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Propagation of the electromagnetic ion-cyclotron wave in a fusion plasma

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Abstract. The propagation of the electromagnetic ion-cyclotron wave in a fusion plasma described by a loss-cone structure is discussed. The wavelength is assumed to be much larger than the ion Larmour radius and the ion plasma frequency $\gg$ the ion-cyclotron frequency. The two modes that propagate in the plasma interact strongly and fuse together under certain conditions making the plasma unstable. The coalescence of the modes is found to decrease with an increase in electron temperature.

Keywords. Dispersion relation; ion-cyclotron wave; fusion plasma; loss-cone structure.

1. Introduction

The instabilities produced by generalised loss-cone distributions play an important role in mirror-magnetic configurations. For example, the electrostatic ion-cyclotron instability causes anomalous end-losses in mirror-machines (Baldwin 1977). Several aspects of the instabilities which occur in loss-cone plasmas have already been considered: the case of warm plasmas (Harris 1961; Dory *et al* 1965; Rosenbluth and Post 1965), the case of mixed warm-cold plasmas (Pearlstein *et al* 1966; Farr and Budwine 1968; Gomberoff and Cuperman 1976, 1981) and the correction due to electromagnetic effects (Callen and Guest 1971, 1973). The dispersion relations using loss-cone distributions are very complex and hence past analyses were essentially numerical (Himmell 1971; Cordey and Farr 1972).

We have derived in a simple form the dispersion relation for the near perpendicular propagation of the electromagnetic ion-cyclotron wave for wavelengths larger than the ion-Larmour radius γ_L and large ion plasma frequencies ($\omega_{p+}^2 \gg \Omega_+^2$). Our analysis shows that two modes can propagate in the plasma. These modes interact strongly fusing together in a number of cases and making the plasma unstable. This coalescence between the modes is found to decrease with an increase in electron temperature.

2. Dielectric tensor

Consider a plasma of uniform density N, in a uniform magnetic field $\mathbf{B}_{oz}$, directed along the z-axis. The Vlasov equations can be solved and a dispersion equation obtained if a Fourier-Laplace transform exists for the perturbed distribution function f, electric field **E** and magnetic field **B** (Montgomery and Tidman 1964). Setting $k_y = 0$ ($k_x \equiv k_\perp$; $k_z \equiv k_\parallel$), we get (Stix 1962)

$$\begin{vmatrix} -n_\parallel^2 + K_{xx} & K_{xy} & n_\perp n_\parallel + K_{xz} \\ K_{yx} & -n_\parallel^2 - n_\perp^2 + K_{yy} & K_{yz} \\ n_\perp n_\parallel + K_{zx} & K_{zy} & -n_\perp^2 + K_{zz} \end{vmatrix} = 0. \qquad (1)$$

In (1) $n_\parallel$ and $n_\perp$ are the refractive indices parallel and perpendicular to the magnetic field respectively and K_{ik} ($i, k = x, y, z$) are the elements of the dielectric tensor.

In this paper we assume that the equilibrium distribution function f_0 has a loss-cone structure given by

$$f_0 = \frac{1}{j!\, \pi^{3/2}\, W^{2J+2}\, U} [v_\perp]^{2J} \exp\left[-\frac{v_\parallel^2}{U^2} - \frac{v_\perp^2}{W^2}\right] \qquad (2)$$

where $W^2 = 2\, T_\perp / [m\, (j+1)]$,

$U^2 = 2\, T_\parallel / m$,

j is the loss-cone index. A loss-cone structure is assumed for both electrons and ions to simplify the algebra.

Substituting (2) into the elements of the dielectric tensor (Landau and Cuperman 1971; hereinafter referred to as I), carrying out the $dv_\perp$ (Chandu Venugopal and Viswanathan 1982) and $dv_\parallel$ (Appendix A of I) integrations gives us the following expressions for K_{ik}:

$$\begin{matrix} K_{xx} - 1 \\ K_{xy} \\ K_{yy} - 1 \end{matrix} = \sum_{'+','-'} C \sum_{n=-\infty}^{+\infty} \begin{matrix} n^2/a^2 \\ in/a \\ 1 \end{matrix} \left\{ \begin{matrix} I_P^{(J)} \\ I_{aP}^{(J)} \\ Q_P^{(J)} \end{matrix} \left[\frac{1-E}{n-z} - \frac{AE}{z}\right] - j\left[\frac{1-E}{n-z} - \frac{E}{z}\right] \begin{matrix} I_P^{(J-1)} \\ I_{aP}^{(J-1)} \\ Q_P^{(J-1)} \end{matrix} \right\}$$

$$\begin{matrix} K_{zx} \\ K_{zy} \end{matrix} = \sum_{'+','-'} C \sum_{n=-\infty}^{+\infty} \begin{matrix} n/\theta a^2 \\ i/\theta a \end{matrix} \left\{ \begin{matrix} I_P^{(J)} \\ I_{aP}^{(J)} \end{matrix} \left[E + \frac{A\,(n-z)\,E}{z}\right] - j\left[E + \frac{(n-z)\,E}{z}\right] \begin{matrix} I_P^{(J-1)} \\ I_{aP}^{(J-1)} \end{matrix} \right\}$$

$$K_{zz} - 1 = \sum_{'+','-'} C \sum_{n=-\infty}^{+\infty} 1/\theta^2 a^2 \left\{ I_p^{(J)} \left[\frac{nA}{z} + \frac{W^2}{U^2}\right] (z-n)\, E - j\left[\frac{n(z-n)E}{z}\right] \cdot I_p^{(J-1)} \right\}$$

with $K_{xz} = K_{zx}$ and $K_{yz} = -K_{zy}$. (3)

In (3) '+' refers to ions and is assumed on all quantities and, in general, should be understood, as added when not specifically stated and '−' refers to electrons.
Also

$$\Omega = eB_0/(mc),\ \omega_p^2 = 4\pi Ne^2/m,\ \overline{\omega_p^2} = \omega_p^2/\Omega^2,$$

$$z = \omega/\Omega, \quad \theta = k_{\|}/k_{\perp}, \quad a = k_{\perp}/\Omega \text{ and } A = 1 - \frac{W^2}{U^2} \tag{4a}$$

$$C = 4\,[j!\ W^{2J+2}]^{-1}\,\bar{\omega}_p^2/z. \tag{4b}$$

$$I_p^{(j)} = \frac{(-1)^j}{W^2}\frac{d^j I}{dp^j};\ I_p^{(j-1)} = (-1)^{j-1}\frac{d^{j-1} I}{dp^{j-1}}. \tag{4c}$$

$$I_{ap}^{(j)} = \frac{(-1)^j}{2W^2}\frac{d}{da}\left(\frac{d^j I}{dp^j}\right);\ I_{ap}^{(j-1)} = \frac{(-1)^{j-1}}{2}\frac{d}{da}\left(\frac{d^{j-1} I}{dp^{j-1}}\right). \tag{4d}$$

$$Q_p^{(j)} = \frac{(-1)^j}{W^2}\frac{d^j Q}{dp^j};\ Q_p^{(j-1)} = (-1)^{j-1}\frac{d^{j-1} Q}{dp^{j-1}}. \tag{4e}$$

where $$p = 1/W^2 \text{ and } Q = \left.\frac{d^2 I(\alpha,\beta)}{d\alpha\, d\beta}\right|_{\alpha=\beta}. \tag{4f}$$

$I = I(\alpha, \beta)$ arises from integrals of the type (Gradshteyn and Ryzhik 1965)

$$\int_0^\infty \exp(-\Delta^2 x^2)\, J_n(\alpha x)\, J_n(\beta x)\, x\, \mathrm{d}x = I(\alpha, \beta)$$

$$= \frac{1}{2\Delta^2}\exp\left[-\frac{(\alpha^2+\beta^2)}{4\Delta^2}\right] I_n(\alpha\beta/2\Delta^2). \tag{5}$$

In our case $1/\Delta^2 = W^2$. In (4c) we have $\alpha=\beta$ in $I(\alpha, \beta)$, in (4d) we differentiate $I(\alpha, \alpha)$ with respect to α and in (4e) we differentiate $I(\alpha, \beta)$ with respct to α and β and finally set $\alpha=\beta$. They are then differentiated j or $(j-1)$ times with respect to p. The arguments of the Bessel functions are $I_n = I_n(l_\perp)$ where

$$l_\perp = \frac{2k_\perp^2 T_\perp}{\Omega^2 m(j+1)} = \frac{2}{(j+1)}\, l'_\perp. \tag{6}$$

The E-function is defined as

$$E(S') = -\tfrac{1}{2}Z'(s'/\sqrt{2}),$$

where $$S' = (z-n)/(\theta\sqrt{l_{\|}}),$$

and $$l_{\|} = k_\perp^2 T_{\|}/(\Omega^2 m). \tag{7}$$

Z' is the derivative of the plasma dispersion function of Fried and Conte.

3. The approximation scheme

We choose an ordering scheme (in terms of a small parameter ϵ) that reduces the electron contribution to a minimum as we are interested only in the ion-cyclotron wave propagating with a small range of frequencies around its first harmonic. We thus choose

$$\gamma = 1 - z_+^2 \sim \epsilon; \quad l_\perp, \; l_\|, \; \theta \text{ and } 1/\bar{\omega}_p^2 \sim \epsilon,$$

$$T_+/T_- \text{ and } W^2/U^2 \sim 1 \text{ and } m_-/m_+ \sim \epsilon^2. \tag{8}$$

Since $l_\perp \ll 1$ we can use relations (4c) to (4e) combined with a series expansion of both terms in (5) to get expressions for

$$I_p^{j,(j-1)}, \quad I_{ap}^{j,(j-1)} \text{ and } Q_p^{j,(j-1)}.$$

These are given in Appendix 1.

From (8), $s' \gg 1$ for all n. The asymptotic expansion of Z, in terms of the E-function, thus needed is given by (I)

$$E(S') = -\frac{\theta^2 l_\|}{(n-z)^2} - 3\frac{(\theta^2 l_\|)^2}{(n-z)^4} \dots + 2i(z-n)\, e_n$$

where $$e_n = \left(\frac{\pi}{8\theta^2 l_\|}\right)^{1/2} \exp\left[-(z-n)^2/2\theta^2 l_\|\right], \tag{9}$$

e_n for $n = 0, 1$ and $2 \ll 1$ and hence will be neglected.

4. Explicit evaluation of the tensor elements

We calculate the dielectric tensor elements retaining terms of the order of $1/\epsilon$, 1 and ϵ. Only the n=0, 1, 2 ion terms and the n=0 electron terms contribute; all electron terms are written down in terms of the ion terms.

We demonstrate the calculation of the element K_{xx}. Expanding the first square bracket of (3) using (9) and summing over n, we get

$$\sum_{n=-\infty}^{\infty} n^2 \left[\frac{1-E}{n-z} - \frac{AE}{z}\right] = 2\sum_{n=1}^{\infty} n^2 \left\{\frac{z}{n^2 - z^2} + (\theta^2 l_\|)\frac{z(3n^2 + z^2)}{(n^2 - z^2)^3}\right.$$

$$\left. + 3(\theta^2 l_\|)^2 \frac{z(5n^4 + \text{ion}^2 z^2 + z^4)}{(n^2 - z^2)^5} + \frac{A}{z}(\theta^2 l_\|)\frac{n^2 + z^2}{(n^2 - z^2)^2}\right\}. \tag{10}$$

The other sum (labelled 10a), for the square bracket multiplying j in (3), can be got from (10) by putting A=1. The sums (10) and (10a) are then multiplied by the appropriate expressions for $I_p^{(j)}$ and $I_p^{(j-1)}$ for $n=1$ and 2 and then added. The resultant expression for K_{xx} can be further simplified by using (A3). We finally get

$$\frac{K_{xx}}{\bar{\omega}_p^2} = \frac{1 - l'_\perp}{\gamma} + 4\frac{\theta^2 l_\parallel}{\gamma^3} + \frac{\theta^2 l_\parallel}{\gamma^2}[(2A-1)(j+1) - j]$$

$$+ 48\frac{\theta^4 l_\parallel^2}{\gamma^5} - 4\frac{\theta^2 l_\parallel l'_\perp}{\gamma^3} + \frac{5}{16}\frac{l'^2_\perp}{\gamma}\frac{(j+2)}{(j+1)} + \frac{l'_\perp}{3} + \frac{1}{\bar{\omega}_p^2}. \tag{11a}$$

The other elements can be derived in a similar manner. The expressions are:

$$K_{yy}\frac{z_+^2}{\bar{\omega}_p^2} = \frac{1 - 3l'_\perp}{\gamma} + 4\frac{\theta^2 l_\parallel}{\gamma^3} - 1 + \frac{\theta^2 l_\parallel}{\gamma^2}[(2A-5)(j+1) + 3j]$$

$$+ 48\frac{\theta^4 l_\parallel^2}{\gamma^5} - 12\frac{\theta^2 l_\parallel l'_\perp}{\gamma^3} + \frac{37}{16}\frac{l'^2_\perp}{\gamma}\frac{(j+2)}{(j+1)} + 2\left[\frac{2}{3} - \frac{T_{\perp,-}}{T_{\perp,+}}\right] l'_\perp + \frac{1}{\bar{\omega}_p^2}. \tag{11b}$$

$$\frac{K_{xy}}{i}\frac{z_+}{\bar{\omega}_p^2} = \frac{1 - 2l'_\perp}{\gamma} + 4\frac{\theta^2 l_\parallel}{\gamma^3} - 1 + \frac{\theta^2 l_\parallel}{\gamma^2}[(2A-3)(j+1) + j]$$

$$+ 48\frac{\theta^4 l_\parallel^2}{\gamma^5} - 8\frac{\theta^2 l_\parallel l'_\perp}{\gamma^3} + \frac{15}{16}\frac{l'^2_\perp}{\gamma}\frac{(j+2)}{(j+1)} + \frac{2}{3}l'_\perp. \tag{11c}$$

$$\frac{K_{xz}}{\bar{\omega}_p^2} = -2\frac{\theta l_\parallel}{\gamma^2} + 2\frac{\theta l_\parallel l'_\perp}{\gamma^2} - 24\frac{\theta^3 l_\parallel^2}{\gamma^4} - \frac{\theta l_\parallel}{\gamma}[A(j+1) - j] \tag{11d}$$

$$\frac{K_{zy}}{i}\frac{z_+}{\bar{\omega}_p^2} = -2\frac{\theta l_\parallel}{\gamma^2} - \theta l_\parallel \frac{m_+}{m_-}\frac{T_{\perp,-}}{T_{\parallel,+}} + 4\frac{\theta l_\parallel l'_\perp}{\gamma^2}$$

$$- 24\frac{\theta^3 l_\parallel^2}{\gamma^4} - \frac{\theta l_\parallel}{\gamma}[(A-1)(j+1)] \tag{11e}$$

and $$K_{zz}(z_+^2/\bar{\omega}_p^2) = -(m_+/m_-). \tag{11f}$$

Only the $1/\epsilon^2$ term has been retained for K_{zz}. The next term is ~ 1.

5. The dispersion relation

The tensor elements (11a) to (11f) will now be used to derive the dispersion relation for the propagation of the electromagnetic ion cyclotron wave. We propose to retain terms only ~ 1. Starting from the basic definition of $n_\perp$ $(= k_\perp C/\omega)$ we can easily show that

$$n_\perp^2 = \frac{l'_\perp}{\beta_\perp} \frac{\bar{\omega}_p^2}{z_+^2} \text{ and } n_\parallel^2 = \theta^2 n_\perp^2,$$

where $$\beta_\perp = 4\pi N T_{\perp,+}/B_0^2. \tag{12}$$

Assuming that, at most, $(l'_\perp/\beta_\perp) \sim 1$, we find from (12) that $(n_\parallel^2/\bar{\omega}_p^2) \sim \epsilon^2$ and thus

$$\frac{K_{xx}}{\bar{\omega}_p^2} + \frac{n_\parallel^2}{\bar{\omega}_p^2} \approx \frac{K_{xx}}{\bar{\omega}_p}$$

and $$K_{yy}\frac{z_+^2}{\bar{\omega}_p^2} + \frac{n_\parallel^2}{\bar{\omega}_p^2} \approx K_{yy}\frac{z_+^2}{\bar{\omega}_p^2}$$

as the least significant terms retained in $K_{xx}/\bar{\omega}_p^2$ and $(K_{yy}\, Z_+^2/\bar{\omega}_p^2)$ are $\sim \epsilon$.

Expanding the determinant (1) in factors of the last column, dividing by $(-n_\perp^2 + K_{zz})$ and multiplying by $z_+^2/\bar{\omega}_p^4$ we get

$$\frac{K_{xx}}{\bar{\omega}_p^2}\left[-\frac{l'_\perp}{\beta_\perp} + K_{yy}\frac{z_+^2}{\bar{\omega}_p^2}\right] + \left[K_{xy}\frac{z_+}{\bar{\omega}_p^2}\right]^2$$

$$+ \left\{\frac{n_\perp n_\parallel + K_{xz}}{-n_\perp^2 + K_{zz}}\left[K_{xy}\frac{z_+}{\bar{\omega}_p^2} K_{yz}\frac{z_+}{\bar{\omega}_p^2} + \left(\frac{\theta}{z_+^2}\frac{l'_\perp}{\beta_\perp} + \frac{K_{xz}}{\bar{\omega}_p^2}\right)\right.\right.$$

$$\left.\left(\frac{l'_\perp}{\beta_\perp} - K_{yy}\frac{z_+^2}{\bar{\omega}_p^2}\right)\right] + \frac{K_{yz}\, z_+}{-n_\perp^2 + K_{zz}}\left[\frac{K_{xx}}{\bar{\omega}_p^2} K_{yz}\frac{z_+}{\bar{\omega}_p^2}\right.$$

$$\left.\left. + K_{xy}\frac{z_+}{\bar{\omega}_p^2}\left(\frac{\theta}{z_+^2}\frac{l'_\perp}{\beta_\perp} + \frac{K_{xz}}{\bar{\omega}_p^2}\right)\right]\right\} = 0. \tag{13}$$

We shall now show that the curly-bracketed terms of (13) do not contribute to the dispersion relation. An examination of the tensor elements (11a) to (11f) reveals that

$$K_{xy} = i\, K_{yy} = i\, K_{xx},$$

and $$K_{yz} \approx -i\, K_{xz},$$

for the most significant term. Thus

$$K_{xy}\, K_{yz} - K_{yy}\, K_{xz} = 0$$

and $$K_{xx}K_{yz}+K_{xy}K_{xz}=0$$

Also $$\frac{n_\perp n_\parallel + K_{xz}}{-n_\perp^2+K_{zz}} \text{ and } \frac{K_{yz}z_+}{-n_\perp^2+K_{zz}} \text{ are } \sim \epsilon^2.$$

Their product with the square-bracketed terms would be at most $\sim\epsilon$ and since we retain only terms ~ 1 the curly-bracketed terms do not contribute to the dispersion relation. Substituting from (11a) to (11c) for the tensor elements, simplifying and finally multiplying by $-\gamma^2$ we get the dispersion relation as

$$\gamma^2-\gamma\left[1-\frac{l'_\perp}{\beta_\perp}-\delta\right]+\frac{l'^2_\perp}{4}\frac{(j-2)}{(j+1)}-4\frac{\theta^2 l_\parallel}{\gamma}\left[1-\frac{l'_\perp}{\beta_\perp}\right]=0, \quad (14)$$

where $$\delta=\left[\frac{8}{3}+2\frac{T_{\perp,-}}{T_{\perp,+}}-\frac{l'_\perp}{\beta_\perp}\right]l'_\perp-\frac{2}{\bar{\omega}_p^2}.$$

If we let $[1-(l'_\perp/\beta_\perp)]$ be ~ 1 we get $\gamma=0$ for the ϵ order term and this is much greater than the other ϵ^2 terms. This result is inconsistent with our ordering as θ^2 cannot now be $\sim\epsilon^2$. Thus to get a consistent ordering we need to set $[1-(l'_\perp/\beta_\perp)]\sim\epsilon$. But the θ^2 term is now of order $\sim\epsilon^3$ and thus does not contribute to the dispersion relation as the other terms are of the order ϵ^2. Thus the dispersion relation reduces to the first three terms.

Due to the tediousness of differentiating (5) previous numerical computations considered only low values of j. For example Cordey and Farr (1972) considered the ion cyclotron instability for $j=0$ and 1 while Himmell (1971) considered it for $j=1$ to 4. In contrast our dispersion relation (14) is a very general one and j can take on any value upwards of zero. As a check on our result we note that for $j=0$ the dispersion relation (14) reduces to that in I which was derived using anisotropic Maxwellian distribution function (the loss-cone distribution function reduces to the anisotropic Maxwellian for $j=0$).

6. Applications

We now plot the dispersion relation (14) for typical fusion conditions (Cap 1976)

$$N=10^{16}\,\text{Cm}^{-3},\ T_+=10^8\,{}^\circ\text{K and } B_0=10^5\text{G}.$$

With these parameters $\beta_\perp=0{\cdot}17343$.

The plot given in figure 1 is the dispersion relation for $j=0$ and 8 and shows that two modes, one with a constant frequency and another with an increasing frequency, can propagate in the plasma. They interact strongly and mode conversion takes place at $k_\perp\gamma_L=0{\cdot}5$, beyond this point the former mode increases in frequency while the latter mode exhibits a constant frequency. Such mode conversion mechanisms are of interest in fusion plasmas; one of the methods suggested for the electron cylotron resonance heating of plasmas in tokomaks is by the mode conversion of the ordinary

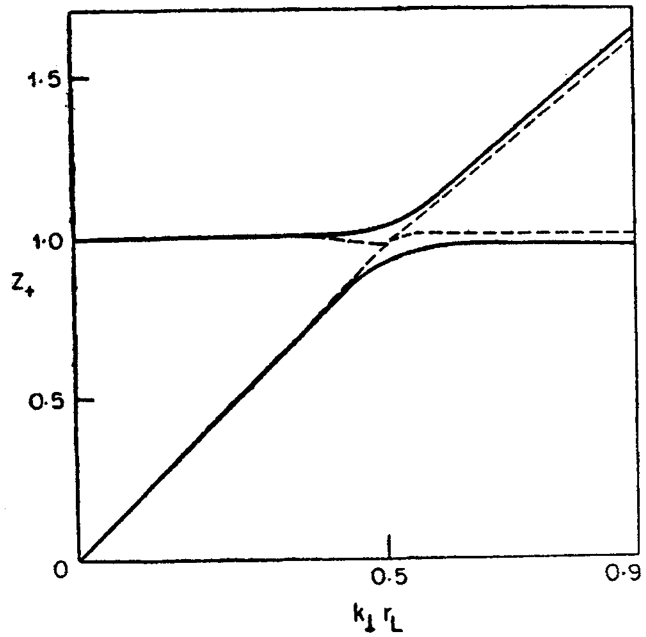

Figure 1. Plot of the dispersion relation for $\beta_\perp = 0{\cdot}17343$ and $T_{\perp,-}/T_{\perp,+} = 1$. The dotted lines are for $j = 8$ and the solid lines for $j = 0$.

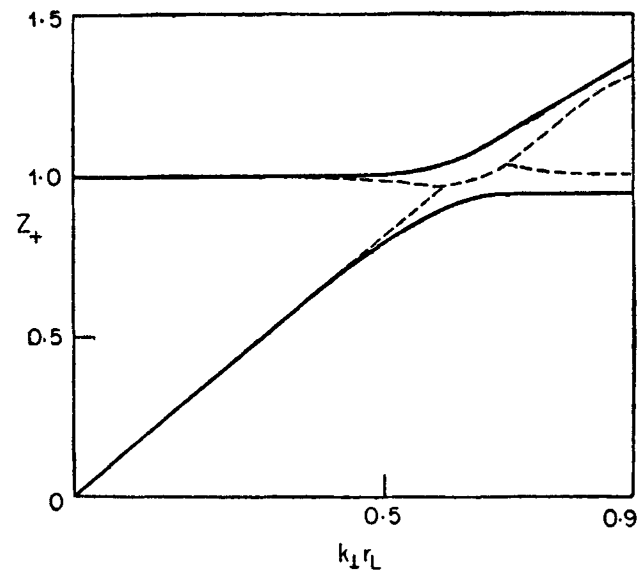

Figure 2. Plot of the dispersion relation for $\beta_\perp = 0{\cdot}17343$ and $T_{\perp,-}/T_{\perp,+} \sim \epsilon$. The dotted lines for $j = 7$ and the solid lines for $j = 0$.

wave into a cyclotron harmonic wave (Cairns and Lashmore-Davies 1982). For $j \geqslant 8$ the two modes coalesce at $k_\perp \gamma_L = 0{\cdot}5$ resulting in a pair of complex conjugate roots for the dispersion relation. This indicates that the plasma is unstable (Cordey and Farr 1972); the values of the imaginary part of z_+ are $\sim 10^{-2}$.

Table 1. Propagation characteristics of the modes.

$\beta_\perp$	$T_{\perp,-}/T_{\perp,+}$	Characteristics
0·17343	2	Modes do not coalesce upto $j = 10$
	1	Modes coalesce for $j \geqslant 8$ at $k_\perp \gamma_l = 0{\cdot}5$ Real $z_+ = 0{\cdot}974$
	0·1	Modes coalesce for $j \geqslant 3$ at $k_\perp \gamma_l = 0{\cdot}6$ Real $z_+ = 0{\cdot}985$.
	$\sim \epsilon$	Modes coalesce for $j \geqslant 4$ at $k_\perp \gamma_l = 0{\cdot}6$ Real $z_+ = 0{\cdot}976$. For $j \geqslant 7$ they fuse from $k_\perp \gamma_l = 0{\cdot}6$ to $0{\cdot}7$. Real $z_+ = 0{\cdot}976$ to $1{\cdot}045$ (figure 2).
1·7343	2	Modes do not coalesce upto $j = 10$.
	1	Modes coalesce for $j \geqslant 5$ at $k_\perp \gamma_l = 0{\cdot}6$. Real $z_+ = 0{\cdot}97$. For $j \geqslant 8$ they fuse from $k_\perp \gamma_l = 0.6$ to $0{\cdot}7$. Real $z_+ = 0{\cdot}97$ to $1{\cdot}048$.
	0·1	Modes coalesce for $j \geqslant 3$ at $k_\perp \gamma_l = 0{\cdot}8$. Real $z_+ = 0{\cdot}996$. For $j \geqslant 4$ they fuse from $k_\perp \gamma_l = 0{\cdot}8$ to $0{\cdot}9$. Real $z_+ = 0{\cdot}996$ to $1{\cdot}056$.
	$\sim \epsilon$	Modes coalesce for $j \geqslant 3$ in the region $k_\perp \gamma_l = 0{\cdot}8$ to $0{\cdot}9$. Real $z_+ = 0{\cdot}979$ to $1{\cdot}037$.

Table 1 depicts the characteristics of the roots of the dispersion relation when the parameters are altered.

The table shows that the two modes tend to coalesce over a larger frequency range as the electron temperature tends to zero. Also the modes are unstable around $\omega \approx \Omega_+$ in agreement with the results of Cordey and Farr (1972).

7. Conclusion

The dispersion relation for the near perpendicular propagation of the electromagnetic ion cyclotron wave has been derived. Two modes, which interact strongly, can propagate in the plasma. These modes coalesce under certain conditions making the plasma unstable. A possible method of stabilisation is to increase the electron temperature as the modes do not coalesce for electron to ion temperature ratios greater than 2.

Appendix 1

The expressions for $I_p^{(j)} \cdot I_{ap}^{(j)}$ and $Q_p^{(j)}$ derived as indicated earlier are:

where $\quad j_{(x)} = (j + x)!/[j!\, x!] \quad x = 1, 2, 3, \ldots$ (A1)

n	$\frac{4}{j!\,W^{2j+2}\,a^2} I_p^{(j)}$	$\frac{4}{j!\,W^{2j+2}\,a} I_{ap}^{(j)}$	$\frac{4}{j!\,W^{2j+2}} Q_p^{(j)}$
0	$\frac{2}{l_\perp} - j_{(1)} + \frac{3}{8} l_\perp j_{(2)}$	$-j_{(1)} + \frac{3}{4} l_\perp j_{(2)} - \frac{5}{16} l_\perp^2 j_{(3)}$	$l_\perp j_{(2)} - \frac{3}{4} l_\perp^2 j_{(3)}$
1	$\frac{j_{(1)}}{2} - \frac{l_\perp}{4} j_{(2)} + \frac{5}{64} l_\perp^2 j_{(3)}$	$\frac{j_{(1)}}{2} - \frac{l_\perp}{2} j_{(2)} + \frac{15}{64} l_\perp^2 j_{(3)}$	$\frac{j_{(1)}}{2} - \frac{3}{4} l_\perp j_{(2)} + \frac{37}{64} l_\top^2 j_{(3)}$
2	$\frac{l_\perp}{16} j_{(2)} - \frac{l_\perp^2}{32} j_{(3)}$	$\frac{l_\perp}{8} j_{(2)} - \frac{3}{32} l_\perp^2 j_{(3)}$	$\frac{l_\perp}{4} j_{(2)} - \frac{l_\top^2}{4} j_{(3)}$

Expressions for

$$I_p^{(j-1)},\ I_{ap}^{(j-1)} \text{ and } Q_p^{(j-1)}$$

(labelled A-2) can be derived from the above by dividing the quantities by W^2 and replacing the j-factors by g-factors, these being defined by

$$g_{(y)} = (j+y)!/[j!\,(y+1)!] \quad y = 0, 1, 2, \ldots$$

We also have the relations

$$j_{(1)} - j\,g_{(0)} = 1,$$

$$j_{(2)} - j\,g_{(1)} = (j+1),$$

$$j_{(3)} - j\,g_{(2)} = \frac{(j+1)(j+2)}{2}. \tag{A3}$$

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