3 + \eta a_4) - \eta \sqrt{\pi/2} / \sqrt{\pi/2} (a_3 + \eta a_4)^2].$$

At the spin-glass, ferromagnetic boundary this second solution vanishes. This leads to an equation for the critical concentration after elimination of η :

$$F(c_0) = (\sqrt{\frac{1}{2}\pi} - a_1)(\sqrt{\frac{1}{2}\pi} - a_4) - a_2 a_3 = 0, \quad (14)$$

$$a_1 = T_{11}/[2(T_{\sigma 1}^2 + T_{\sigma 2}^2)^{1/2}], \quad a_2 = T_{12}/[2(T_{\sigma 1}^2 + T_{\sigma 2}^2)^{1/2}], \quad a_3 = T_{21}/[2(T_{\sigma 3}^2 + T_{\sigma 4}^2)]^{1/2},$$

and $a_4 = T_{22}/[2(T_{\sigma 3}^2 + T_{\sigma 4}^2)]^{1/2}$

At $c_A = 1$, $F(c) > 0$ and $m_A \neq 0$, this is expected since we do expect a ferromagnetic low temperature phase for the pure ferromagnetic A constituent ($J_{AA} > 0$). However, for different values of the three couplings J_{AA} , J_{AB} and J_{BB} (14) provides a simple equation for the determination of c_0 .

In the nearest neighbour interaction case the determination of the spin-glass anti-ferromagnetic boundary is exactly similar to the above and we have to replace the $J_{\alpha\alpha'}$, by $-J_{\alpha\alpha'}$.

In the remainder of the c - T plane we cannot obtain such simple solutions, but knowing the solutions at the paramagnetic boundary and the $T=0$ line, we may iteratively solve the integral equations (9) and fill out the phase diagram.

4. Results and discussion

We display two model cases: a symmetric one with $J_{AA}=1$, $J_{AB}=0.8$ and $J_{BB}=-1$ (in arbitrary units, $k_B=1$) and a asymmetric case with $J_{AA}=1$, $J_{AB}=2$, $J_{BB}=-6$. In each of the cases we take three possible ratios $K_{\alpha\alpha'}/J_{\alpha\alpha'}=0.65$, 0.75 and 0.8 [$J_{\alpha\alpha'} = \sum_R J(R)^{\alpha\alpha'}$ and $K_{\alpha\alpha'}^2 = \sum_R J^2(R)^{\alpha\alpha'}$]. The phase diagrams are displayed in figures 1 to 3.

Let us first discuss the symmetric case, and compare the phase diagram of a similar case discussed by Katsura *et al* (1979). Qualitatively the two are very much alike. However, there are several differences. In our case the paramagnetic-spin glass phase boundary *i.e.* $T_g(c)$ is dependent on the ratio K/J . This is to be expected, since the K 's determine the spread in the distribution of the internal field, which is finally responsible for the spin glass phase. For the ratios 0.8 and 0.75 in a concentration range around $c=0.5$ this curve lies above either the para-ferro boundary $T_c(c)$ or the para-antiferro boundary $T_N(c)$. In both these cases, therefore, there is a possibility of transition from the paramagnetic to the spin glass phase at these concentrations. For the ratio 0.65 (and all smaller ratios) the $T_g(c)$ lies totally below the $T_c(c)$ and $T_N(c)$ curves, and therefore we do not expect a direct transition from the para to the spin glass phase. This is to be compared with figure 1 (b) of Katsura, where the ratio is apparently 0.5 . Here too, the spin glass boundary lies below T_c and T_N , but it appears to be independent of c . Moreover, Katsura's T_c , T_N curves when extrapolated do not pass through 0 and 1 , an exact feature of our solutions (11) and (12).

In such a case the question arises: is there a spin glass transition? Katsura's answer seems to be that the spin-glass phase is not realised. However, if one examines the

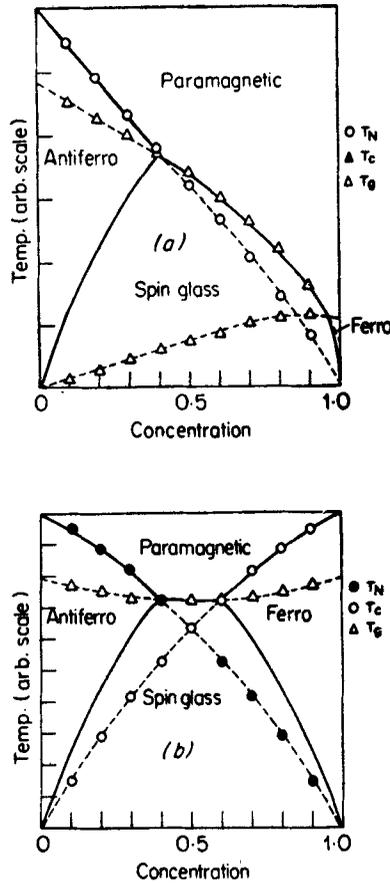


Figure 1. Phase diagram for k/J ratio 0.8 (a) $J_{AA} = 1, J_{AB} = 2, J_{BB} = -6$ (b) $J_{AA} = 1, J_{AB} = 0.8, J_{BB} = -1$.

$T=0$ solutions, we see that in the concentration region between $c=0.44$ and 0.56 the only possible solution is a spin glass with $q=1, m=0$. We can therefore surmise that at low temperatures in these concentration regime there is a spin glass region. Its upper boundary is not the curve $T_g(c)$, as at the actual boundary q will not be small and the approximations involved in deducing T_g will be invalid. However, this boundary may be found from the numerical solutions of (9). We may thus have a transition from a ferromagnetic to a glassy phase as we lower the temperature at these concentrations. There are several experimental evidences of such transitions (Mookerjee and Roy 1983).

For the ratio 0.8, no long-ranged ordered solution is possible at $T=0$, except at $c=0$ and 1. Thus the spin glass phase occupies the entire concentration regime at low temperatures.

Note that if instead of the approximations taken at $T=0$, we had done a numerical solution at $T=0$, the spin glass phase would still occupy a very large portion of the concentration range. A preliminary work done subsequently indicates that at $T=0$ spin glass phase spreads over $c_1 < c < c_2$, where $c_2 \sim 0.9$ and $c_1 \sim 0.1$ for $k/J=0.8$. A similar situation was reported by Medvedev.

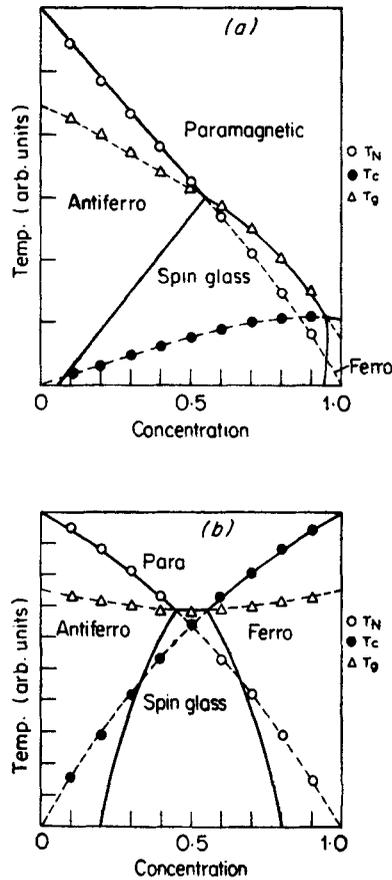


Figure 2. Phase diagram for the k/J ratio 0.75 (a) $J_{AA} = 1$, $J_{AB} = 2$, $J_{BB} = -6$ (b) $J_{AA} = 1$, $J_{AB} = 0.8$, $J_{BB} = -1$.

The physical reason behind this may be the following: what really drives the breakdown of the ferro (or antiferro) magnetic phases and transition to a spin glass phase, even for very small concentrations of the B (or A) atom is the large spread of the distribution of the local molecular field. The spread or standard deviation of the local molecular field is measured by K and the mean by J , so for ratios $K/J \sim 1$, the spread is large compared with the mean, and large deviations away from the mean are probable. This may be responsible for the large spread of the spin glass phase. However, in any realistic systems such large ratios may not be realisable (e.g. RKKY simple cubic case quoted in paper I, $K/J \approx 0.08$ and nearest neighbour simple cubic $K/J = 0.4$ so that the question may remain academic or mathematical one.

The asymmetric case does not provide any new features other than the ones already described, although the shapes of the boundaries are quite different.

From the qualitative similarity of the phase diagrams with that of Katsura's it is obvious that corrections to the simple mean field (e.g. Bethe approximation) will not yield any essentially new feature.

We have thus examined the phases of a random Ising model with competing interactions. We assert that a spin glass phase always exists, definitely at low temperatures

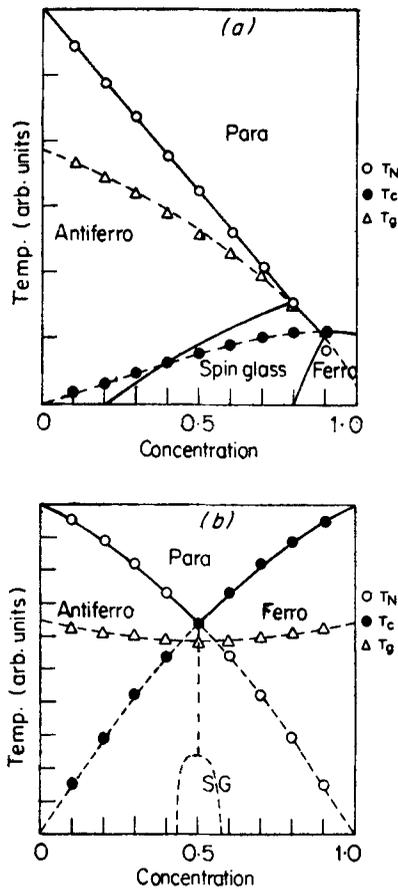


Figure 3. Phase diagram for the k/J ratio 0.65 (a) $J_{AA} = 1$, $J_{AB} = 2$, $J_{BB} = -6$ (b) $J_{AA} = 1$, $J_{AB} = 0.8$, $J_{BB} = -1$.

for some concentration range; although a direct transition from a paramagnetic to the glass phase may not always be possible. This is at variance with some earlier studies (Katsura *et al* 1979), who maintain that a glassy phase is only possible for particular ratios of J_{AA} , J_{AB} and J_{BB} . Generalisation to the Heisenberg model will not only involve replacing $\tanh(x)$ by the Brillouin functions $B_G(x)$, but will also introduce instead of one glass order parameter q , two parameters q_{\parallel} and q_{\perp} . We shall study the Heisenberg model in a further communication and elaborate more on the mixed phase reported by many experimentalists (Eggarter and Eggarter 1977; Mookerjee and Roy 1982).

References

- Aharony A 1975 *Phys. Rev. Lett.* **34** 590
 Aharony A 1978 *J. Magn. Mater.* **7** 198
 Aitken R G, Cheung T D and Kouvel J S 1982 *J. Appl. Phys.* **53** 2239
 Edwards S F and Anderson P W 1975 *J. Phys.* **F5** 965

- Eggarter E and Eggarter T P 1977 *Phys. Rev.* **B15** 3804
Kaneyoshi T 1976 *J. Phys.* **C9** L289
Katsura S and Matsubara F 1974 *Can. J. Phys.* **52** 120
Katsura S, Fujiki S and Inawashiro S 1979 *J. Phys.* **C12** 2839
Klein M 1968 *Phys. Rev.* **B173** 552
Kuzmin N N and Menshikov A Z 1979 (work reported by Medvedev M V 1979 *Fiz Tverd Tela (Leningrad)* **21** 3356)
Luttinger J M 1976 *Phys. Rev. Lett.* **37** 778
Matsubara F 1974 *Prog. Theor. Phys.* **51** 378, 1694
Medvedev M V and Zaborov A V 1982a *Phys. Stat. Sol.* **B109** 773
Medvedev M V and Zaborov A V 1982b *Phys. Stat. Sol.* **B110** 387
Menshikov A Z and Teplykh A Y 1977 *Fiz. Met. Metalloved* **44** 1215
Menshikov A Z and Kuzmin N N 1980 *Fiz. Met. Metalloved* **49** 433
Menshikov A Z, Burtlet P, Chamberod A and Tholence J L 1981 *Sol. State Commun.* **V39** 1093
Mookerjee A 1978 *Pramana* **11** 223
Mookerjee A 1979 *Pramana* **14** 11
Mookerjee A and Roy S B 1983 *J. Phys.* **F13** 1945
Morita and Horiguchi 1976 *Solid State Commun.* **19** 833
Oguchi T and Ueno Y 1979 *J. Phys. Soc. Jpn.* **46** 729
Plefka T 1976 *J. Phys.* **F6** L327
Rode V Y, Deryabin A V and Damashke G 1975 *Fiz. Met. Metalloved* **40** 429
Shull R D and Beck P A 1974 *AIP Conf. Proc.* **24** 95