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Elastic scattering of positrons by helium and lithium

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Abstract. We present differential and integrated cross-sections for elastic scattering of positrons by helium and lithium at intermediate and high energies, calculated using the two-potential eikonal approach. The present calculations are compared with the available theoretical results.

Keywords. Positron-atom; electron-atom; scattering; two-potential eikonal approximation; Schrödinger equation.

1. Introduction

During recent years, a great deal of theoretical and experimental efforts have been devoted to the study of positron-atom scattering. Due to the distinguishability of the positron from the target electrons, there is no exchange interaction for positrons. The static interaction for positrons is of opposite sign to that for electrons while the polarisation interaction is attractive for both the projectiles. Annihilation of positrons by electrons introduces two additional phenomena (annihilation and positronium formation) for positron collisions which do not arise in electron collisions. Positron-atom scattering can provide a test for the accuracy and validity of the theoretical approximations that have been developed for electron-atom scattering.

The first Born and Glauber amplitudes are independent of the sign of interaction potential and therefore do not distinguish between electron and positron amplitudes. The problem of positron-atom scattering has been studied by a number of workers within the framework of eikonal and eikonal-related approximations. The recent review articles by Byron and Joachain (1977a); Chan *et al* (1979) give a detailed account of these approximations. Byron and Joachain (1977c) applied eikonal Born series (EBS) approach to investigate the positron-hydrogen and helium collisions. Byron and Joachain (1977b) have also used the eikonal-optical model for the calculation of positron-helium and neon scattering cross-sections. Gien (1977) employed the modified Glauber approach to study positron-hydrogen scattering. The problem of positron-hydrogen collision was also studied by Lal and Srivastava (1979) using the two-potential eikonal (TPE) approximation of Ishihara and Chen (1975). Willis *et al* (1981) investigated the scattering of positrons by helium at intermediate energies using the three-state close coupling, Born, Glauber, eikonal Born series, optical model and a distorted-wave Born model. They compared positron cross-sections with electron impact results. In the case of positron scattering the results are found to be considerably lower than the corresponding electron case. At present there does not exist much work on positron-lithium elastic scattering at intermediate energies. Sarkar *et al* (1973) applied polarised first Born approximation and the modified eikonal method to calculate positron-lithium cross-sections over a wide range of energy. Tayal *et al* (1981) calculated positron-lithium elastic scattering cross-sections in the energy ranges of 10–200 eV in the corrected static approximation and in an approximation which combines the contribution of the non-static parts of the higher-order ($n \geqslant 3$) terms in the Glauber sense with the corrected static contribution.

In an earlier paper Tayal *et al* (1980) applied the TPE approach of Ishihara and Chen (1975) to investigate the elastic scattering of electrons by helium and lithium at intermediate energies. In view of the simplicity of the approach and the good results obtained for electron scattering, we have extended our calculations to the study of positron scattering. We have obtained the elastic differential and integrated cross sections for e^+—He and e^+—Li collisions in the energy regions 100–500 eV and 10–200 eV respectively.

2. Theory

Consider the elastic scattering of positrons by a neutral atom of atomic number Z. We denote by $\mathbf{r}$ and $\mathbf{r}_1, \mathbf{r}_2, \ldots \mathbf{r}_z$ the coordinates of the incident positron and of the atomic electrons respectively relative to the target nucleus. The initial and final wavevectors are denoted by $\mathbf{K}_i$ and $\mathbf{K}_f$, respectively and $\mathbf{q} = \mathbf{K}_i - \mathbf{K}_f$ is the momentum transfer. The total interaction potential between the projectile and the target is given by (using atomic units)

$$V(\mathbf{r}, \mathbf{r}_1, \mathbf{r}_2, \ldots \mathbf{r}_z) = \frac{Z}{r} - \sum_{i=1}^{Z} \frac{1}{|\mathbf{r} - \mathbf{r}_i|}. \tag{1}$$

An arbitrary potential V_1 is chosen to be equal to the static potential of the target atom in the ground state and is subtracted from the interaction V, so that the remaining part $V_0 = V - V_1$ is quite smooth and can be treated in Glauber approximation. The contribution of the disturbing part V_1 is obtained exactly by solving the radial Schrödinger equation.

The scattering amplitude corresponding to a transition of the target atom from the initial state i with wavefunction ϕ_i to some final state f with wavefunction ϕ_f is given by (8)

$$F_{fi} = \frac{k_i}{2\pi i} \int \mathrm{d}^2 b\, e^{i\mathbf{q}\cdot\mathbf{b}} [\Gamma_{fi}(b) - 1] + \frac{1}{k_i} \sum_l (2l+1) P_l(\cos\theta)\, e^{i\delta_l} \sin\delta_l \times \int \frac{\mathrm{d}\phi_b}{2\pi} \Gamma_{fi}(\mathbf{b}_l), \tag{2a}$$

$$F_{fi} = F^{(1)} + F^{(2)}, \tag{2b}$$

where **b** is the impact parameter, δ_l is the phase shift for the central potential V_1 for the lth partial wave.

$$\Gamma_{fi}(\mathbf{b}) = \langle \phi_f \,|\, \exp\,[i\,\chi\,(\mathbf{b}, \mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_z)]\,|\phi_i\rangle, \tag{3}$$

$$\chi\,(\mathbf{b}, \mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_z) = \chi_0\,(\mathbf{b}, \mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_z) + \Delta\,\chi\,(\mathbf{b}, \mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_z), \tag{4}$$

$$\chi_0\,(\mathbf{b}, \mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_z) = -\frac{1}{k_i}\int_{-\infty}^{\infty} \mathrm{d}z\, V_0\,(\mathbf{r}, \mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_z), \tag{5}$$

$$\Delta\,\chi\,(\mathbf{b}, \mathbf{r}_1, \mathbf{r}_2, \ldots, \mathbf{r}_z) = \frac{2}{k_i}\,V_0\,(z = 0)\int_0^{\infty} \mathrm{d}z \left(1 - \frac{z}{[z^2 - (V_1\,|\,E)\,r^2]^{1/2}}\right), \tag{6}$$

and $b_l = (l + \tfrac{1}{2})/k_i$.

In (2), the summation over l is carried out up to $l = 10$ and 20 for helium and lithium respectively. The first term $F^{(1)}$ in the scattering amplitude (equation (2b)) is the same for electron-atom and positron-atom scattering. The difference between the electron-atom and positron-atom scattering cases comes from the second term, $F^{(2)}$, in equation (2b) which contains δ_l and its interference with the first term.

3. Results and discussion

The differential cross-sections (DCS) for positron-helium scattering have been calculated at 100, 200 and 400 eV. Figure 1 shows our results for the positron-helium angular distributions. They are compared with those in the Glauber approximation (which predicts results identical to those for electron-atom direct scattering) and the available theoretical calculations in optical model (2), eikonal Born series (EBS) approach (3) and of Gupta and Mathur (1979) in Coulomb-projected Born polarisation approximation. The present TPE results always lie lower than the Glauber results. This difference in the TPE and the Glauber results is mainly due to the better treatment of close encounter collisions. At 100 eV, the optical model-results are less than the present results at all scattering angles. As the impact energy increases the difference between the present and optical-model results narrows down and at 400 eV the present TPE results merge with the optical model calculations. The differential cross-section in EBS approximation has not been shown, as it lies close to the present calculation. The results of Gupta and Mathur (1979) do not agree with the present calculations in the lower angular region and show a dip at 20° at 100 eV and at about 10° at 200 eV.

In figures 2 and 3 we display our results for e^+—Li DCS at 10 and 60 eV and compare them with the elastic scattering calculation of Tayal *et al* (1981) using corrected static and improved corrected static approximation. In figure 2 we also include the results of Sarkar *et al* (1973) obtained in eikonal and polarised first Born approximation at 10 eV. It may be seen that the present TPE cross-sections are much smaller

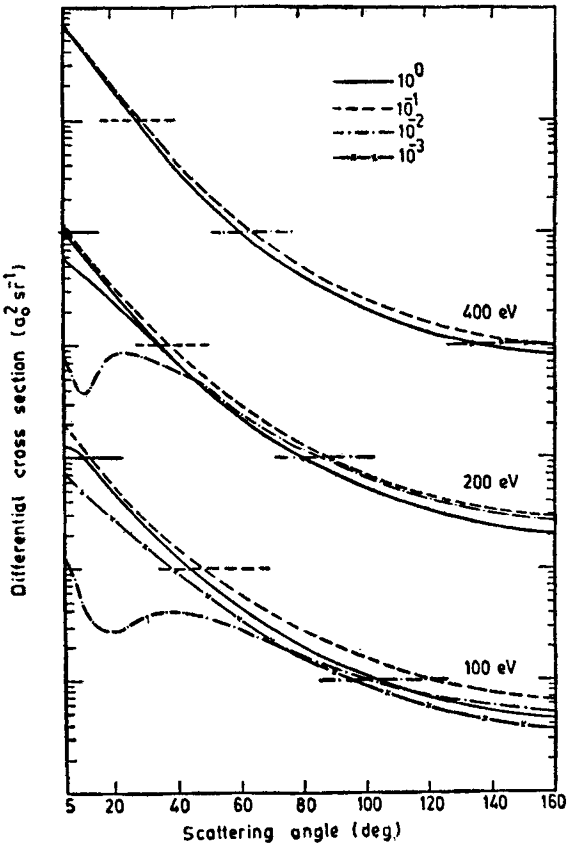

Figure 1. Differential cross-section for e^+—He elastic scattering at 100, 200 and 400 eV. Solid curve: present calculation in TPE approximation; dashed curve: present calculation in the Glauber approximation; dash-cross curve: results of Gupta and Mathur (1979); dash-dot curve: optical model results.

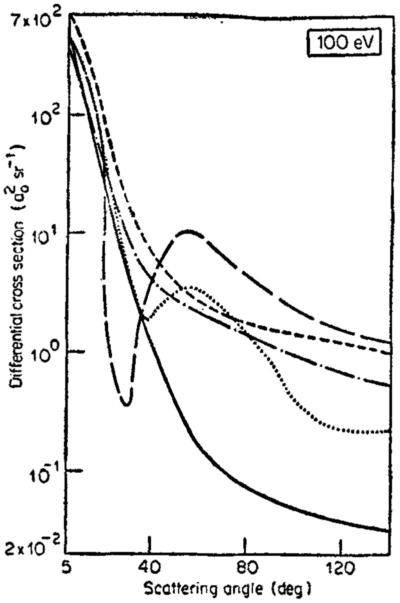

Figure 2. Differential cross-section for e^+—Li elastic scattering at 100 eV. Solid curve: present results in TPE approximation; dash-dot curve: improved corrected static results; dash curve: corrected static results; long dash curve: polarised first Born calculation of Sarkar *et al* (1973); dotted curve: eikonal approximation results of Sarkar *et al* (1973).

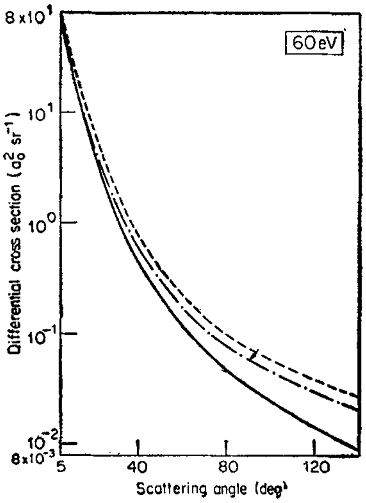

Figure 3. Same as figure 2 but at 60 eV.

than those obtained in other theoretical approximation, except at small angles where they lie above the eikonal, polarised first Born and improved corrected static results. As the incident energy increases the difference between different calculations narrows down but still remains noticeable at large scattering angles. The cross-section of Sarkar *et al* (1973) shows a minimum around 36° in eikonal approximation and around 30° in polarised first Born approximation calculation. Guha and Ghosh (1981) have also reported a similar structure in the differential cross-section in their positron-lithium scattering calculation using two-state approximation.

Table 1 shows our TPE results for total elastic cross-sections for e^+—He scattering in the energy range 100–500 eV and compare them with those obtained in the optical model theory and the EBS approach. We have also included the results of Mukherjee and Sural (1979) obtained in an integral equation approach to the second order potential method. The recent three-state close coupling calculations of Willis *et al* (1981) including $1\,^1S$, $2\,^1S$ and $2\,^1P$ states of helium are also shown at 100 and 200 eV. The EBS approach results stand higher at 100 eV but show a good agreement with the present calculation at higher energies ($\geqslant 200$ eV). The cross-sections obtained in the optical model approach lie lower at 100 and 200 eV. However, at higher energies our results closely agree with the optical model calculations.

We have tabulated our total elastic cross-sections at 10, 20, 60, 100 and 200 eV for e^+—Li scattering in table 2. They are compared with those of Mukherjee and Sural (1979) and Tayal *et al* (1981). Our results are more or less in agreement with the second-order potential calculation of Mukherjee and Sural and the improved corrected static results of Tayal *et al* (1981). The positron cross-sections show important differences when compared with the corresponding electron scattering results. In the absence of any experimental data at present it is rather difficult to comment on the accuracy of the various approaches. In the case of electron-atom scattering the present approach provides a good agreement with the experimental data. However, the present approach does not include long-range polarisation effects. We therefore believe that the present approach would provide reasonable description of the positron-atom scattering, particularly if the target polarisability is low.

Table 1. Total elastic cross-sections (in units of πa_0^2) for positron scattering by helium.

Energy (eV)	Present results	Optical-model theory (Byron and Joachain 1977b)	EBS approach (Byron and Joachain 1977c)	Willis *et al* (1981)	Results of Mukherjee and Sural (1979)
100	0.267	0·172	0·366	0.242	0.193
200	0·162	0·131	0.162	0.162	0.151
300	0·108	0.105	0.115	—	0.112
400	0·086	0·086	0·092	—	0·088
500	0·073	0·073	0.076	—	0·073

Table 2. Total elastic cross-sections (in units of πa_0^2) for positron scattering by lithium.

Energy (eV)	Present results	Results of Mukherjee and Sural (1979)	Results of Tayal *et al* (1981)	
			Improved corrected static	Corrected static
10	19.73	21.80	23.95	53.44
20	9.22	9.63	9.14	16·87
60	3.44	—	3.33	4·20
100	2.19	2·15	2.13	2.45
200	1.22	1.17	1.14	1.22

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