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Charge imbalance due to thermoelectric effects in superconductors: A two fluid description

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Abstract. Using a generalised two-fluid picture for the charge of a superconductor, the generation of charge imbalance by a temperature gradient under different conditions is studied. The voltage developed by a temperature gradient in the presence of supercurrent is estimated. The results agree with experiment. The results obtained for the generation of charge imbalance in the absence of imposed current agree with those obtained using other techniques.

Keywords. Two-fluid model; charge imbalance; temperature gradient; quasiparticle charge; charge currents.

1. Introduction

In recent years the generation of charge imbalance in a superconductor due to a temperature gradient has received considerable attention. Falco (1977) attempted to measure the chemical potential difference between the normal and superfluid components generated by a temperature gradient ∇T alone. One of the difficulties in this experiment is that the effects to be detected are small. The voltages detected are of the order of 10^{-12} to 10^{-13} volts. Pethick and Smith (1979 b), using the Boltzmann equation approach developed by them earlier (Pethick and Smith 1979 a), suggested that a much larger charge imbalance voltage could be developed by the simultaneous application of a temperature gradient and an imposed current. They also predicted that an experiment of the type of Falco, but with an externally imposed current, would lead to a charge imbalance proportional to $v_s \cdot \nabla T$, where v_s is the superfluid velocity associated with the imposed current. In the clean limit for $\Delta \ll k_B T$ (Δ is the gap parameter) they found

$$\overset{*}{Q} = n\alpha \left\{(v_s \cdot \nabla T)/T\right\} \tau. \tag{1}$$

Here τ is the characteristic time for charge-imbalance relaxation, α, the dimensionless parameter tends to $\pi\Delta/4k_BT$ as the temperature approaches the transition temperature T_c,$n=(2/3)\,N(0)\,m\,v_F^2$ is the total electronic density, with v_F being the Fermi velcocity, m the effective mass and $N(0)$ the density of states at Fermi surface for one-spin population. Clarke *et al* (1979a) subsequently measured the Q^* in superconducting thinfilm strip. Their observations were in qualitative agreement with the predictions of Pethick and Smith with the reduced potential diverging as $(1-t)^{-1}$ for a given value of v_s and ∇T. However, the detected voltage is two or three orders of magnitude smaller than the predicted one. Further experimental evidence of these effects have been reported recently by Fjordboge *et al* (1981). Pethick and Smith (1979b) also calculated the charge imbalance in the absence of imposed current, exhibiting a term proportional to $\nabla^2 T$ by using the Ginzburg (1944) condition $J_s = -J_n$. Entin-Wohlman and Orbach (1980) using the microscopic approach and the second Ginzburg condition $J_n = -L_s \nabla T$, calculated the charge imbalance in the absence of imposed current. Here L_s is the thermoelectric coefficient in the superconductor given by (Bardeen *et al* 1959):

$$L_s/L_n = G\,(\Delta/k_B T), \tag{2}$$

where $$G\,(\Delta/k_B T) = 1 - (\tfrac{1}{2}\pi^2)\,(\Delta/k_B T)^3 \text{ for } T \simeq T_c. \tag{3}$$

The charge imbalance was found to depend on $\nabla^2 T + (\nabla T)^2$ and not solely on $(\nabla T)^2$. Using macroscopic approach, Tinkham (1980) incorporated the $\nabla^2 T$ term, which he argued arose from the divergence of normal current.

The purpose of the present paper is to combine features of a two-fluid description with microscopic calculations. In §2 we briefly review the Pethick and Smith (1979b) theory and extend it to account for experimental observations. In §3, the theory is extended to calculate charge imbalance due to non-uniform temperature gradient in the absence of imposed current. The results agree with those obtained using other techniques.

2. Generation of charge imbalance in a current carrying superconductor

The rate at which charge imbalance is generated in a current carrying superconductor due to a temperature gradient, taking into account the phonon scattering processes only, can be obtained by solving the Boltzmann equation:

$$(\mathrm{d}Q_n/\mathrm{d}t)_{\text{gen.}} = (\mathrm{d}Q_n/\mathrm{d}t)_{\text{coll.}}, \tag{4}$$

where Q_n is the charge associated with the normal component in thermal equilibrium and is given by (Mattoo and Singh 1982):

$$Q_n = \sum_{p,\,\sigma} q_p^0 f_{p,\sigma} \tag{5}$$

where q_p^0 is the equilibrium value of the effective charge of quasiparticles, given by (Pethick and Smith 1979a):

$$q_p = \zeta_p/E_p = (E_p^2 - \Delta^2)^{1/2}/E_p. \tag{6}$$

Here ζ_p is the normal state quasiparticle energy ϵ_p relative to chemical potential of the condensate μ_s and E_p is the quasiparticle energy:

$$E_p = (\zeta_p^2 + \Delta^2)^{1/2} = \{(\epsilon_p - \mu_s)^2 + \Delta^2\}^{1/2}. \tag{7}$$

The rate of quasiparticle charge generation is shown to be (Pethick and Smith 1979b):

$$(\mathrm{d}Q_n/\mathrm{d}t)_{\text{gen.}} = -\, n\, \alpha\, \{(v_s \cdot \nabla T)/T\}. \tag{8}$$

The collision term in (4) is written phenomenologically as:

$$(\mathrm{d}Q_n/\mathrm{d}t)_{\text{coll.}} = -\, \overset{*}{Q}/\tau. \tag{9}$$

where $\overset{*}{Q}$ is the deviation of Q_n from its local equilibrium value, corresponding to what has been referred to as $\delta Q_n^{\text{l.e.}}$ in the literature and defined as (Pethick and Smith 1979a):

$$\overset{*}{Q}_{\text{l.e.}} = \sum_{p,\,\sigma} q_p\, \delta f_p^{\text{l.e.}}. \tag{10}$$

Here $\delta f_p^{\text{l.e.}}$ is the deviation of the quasiparticle distribution function from its local equilibrium value. The value of μ_s is determined by the requirement that the total electronic density be the same in the presence of charge imbalance as in thermal equilibrium. Waldram (1975) showed that this condition requires

$$\overset{*}{Q} = -\, 2N(0)\, \delta\mu_s \tag{11}$$

where $\delta\mu_s$ is the difference between the actual value of μ_s and its value in thermal equilibrium. On combining (8) and (9) one gets an expression for charge imbalance given by (1). The potential developed between the normal and superfluid components due to charge imbalance is obtained from (Tinkham 1972):

$$eV = \overset{*}{Q}/2N(0)\, g_{\text{NS}} \tag{12}$$

where g_{NS} is the normalised tunnel conductance. τ_{in} (9) is the usual charge imbalance relaxation time due to inelastic scattering process, *i.e.*

$$\tau^{-1}_{\overset{*}{Q}} = (\pi\Delta/4k_B\, T)\, \tau^{-1}_{\text{in.}}\ (0) \tag{13}$$

where τ_{in} (0) is the normal state electron-phonon scattering time at the Fermi energy E_F. Thus (1) reduces to:

$$eV/E_F = (2/3e)\, \tau_{\text{in.}}\ (0)\ [(J_s \cdot \nabla T)/n_s\, g_{\text{NS}}\, T] \tag{14}$$

Here $J_s = e\, n_s\, v_s$ is the supercurrent with n_s being the superfluid density. Close to T_c, the temperature dependence is dominated by $n_s^{-1} \sim (1-t)^{-1}$, in accordance with experimental observations of Clarke *et al* (1979b). However, the estimated voltage exceeds the experimental values by two to three orders of magnitude. Further the characteristic relaxation time τ calculated from the experimental results is considerably less than its estimated value from phonon scattering processes alone. This discrepancy may be due to the dominance of impurity scattering processes in the experimental samples. If the relaxation time τ in (1) is replaced by temperature-independent impurity scattering time $\tau_{\text{imp.}}$ the order of magnitude is approximately the same at temperature $T=0{\cdot}9\ T_c$, with $(1-t)^{-1/2}$ type temperature dependence.

In defining (10) it is assumed that μ_s and hence q_p have their equilibrium values, like in injection type experiment and the change in distribution function alone gives change in quasi-particle charge Q_n or $\overset{*}{Q}$. The temperature gradient in the presence of supercurrent shifts the chemical potential of the condensate resulting in the shift of excitation spectrum in such a way that in the absence of inelastic scattering process a given quasiparticle remains at the same value of μ_s but changes its excitation energy and hence the effective charge q_p. In such a non-equilibrium state (10) is modified to a more general form:

$$\overset{*}{Q} = \sum_{p,\,\sigma} \{q_p\, \delta f_p^{\text{l.e.}} + \delta\, q_p\, f_p^{\text{l.e.}}\}, \tag{15}$$

where the deviation of the quasiparticle charge from its equilibrium value due to shift in chemical potential of condensate is given by (Kadin *et al* 1980):

$$\delta q_p = \delta\mu_s\, (\partial/\partial\mu_s)\, (\zeta_p/E_p) = -\ \delta\mu_s\, (\Delta^2/2E_p^3) \tag{16}$$

From equations (11), (15) and (16), we have:

$$\overset{*}{Q} = \sum_{p,\,\sigma} q_p\, \delta f_p^{\text{l.e.}} \Big/ \left\{1 - \frac{1}{N(0)} \sum_{p,\,\sigma} f_p^{\text{l.e.}} \frac{\Delta^2}{E_p^3}\right\} \tag{17}$$

For $T \sim T_c$, the denominator is small but positive:

$$1 - \frac{1}{N(0)} \sum_{p,\,\sigma} f_p^{\text{l.e.}} \frac{\Delta^2}{E_p^3} = 1 - Z\,(T) \simeq \pi\, \Delta/4k_B\, T. \tag{18}$$

where the temperature function $Z(T)$ is discussed by Clarke *et al* (1979b). Using equations (1), (13), (17) and (18) the charge imbalance near T_c takes the form:

$$\overset{*}{Q} = \frac{a}{[1 - Z\,(T)]^{-1}}\, n\, \{(v_s \cdot \nabla T)/T\}\, \tau,$$

$$= \frac{\pi\,\Delta}{4\, k_B T}\, n\, \{(v_s \cdot \nabla T)/T\}\, \tau_{\text{in.}}\,(0). \tag{19}$$

This shows that close to T_c, the dependence of $\overset{*}{Q}$ on phonon scattering is weak. However, away from T_c the phonon scattering should play an essential role in charge imbalance relaxation process. This is in accordance with the result obtained by Schmid and Schon (1979), using microscopic approach.

As mentioned earlier the impurity scattering processes dominate over other scattering processes in Sn film used for charge imbalance relaxation experiments. It is worth noting that in absence of supercurrent scattering from impurities cannot change charge imbalance if the energy gap is isotropic. This follows from the fact that scattering occurs between states which are on the same branch of the spectrum and hence have the same energy E_p. Thus the effective charge q_p associated with these states remains unaltered. In the presence of supercurrent, however, the scattering occurs between states having same energy in the laboratory frame E_p' (where the superfluid moves with velocity v_s *i.e.* $E_p'=E_p+P\cdot v_s$) and hence the effective charge q_p will change in collision process. In clean superconductors, where the impurity concentration is small so that the mean free path is larger than the zero temperature coherence length, the impurity scattering process produce no change in quasiparticle charge for the isotropic energy gap. However, if the gap is anisotropic, the impurity scattering processes in which quasiparticles remain on the same branch and branch-mixing processes give rise to change in quasiparticle charge. In order to calculate the effect of impurity scatterings, one must solve the Boltzman equation by including all the coherence factors in the scattering matrix elements and allowing for scatterings by both magnetic and nonmagnetic impurities. In practice it is a complicated task, as one never enters into a domain where the scattering by impurities is the only important mechanism. It is necessary to consider the combined effects of impurities and phonons simultaneously. Schmid and Schon (1979) and Beyer Nielson *et al* (1980) solved the Boltzmann equation in various limits. Their solutions are valid mostly in the clean limit, where $1/\tau_{\text{imp.}}$ is small. We consider the simple case for $1/\tau_{\text{imp.}} \ll 1/\tau_{\text{in.}}(0)$, which can be treated by the perturbation theory starting from the solution to the problem of phonon scattering. We find

$$1/\tau=\frac{\pi\Delta}{4k_B T}\frac{1}{\tau_{\text{in.}}(0)}+\frac{\Delta}{k_B T}\frac{1}{\tau_{\text{imp.}}}. \tag{20}$$

Consider first the limit where further electron phonon scattering is completely neglected, $1/\tau_{\text{in.}}(0)=0$. Under these conditions (19) simplifies to:

$$\overset{*}{Q}=2F(\Delta, T)\, n\, \{(v_s\cdot\nabla T)/T\}\, \tau_{\text{imp.}} \tag{21}$$

where $F(\Delta, T)=(k_B T/2\Delta)^{-1}$ is the dimensionless parameter. As a first approximation we can take the impurity scattering mean free path $l_{\text{imp.}}$ equal to its normal state value $v_F\tau_{\text{imp.}}$, we then get:

$$\frac{\overset{*}{Q}}{2N(0)}=eV\, g_{\text{NS}}=(2/3)\, P_F\, l_{\text{imp.}}\, \{(v_s\cdot\nabla T)/T\}\, F(\Delta, T). \tag{22}$$

Near T_c Schmid and Schon (1979) derived an analytical result for the voltage, which corresponds to $F=(1/\pi)\ln(8\Delta\tau_{\text{in.}})$. Our numerical results are in good agreement with this value, since at $T = 0{\cdot}99\ T_c$ one has $(1/\pi) \ln (8\Delta\, \tau_{\text{in.}}). \simeq 1{\cdot}8$ for Sn.

To facilitate comparison with experiments we divide our calculated voltage with the temperature gradient and the total current $I = A\, n_s\, e\, v_s$, where A is the cross-sectional area and obtain:

$$\frac{V g_{NS}\, T}{I\, \nabla T} = \frac{1}{\pi}\, \frac{\hbar}{e^2\, N(0)}\, \frac{F(\Delta, T)}{\Delta \tanh\, (\Delta/2T)}\, \frac{1}{A} \tag{23}$$

after using

$$n_s = \frac{\pi}{2}\, \Delta^2\, \sigma\, m/e^2\, T_c \tag{24}$$

near T_c. Here σ is the normal state conductivity.

In figure 1 we plot the left side of (23) *versus* $(1 - t)$ for the sample given in § 4 of Clarke *et al* (1979 a) to check the temperature dependence, by setting $v_F = 0{\cdot}65 \times 10^8$ cm/sec., measured value of $l_{\text{imp.}} = v_F\, \tau_{\text{imp.}} = 4300$ A. U. and the cross-sectional area $A = 2{\cdot}9 \times 10^{-7}$ cm². In figure 2 the $V g_{NS}/I\, \nabla T$ *versus* reduced temperature for the above mentioned sample is plotted. The agreements are fairly good. In figure 3 we plot $V\, g_{NS}\, T(1 - t^4) / I \cdot \Delta\, T$ *versus* reduced temperature for sample § 3 of Fjordboge *et al* (1981) for which our estimates of V/I are expected to be more reliable. In the above calculations $N(0)$ was taken to be equal to three times the free electron value. The value of $N(0)$ deduced from heat capacity measurements is 1·3 times the free electron value, which would lead to the theoretical value 2·3 times larger than the one shown in figure 3. Clearly the band effects are

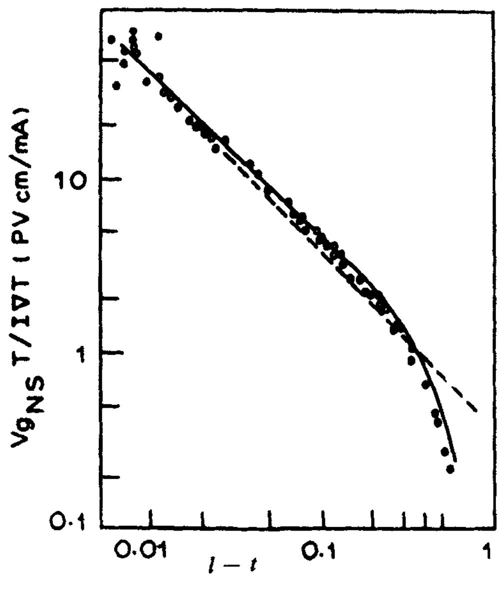

Figure 1. $Vg_{NS}\ T/I \cdot \nabla T$ *vs* $(1 - t)$, where $t = T/T_c$ for sample § 4 of Clarke *et al* (1979a). (·) represents experimental data, (- - -) has a slope of −1, (—) is equation 23 fitted at one point.

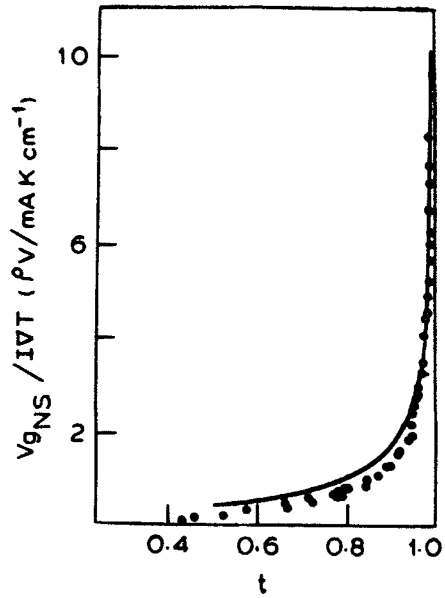

Figure 2. $Vg_{NS}/T \cdot \nabla T$ *vs* reduced temperature. (—) represents equation (23) and (·) are the experimental data for sample § 4 of Clarke *et al* (1979a).

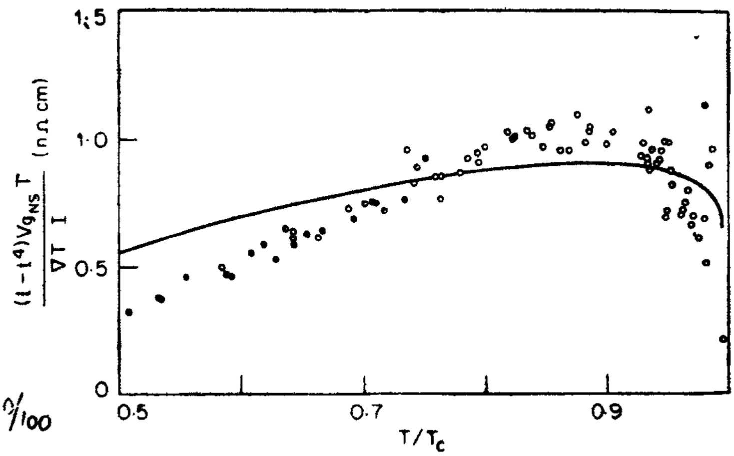

Figure 3. Comparison of theoretical and experimental values of $(1-t^4)\ Vg_{NS}\ T/I \cdot \nabla T$ for sample § 3 of Fjordboge *et al* (1981). The factor $(1 - t^4)$ is included to remove most rapid temperature dependence.

important in an isotropic material like Sn and it would not be reasonable to expect much better agreement than this within our simple model. It is worth noting that our estimated voltage is π times that of Clarke and Tinkham (1980), who used a quite different approach to derive results for arbitrary temperature.

3. Generation of charge imbalance in absence of imposed current

When spatial variations in the quasiparticle distribution function are present, Q_n will change locally for transport of normal charge from adjacent regions. Thus equation (4) should be generalised to:

$$(\mathrm{d}\, Q_n/\mathrm{d}t)_{\mathrm{gen.}} + \nabla \cdot J_n^Q = (\mathrm{d}\, Q_n/\mathrm{d}t)_{\mathrm{coll}}. \tag{25}$$

where J_n^Q is the current associated with Q_n, referred to as the current of normal charge, given by (Pethick and Smith 1979a):

$$J_n^Q = \sum_{p,\,\sigma} q_p\, v_p\, \delta f_p^{\mathrm{l.e.}} \tag{26}$$

The presence of J_n^Q in (25) indicates that the quasiparticle charge can change in a given region not only through scattering processes but also through net inflow and outflow of quasiparticle charge. Thus the divergence term in (25) acts to produce a direct change in distribution function so that its contribution is subjected to an enhancement factor of $4\, k_B T/\pi\Delta$ near T_c. Further, in steady state situation the change in distribution function alone corresponds to change in Q_n. However, in situations where spatial variations are also present, μ_s and hence q_p change from their equilibrium values and hence contribute to the total change in Q_n. From equation (17) it is evident that this contribution of q_p for total change in Q_n is due to the response of the superfluid, which in turn is responsible for maintaining electroneutrality. This acts to enhance the change in Q_n due to the change in distribution function alone by a factor $4\, k_B T/\pi\,\Delta$. Thus (25) must be generalised to (Kadin *et al* 1980):

$$\left(\frac{\mathrm{d}\, Q_n}{\mathrm{d}t}\right)_{\mathrm{gen.}} = -\frac{4\, k_B T}{\pi\,\Delta}\{(\overset{*}{Q}/\tau) + (\nabla \cdot J_n^Q\} \tag{27}$$

In a superconductor a temperature gradient leads to a flow of excitations and hence to a quasiparticle current by the usual mechanism which generate the thermoelectric currents in the normal metal. This in turn will give rise to a change in quasiparticle charge. Since our interest is in the quasiparticle charge current and not the total current of quasiparticles, one may write phenomenologically

$$J_n^Q = -\, L_s^Q\, \nabla T. \tag{28}$$

It is obvious that for a given temperature, L_s^Q should be less than L_s as $J_n^Q < J_n$. The current of normal charge associated with deviation from local equilibrium due to a thermal gradient is given by:

$$J_n^Q = e \sum_{p,\,\sigma} q_p\, v_p\, (E/T) \frac{\partial f^0}{\partial E}\, v_p \cdot \nabla T\, l_{\mathrm{imp.}} \tag{29}$$

where $l_{\mathrm{imp.}}$ is the transport mean free path due to impurity scattering, and is the same in the superconducting and normal state (Bardeen *et al* 1959). In (29) $eq_p \cdot v_p =$ $= e\, q_p^2\, p/m$ (as $v_p = q_p\, p/m$) is the electric current carried by quasiparticles constituting the normal charge and $(E/T)\ \partial f^0/\partial E$ originates from the driving term $v \cdot \nabla f$ in the Boltzmann equation for the case of a temperature gradient. Converting the sum into an integral over the normal state energy variable ξ, one finds (28) with

$$L_s^Q = (1/T) \int_{-a}^{a} d\xi\ (\xi^3/E)\ (-\ \partial f^0/\partial E)\ H(\xi)\ \text{sgn}\ \xi. \tag{30}$$

Here $H(\xi) = 2/3\{P(\xi)/m\} N(\xi)\ l_{\text{imp.}}$. $N(\xi)$ is the density of states and sgn ξ reflects the fact that quasiparticle velocity changes the sign at Fermi surface. Expanding H about $\xi = 0$, one finds:

$$L_s^Q = \beta T\ \tilde{G}\ (\Delta/k_B T), \tag{31}$$

where $\beta = (\pi^2/3)\ H(0)\ k_B^2$, which is related to the thermoelectric coefficient of the normal component by $L_n^Q = \beta T$ and

$$\tilde{G}\ (\Delta/k_B T) = 1 - (3/\pi^2)\ (\Delta/k_B T)^2 \text{ for } T \sim T_c \tag{32}$$

The presence of J_n^Q in (27) changes the quasiparticle charge density at the rate of $-\ \nabla \cdot J_n^Q$, which in a steady-state situation must be balanced by the rate at which the quasiparticle charge is converted into condensate charge. However, when the spatial variations are present the change in quasiparticle charge density will produce a corresponding change in superfluid charge density thus constituting a superfluid charge current. In the absence of imposed current the flow of J_n^Q, driven by a temperature gradient is locally cancelled by counterflow of superfluid charge current, *i.e.* (Mattoo and Singh 1983):

$$J_n^Q = -\ J_s^Q. \tag{33}$$

Here J_s^Q is the current associated with the Q_s, referred to as superfluid charge current. This consists of the usual supercurrent J_s plus the current associated with quasiparticle charge, *i.e.* which at temperature close to T_c can be appraximated as

$$J_s^Q = n_s\, v_s + n_n\, v_s = n\, v_s. \tag{34}$$

Here n is the total density and n_n is the density of the normal component. Using (8), (28), (33) and (34), (27) leads to:

$$\overset{*}{Q} = \alpha\, \tau_{\text{in.}}\ (0)\ L_s^Q\ (\nabla T)^2/T + \tau \left\{ L_s^Q\ \nabla^2\ T + \frac{\partial L_s^Q}{\partial T}\ (\nabla T)^2 \right\}, \tag{35}$$

where τ is the inelastic charge imbalance relaxation time given by (13). Using (31)–(33), equation (35) takes the form:

$$\overset{*}{Q} = L_n^Q\, \tau\ [\{\nabla^2\, T + (\nabla T)^2/T\} - (3/2)\ (\Delta/k_B T)^2 \times \{\nabla^2\, T - (\nabla T)^2/T\} - (3/16)\ (\Delta/k_B T)^4\ (\nabla T)^2/T]. \tag{36}$$

At temperature close to T_c, $L_n^Q \simeq L_n$ and contribution due to the last term is negligible. Equation (36) reduces to an expression obtained by Entin-Wohlman and Orbach (1980) using microscopic approach.

The second term in (35) appears to be due to divergence of normal charge current. Since $\partial L_s^Q/\partial T$ is expected to be of the order of L_s^Q/T_c, the $(\nabla T)^2$ term is smaller than $\nabla^2 T$ term by a factor of order

$$(\nabla T)^2/T_c \nabla^2 T,$$

and is normally negligible. Thus (35) simplifies to:

$$\overset{*}{Q} = \tau L_n^Q \left\{\frac{\pi \Delta a}{4 k_B T} \frac{(\nabla T)^2}{T} + \nabla^2 T\right\} \tag{37}$$

which shows that the coefficient of $(\nabla T)^2$ term is roughly down by a factor $(\pi\Delta/4 k_B T)^2$ and hence close to T_c, we have

$$\overset{*}{Q} = \tau L_n \nabla^2 T. \tag{38}$$

This is similar to the result obtained by Tinkham (1980). It is worth noting that the voltage observed by Falco (1977), whose sign and approximate magnitude remains invariant for reversal of temperature gradient and which is linear in $|\nabla T|$, cannot be explained on the basis of the presence of uniform temperature gradient. The outcome of Falco's experiment may be explained by a non-uniform temperature gradient resulting in a charge imbalance similar to equation (38). However, it needs further experimental investigations to test the hypothesis.

4. Conclusions

In this paper we have extended a formalism based on two-fluid model for the charge of a superconductor, developed by Pethick and Smith (1979 a, b), to account for latest experimental observations and theoretical expressions.

As far as comparison with experiments is concerned, we have shown that in case of charge imbalance generated by a temperature gradient and supercurrent, our results agree reasonably well with the measurements of Clarke *et al* (1979a) and Fjordboge *et al* (1981).

For the generation of charge imbalance by a temperature gradient in the absence of imposed current, our results are in good agreement near T_c, with those obtained by Entin-Wohlman and Orbach (1980) and Tinkham (1980) under quite different approaches.

The approach outlined in this paper combines the two-fluid model with microscopic calculations. It complements the general Green's function approach, which has played a key role in understanding various non-equilibrium effects in superconductors.

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References

Bardeen J, Rickayzen G and Tiwardt L 1959 *Phys. Rev.* **113** 982
Beyer Nielson J, Ono Y A, Pethick C J and Smith H 1980 *Solid State Commun.* **33** 925
Clarke J, Fjordboge B and Lindelof P E 1979a *Phys. Rev. Lett.* **43** 642
Clarke J, Eckren U, Schmid A, Schon G and Tinkham M 1979b *Phys. Rev.* **B20** 3933
Clarke J and Tinkham M 1980 *Phys. Rev. Lett.* **44** 106
Entin-Wohlman O, Orbach R 1980 *Phys. Rev.* **B22** 4271
Falco C M 1977 *Phys. Rev. Lett.* **39** 660
Fjordboge B, Lindelof P E and Clarke J 1981 *J. Low Temp. Phys.* **44** 535
Ginzburg V L 1944 *J. Phys.* (*USSR*) **8** 148
Kadin A M, Smith L N and Skocpol W J 1980 *J. Low Temp. Phys.* **38** 497
Mattoo B A and Singh Y 1982 *Pramana* **19** 483
Mattoo B A and Singh Y 1983b *Prog. Theor. Phys.* (*Jpn.*) **70** 1
Pethick C J and Smith H 1979a *Ann. Phys.* (*NY*) **119** 133
Pethick C J and Smith H 1979b *Phys. Rev. Lett.* **43** 640
Schmid A and Schon G 1979 *Phys. Rev. Lett.* **43** 793
Tinkham M 1980 *Phys. Rev.* **B22** 2594
Tinkham M 1972 *Phys. Rev.* **B6** 1747
Waldram J R 1975 *Proc. R. Soc. London* **A345** 231