

Local charge compensation in the quark-cascade jet-production model

S P MISRA and B K PARIDA*

Institute of Physics, Bhubaneswar 751 005, India

*Present address: Department of Physics, Utkal University,
Vani Vihar, Bhubaneswar 751 004, India

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Abstract. The charge correlations and the local charge compensation mechanism in rapidity space for quark jets are analysed in the framework of the quark-cascade jet-production model. The calculated results are compared with those observed experimentally in antineutrino-nucleon processes, and for the short range charge correlations observed in e^+e^- annihilation experiments. The results appear consistent with the quark-cascade model.

Keywords. Quark-cascade; jet-production model; charge correlations; rapidity distributions; local charge compensation.

1. Introduction

Local charge compensation (Krzywicki and Weingarten 1974; Bromberg *et al* 1975) mechanism in rapidity has been recently examined (Orava 1981; 1982) for the quark jets in antineutrino nucleon collisions. The same has also been analysed for hadron production in e^+e^- annihilation experiments (Brandelik *et al* 1981), which naturally corresponds to hadronization of quark-antiquark pairs. The theoretical basis for these hadronic collisions had been models (Snider 1975; Pinsky *et al* 1973) based on Muller-Regge ideas or multiperipheral analysis.* A cluster emission model had also been proposed (Quigg *et al* 1975). However, these are not very appropriate for consideration *e.g.* the quark jets in antineutrino-nucleon collisions, or in e^+e^- annihilation to hadrons. For such processes, the quark fragmentation model (Krzywicki and Peterson 1972; Finkelstein and Peccei 1972; Niedermayer 1974; Field and Feynman 1978; Sukhatme 1978; Misra and Panda 1980) may be relatively more appropriate, particularly in the context of the calculations and measurements of quark charge (Berge *et al* 1981). We shall consider here the mechanism of local charge compensation in the framework of the quark-cascade jet model and compare the results with the observations for quark jets (Orava 1981), and the short-range charge correlations in e^+e^- annihilation experiments (Brandelik *et al* 1981). The observation here is that the local charge compensation comes naturally in the quark-cascade picture of hadronization.

The paper is organised as follows. In § 2, we give the definitions for the analysis of the data indicating local charge compensation in rapidity developed earlier for hadron-hadron collisions (Bromberg *et al* 1975). We further discuss qualitatively the

*The treatment of Snider (1975) is essentially based on Chew and Pignotti (1968). Pinsky *et al* (1973) showed the similarity of multiperipheral and Muller-Regge concepts.

quark-cascade model in the above context, and note the gross expected nature of the experimental observations in this model. In § 3 we re-express the quark-cascade model in terms of rapidity and examine the charge correlations in a quark jet. Section 4 gives the application of the results and a comparison with experiments.

The local charge compensation mechanism as it arises from an extension of the quark-cascade jet-production model has been discussed (Misra *et al* 1980, 1981, 1982). Another explanation for the observed local charge compensations has been resonance production (Hayot 1975).[†] It is also otherwise obvious that such effects will be present. However, more specific conclusions can be drawn when one has an adequately reliable estimate of resonance production as well as the corresponding rapidity distributions, in addition to the effects of the quark-cascade model.

2. Local charge compensation

Local compensation of charge in rapidity has been known for a long time for hadronic collisions (Krzywicki and Weingarten 1974; Bromberg *et al* 1975). The presence of this mechanism is best illustrated with a zone analysis (Bromberg *et al* 1975) where hadrons are first arranged according to their rapidities in a definite order and the cumulative hadronic charge is determined starting from one end of the hadronic chain. The region where the cumulative charge is nonzero is defined as a zone and that where it is zero is defined as a gap.* A zone has two oppositely-charged hadrons at the two end points, and contains an even number of charged particles, and is flanked by gaps. For the quark jets in antineutrino collisions (Orava 1981), the forward zones I are defined as those zones which have only positive rapidity in the centre of mass frame of reference of the hadronic subsystem. When one of the zones is further allowed to contain the zero rapidity region, they are known as forward zones II (Orava 1981, 1982). Obviously, all these zones essentially 'belong' to the quark jet.

The relevance of the zones lies in the fact that they reflect clearly the local charge compensation mechanism (Bromberg *et al* 1975; Orava 1981). In the present context we first illustrate this with a trivial version of the quark-cascade jet model. Here, we consider only u or d quarks producing pions, such that they take a constant fraction of the momentum of the fragmenting quark. This implies that the pions are produced uniformly in rapidity with a spacing of say y_0 . Further, here we shall have exactly *two* charged particles in each zone Z , such that n_Z , the charged multiplicity of the zone Z , is always *two*. Again, as explained later, the fragmentation of a u -quark will yield a π^+ with twice the probability that it will yield a π^0 . Thus, for a large number of zones Z , $\langle \lambda_z \rangle$, the average length in rapidity of the zones, will be given as $\langle \lambda_z \rangle = 1.5 y_0$. We may contrast this result with that of the random charge distribution. It has been shown by Bromberg *et al* (1975) that for such random charges of the pions with no local compensation of charge, $\langle n_Z \rangle$ and $\langle \lambda_Z \rangle$ will continue to rise as the number of zones increases. With a simple combinatorial analysis, it can be shown for instance that with a total of ten charged particles in a jet, we have for random charges $\langle \lambda_z \rangle = 2.6 y_0$ instead of $1.5 y_0$ as above. This difference can be easily understood from the fact that π^+ , π^0 , π^- follow a π^+ with equal probabilities ($\frac{1}{3}$, $\frac{1}{3}$,

[†]Hayot (1975) related short-range charge correlations to resonance production.

*We follow the notations of Orava (1981) for the zone analysis of quark jets.

$\frac{1}{3}$) for the random charges, whereas in the quark-cascade model, the next rank meson (Field and Feynman 1978; Sukhatme 1978) is π^+ , π^0 or π^- , with (Misra *et al* 1980, 1981, 1982) probabilities $(0, \frac{1}{3}, \frac{2}{3})$ respectively. This obviously implies a faster compensation of charge. The probabilities arise since the d -quark cannot yield a π^+ at the primary level, and further, since π^0 has half the d -quark content as compared to π^- .

We now compare these results with the experimental observations. It is first noted that for quark jets (Orava 1981) $\langle n_z \rangle \simeq 2$ to 2.6 depending on the definitions of the forward zones quoted earlier. Further, y_0 as calculated (Field and Feynman 1978; Misra and Panda 1980) and observed is around 0.7, such that we may expect $\langle \lambda_z \rangle \simeq 1$ which, again, is as observed for quark jets with forward zones II (Orava 1981). These constitute a gross indication that the quark-cascade model may explain the phenomenon of local charge compensation as observed.

The assumption that the meson has a constant fraction of the momentum of the fragmenting quark is really a crude approximation. In fact, with the fragmentation of (u, d) quarks to pions, the fragmentation function (Field and Feynman 1978; Misra and Panda 1980) will be given as (Misra *et al* 1980, 1981, 1982)

$$f_{iJ}(z) = \left(\frac{1}{3} + \frac{2}{3} \tau_1\right)_{iJ} f_{\pi}^{\prime}(z), \quad (1)$$

where isotopic spin dependence is included. As noted earlier, the production of the appropriate charged pions at any stage has *twice* the probability of the neutral pion being produced. The fragmentation function $f_{\pi}^{\prime}(z)$ in (1) can have different approximations and is the basic input in the quark-cascade jet-production models. This has been used to obtain the hadronization to many exclusive channels in Misra *et al* (1980, 1981, 1982). We shall now develop it in the next section to obtain quantitatively the charge compensation which arises from the quark-cascade model. We may note in particular that (Field and Feynman 1978; Sukhatme 1978) $y_0 \simeq \int \ln \{1/(1-z)\} f_{\pi}^{\prime}(z) dz$ yields the rapidity interval per particle, which can be calculated as 0.7.

As stated earlier the results will be further affected by resonance production as well as by quantum chromodynamic 'fragmentations' (Altarelli and Parisi 1977; Field 1981) which will be in addition to effects considered here. As estimated* earlier (Misra *et al* 1981), these will be relevant at high energies (W (hadronic) $\gtrsim 8$ –10 GeV).

3. Charge correlations in the quark-cascade model

We shall now consider charge correlations in rapidity for quark jets in the quark-cascade jet-production model. We shall illustrate the model with equation (1), with only u and d quarks, and with production of pions only. This always forms the

*It has been seen in Misra *et al* (1982) from an analysis of multiplicity structures, that the quark-cascade model is reasonable at moderate energies ($2 \text{ GeV} \lesssim W \lesssim 8$ –10 GeV), but as the energy increases, it needs to be supplemented by quantum chromodynamic effects. See also, Field (1981).

dominant mode for fragmentation. Clearly, $f_\pi(z)$ in (1) is normalised to unity. At high energies, for a meson in the quark jet we have rapidity

$$\eta = \frac{1}{2} \ln [(E + p_{||})/(E - p_{||})] \simeq \ln (2 p_{||}/m_\perp).$$

Let us now consider two successive fragmentations of a quark q of initial momentum P . Then, the momenta of the corresponding mesons are respectively given as $P z_1$, and $P(1 - z_1) z_2$ with a differential probability $f_\pi(z_1) dz_1$, $f_\pi(z_2) dz_2$, and their respective rapidities are $\eta_1 = \ln (2 P z_1/m_\perp)$ and $\eta_2 = \ln [2 P(1 - z_1) z_2/m_\perp]$. This yields that, between two mesons of consecutive ranks, the algebraic rapidity interval $\eta \equiv \eta_1 - \eta_2$ will occur with a probability $g(\eta) d\eta$, where

$$g(\eta) = \int f_\pi(z_1) dz_1 f_\pi [\{z_1/(1 - z_1)\} e^{-\eta}] \{z_1/(1 - z_1)\} e^{-\eta}. \tag{2}$$

In (2), we explicitly take $f_\pi(z) = 0$ for $z < 0$ and for $z > 1$, these conditions will automatically define the appropriate ranges for the integration variables. Clearly, η may be either positive or negative. If η is positive the order in rapidity is the same as the order of fragmentation, and when η is negative, the order in rapidity will get reversed with respect to the fragmentation order. We note that it is the order in rapidity which is observed, and, the order in fragmentation which is responsible for the local charge compensation. We call $g(\eta)$ as the 'primordial' distribution function in rapidity for successive mesons in the fragmentation order. Obviously, $\int_0^\infty g(\eta) d\eta$ is the probability that the rapidity order is the same as the fragmentation order and, $\int_{-\infty}^0 g(\eta) d\eta$ is the probability that these orders are reversed. Clearly, from $\int_0^1 f_\pi(z) dz = 1$, we obtain that $\int_{-\infty}^\infty g(\eta) d\eta = 1$. As we shall see later, the reasonableness of some of the results of § 2 corresponds to the fact that $\int_{-\infty}^0 g(\eta) d\eta$ is relatively small.

For the definition of zones, we need only consider the charged pions, π^0 's being irrelevant. As far as the fragmentation order is concerned, the charged particle following a π^+ is always a π^- . Let the algebraic rapidity interval between a π^+ and the next π^- in this order be η . Then, the following situations may be obtained: π^- may just follow the π^+ or, there may be k π^0 's inbetween, with $k \geq 1$. In general, let us have a π^+ followed by k π^0 's and then by a π^- . This corresponds to the fact that the d -quark following the π^+ fragments k times yielding each time a π^0 , and then fragmenting to a π^- . From equation (1) the probability for the yield of π^0 has a factor of $(\frac{1}{3})$ and, that of π^- , a factor of $(\frac{2}{3})$. Hence, with $\eta_1, \eta_2, \dots, \eta_{k+1}$ ($k \geq 0$) being the respective algebraic rapidity intervals between the successive mesons in the fragmentation order, the above process has the differential probability given by

$$(\frac{1}{3})^k (\frac{2}{3}) g(\eta_1) d\eta_1 g(\eta_2) d\eta_2, \dots, g(\eta_{k+1}) d\eta_{k+1}. \tag{3}$$

Further, the total algebraic rapidity interval between the π^+ and the π^- above is given as $\eta = \eta_1 + \eta_2 + \dots + \eta_{k+1}$. Thus, adding all the above types of contributions we

obtain the corresponding distribution function $G(\eta)$ in rapidity for successive charged pions as

$$G(\eta) = \sum_{k=0}^{\infty} \left(\frac{2}{3}\right)^k \int g(\eta_1) d\eta_1 g(\eta_2) d\eta_2 \dots g(\eta_{k+1}) d\eta_{k+1} \times \delta(\eta - \eta_1 - \eta_2 \dots \eta_{k+1}). \tag{4}$$

We obviously have $\int_{-\infty}^{\infty} G(\eta) d\eta = 1$, as expected. If

$$\tilde{g}(\lambda) = \int_{-\infty}^{\infty} g(\eta) \exp(-i\eta\lambda) d\eta, \tag{5}$$

then we obtain

$$\tilde{G}(\lambda) = \frac{\left(\frac{2}{3}\right) \tilde{g}(\lambda)}{1 - \left(\frac{2}{3}\right) \tilde{g}(\lambda)}, \tag{6}$$

such that the series (4) gives

$$G(\eta) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{2 \tilde{g}(\lambda)}{3 - \tilde{g}(\lambda)} \exp(i\eta\lambda) d\lambda. \tag{7}$$

We may be able to solve (7) with contour integration techniques by considering the equation $\tilde{g}(\lambda) = 3$ in some simple cases. For the general case, however, (4) is much more useful, particularly since the factors $\left(\frac{2}{3}\right)^k$ ensure rapid convergence. Obviously, this rapidity interval distribution function $G(\eta)$ for the charged pions will be relevant to us for a zone analysis. η here may be both positive and negative, whereas for the zone analysis, we need to convert the above information to the observed order in rapidity. We shall do this and consider some applications to quark jets (Orava 1981) as well as the quark-antiquark jets in e^+e^- annihilations (Brandelik *et al* 1981).

4. Applications

We note that the present rapidity analysis of charge correlations is based on the 'primordial' fragmentation function $f_{\pi}(z)$. We illustrate the results of the present model by taking $f_{\pi}(z) = (d\sigma/dz) / \sigma_{\tau}$, as defined earlier (Misra and Panda 1980; Misra *et al* 1980, 1981, 1982; Biswal and Misra 1981). In the context of the present paper this function is merely a phenomenological input, and is plotted in figure 1. The two corresponding primordial distribution functions in rapidity, $g(\eta)$ and $G(\eta)$ are next calculated according to (2) and (4) and plotted in figures 2(a) and 2(b) respec-

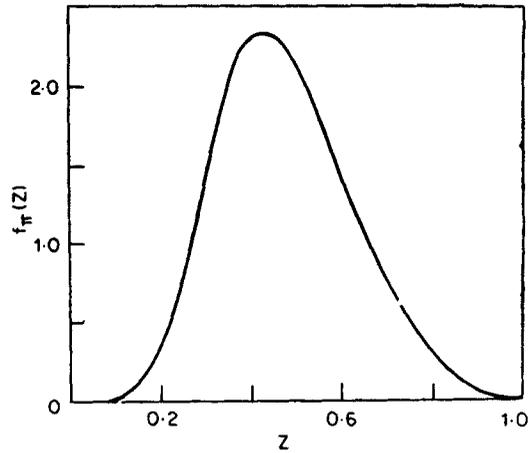


Figure 1. The primordial quark fragmentation function $f_{\pi}(z)$ plotted as a function of z according to Misra and Panda (1980).

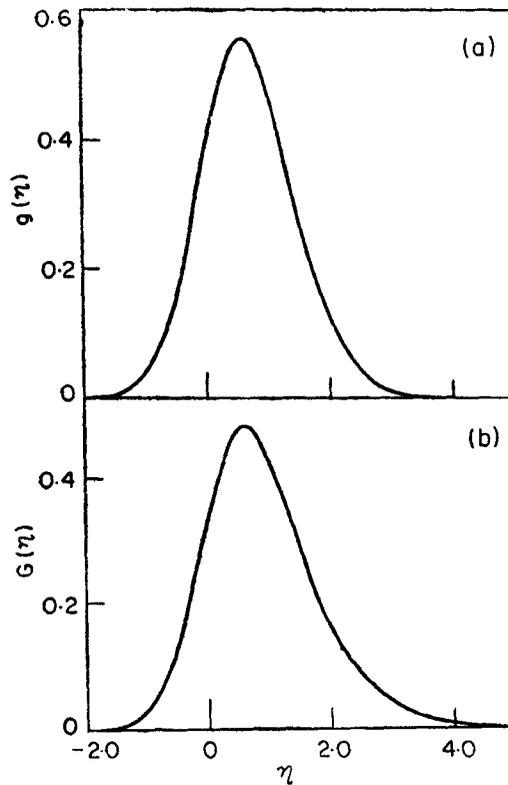


Figure 2. (a) The "primordial" rapidity distribution function as in equation (2) plotted vs the rapidity interval η . (b) The distribution function $G(\eta)$ for charged pions according to equation (4) is plotted as a function of η .

tively. One may see in particular the finite probability for a reversal of fragmentation order and the order in rapidity. In fact, for charged pions $p_1 \equiv \int_0^{\infty} G(\eta) d\eta = 0.845$

gives the probability for the order of fragmentation and the order in rapidity to be the same, whereas $p_2 \equiv \int_{-\infty}^0 G(\eta) d\eta = 1 - p_1 = 0.155$ gives the probability for a reversal of these orders.

We now analyse the zone length for quark jets (Orava 1981). For this purpose we discuss various possible order reversals in figure 3. Arrowhead indicates fragmentation order, and left to right, the order in rapidity. In figure 3(a), we consider the zone-gap-zone structure in rapidity with no order reversal. Here, and for the subsequent diagrams of figure 3, we note that we are discussing a situation where there is a negative charge at *A* and a positive charge at *D* against a constant background. The situation thus corresponds to the quark jet from one end. We note that the probability for figure 3(a) to occur is p_1^3 . Also, here $n_Z=2$. Figure 3(b) has one order reversal and hence has a probability $p_1^2 p_2$. It has however a zone charge $n_Z = 4$. Figures 3(c) and 3(d) also have one order reversal, and thus occur with probabilities $p_1^2 p_2$ but have zone charges $n_Z = 2$. We have similarly illustrated various other possible order reversals in figures 3(e-h), where the respective probabilities and the zone charges have been explicitly noted. With $p_1 = 0.845$ and $p_2 = 0.155$ as above, we next calculate $\langle n_Z \rangle$ with the respective probabilities. This yields that $\langle n_Z \rangle = 2.22$, which may be compared with the experimental values (Orava 1981) of $2 - 2.2$ for forward zones I. We may note, in particular, that the probabilities for $n_Z = 2$

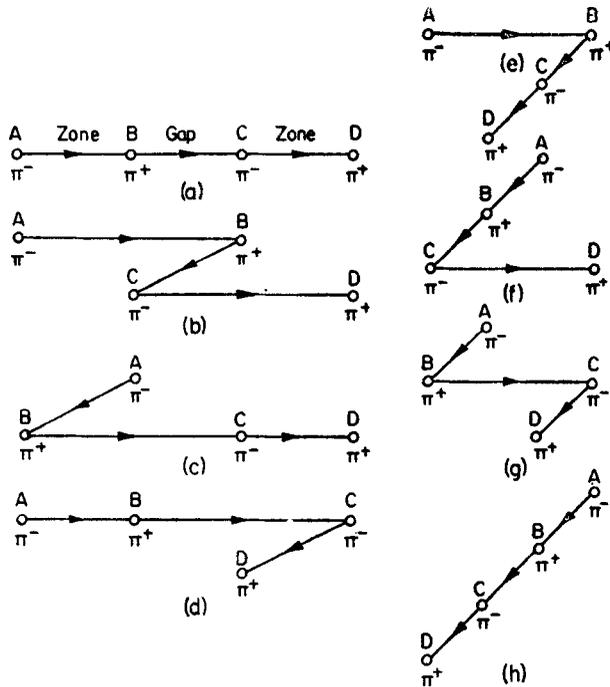


Figure 3. A few illustrative patterns of order reversal for fragmentations are shown. Arrowhead indicates fragmentation order and left to right indicates order in rapidity. The probabilities associated with the patterns (a)-(h) are respectively p_1^3 , $p_1^2 p_2$, $p_1^2 p_2$, $p_1^2 p_2$, $p_1^2 p_2$, $p_1 p_2^2$, $p_1 p_2^2$ and p_2^3 . The respective zone charges n_z are 2, 4, 2, 2, 2, 2, 2, and 2.

and $n_Z = 4$ respectively become 0.889 and 0.111. We now calculate the average rapidity extension of the zones of charge $n_Z=2$, which are abundant and are relatively simple. We note that figures 3(a, e, f) show the similar type of zone-gap-zone structure ($\pi^- \pi^+ \pi^- \pi^+$) and hence cannot be distinguished from one another experimentally. Similar is the case with figures 3(g,h) which correspond to the common zone-gap-zone structure ($\pi^+ \pi^- \pi^+ \pi^-$). Taking this indistinguishability into account we get $\langle \lambda_Z \rangle = 0.67$ units of rapidity whereas the experimental value (Orava 1981) is 0.57 ± 0.01 units of rapidity for forward zones I. The discrepancy can probably be accounted for by resonance production and decay which, however, is not well studied in rapidity space. However, the above analysis considers only a sequence of four charged particles, whereas we can have a larger number of charged particles as the energy increases. This ambiguity is present, but it does not have sizable effect since for a sequence of larger number (≥ 6) of charged particles, the correlation in rapidity is likely to be small. For the present quark jets, the probability of their production is even small. It is obvious that as the energy increases, Monte-Carlo simulation will be needed to retain some details regarding the effects of correlations among a larger number of charged particles. We plot the average number of zones $\langle N_Z \rangle$ vs the average charged multiplicity $\langle n_{ch} \rangle$ in figure 4 against the experimental points (Orava 1981) which further illustrates a reasonable agreement with the results for forward zones I. We note that, by definition, forward zones II include an overlap (Orava 1981, 1982) of the quark jet with the diquark jet in the case of antineutrino-nucleon processes. Thus, for example, the pion arising from the recombination of the residual quark-antiquark pair (antiquark coming from the diquark side) in the quark-cascade picture has to be taken into account for forward zones II. Qualitatively, this appears to lead to a larger $\langle \lambda_Z \rangle$ as expected. However, at high energies the effects of such an overlap will become insignificant, and the corresponding zone structure of the quark jet will be effectively the same as forward zones I. Thus, it appears that the observed local charge compensation for pions in quark jets can be a consequence of the quark-cascade jet-production model.*

We next proceed to show that the same quark-cascade mechanism can also explain the observed (Brandelik *et al* 1981) local charge compensation in the central region

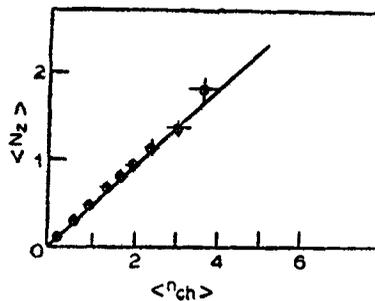


Figure 4. Average number of zones $\langle N_Z \rangle$ vs average charged multiplicity $\langle n_{ch} \rangle$ plotted against the experimental data of Orava (1981).

*It has been seen in Misra *et al* (1982) from an analysis of multiplicity structures, that the quark-cascade model is reasonable at moderate energies ($2 \text{ GeV} \lesssim W \lesssim 8\text{-}10 \text{ GeV}$), but as the energy increases, it needs to be supplemented by quantum chromodynamic effects. See also, Field (1981).

in e^+e^- annihilation experiments. Brandelik *et al* (1981) have defined a function $\tilde{\phi}_r(y, y')$ which is the probability for the charge of a particle produced at rapidity y' to be compensated at rapidity y . They have plotted $\tilde{\phi}_r(y, y')$ vs y for various y' intervals. Their conclusion is that in e^+e^- annihilation, both short range as well as long range charge correlations are present and, that for y' in the central region the correlation is mostly short range. The long-range effects are gradually felt as y' approaches the end region. These obviously arise from the fact that for e^+e^- annihilations, the quark and the antiquark will be oppositely charged and will be responsible for leading hadrons towards the two end regions in the rapidity plots. Clearly, these will not be directly caused by the cascade mechanism discussed, which is a short range effect. The $G(\eta)$ of (4) gives the short range algebraic rapidity interval for the next charged particle in fragmentation order. For simplicity we may assume that in the central region, charge is compensated locally by the nearest neighbours in a symmetric fashion. The expected error due to this assumption will have a probability $p_2^2 = 0.024$, *i.e.* will be about 2.5% only. This yields

$$\tilde{\phi}_r(y, y') = \frac{1}{2} [G(\eta) + G(-\eta)], \quad (8)$$

where $\eta \equiv y \sim y'$. The approximation is reasonable as more than one rapidity reversal will be uncommon as calculated above. We plot $\tilde{\phi}_r(y, y')$ from (8) for the two intervals, $-0.75 \leq y' \leq 0$ and $-1.5 \leq y' \leq -0.75$ in figures 5 (a, b) respectively. The agreement with the experimental results (Brandelik *et al* 1981) may be noted. For

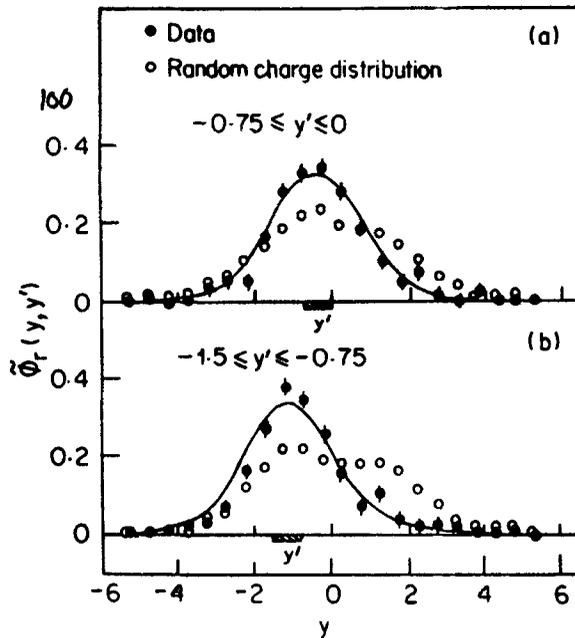


Figure 5. Charge compensation probability $\tilde{\phi}_r(y, y')$ according to equation (8) plotted vs y for (a) $-0.75 \leq y' \leq 0$ and (b) $-1.5 \leq y' \leq -0.75$. The experimental points, denoted by solid circles, are taken from Brandelik *et al* (1981). Also plotted are the results of random charge distributions as open circles (Brandelik *et al* 1981).

other y' intervals approaching the non-central regions, clearly, (8) will be inadequate because the present analysis does not include asymmetric effects due to the meson at y' containing an *end* quark, thus giving rise to long range effects. These long range correlations in the framework of the quark fragmentation model have been however correctly obtained by Brandelik *et al* (1981) and Ritter (1982) with $e^+e^- \rightarrow q\bar{q} + q\bar{q}g$ with a Monte-Carlo simulation of the jets. Our purpose here has been to identify the short range correlation function $G_c(\eta) \equiv \frac{1}{2}[G(\eta) + G(-\eta)]$ without the need for Monte-Carlo calculations.

We have derived the primordial rapidity distribution function from the primordial quark fragmentation function used earlier (Misra and Panda 1980; Misra *et al* 1980, 1981, 1982). The primordial fragmentation function as calculated (Misra and Panda 1980) is given in terms of the wavefunctions of the hadrons. The method thus quantitatively relates the structure of the hadrons explicitly with the generation of hadrons from quarks. It naturally includes an order reversal in rapidity. The calculated function is next used for a zone analysis of the quark jet. Thus, without adjusting the parameters, and, interestingly from the wavefunction of the pion, the experimental observations of zone analysis get generated. It may be conjectured that the forward zones II (Orava 1981) appear to be temporary low-energy effects which will gradually merge into forward zones I as energy increases. We have applied the same analysis to the quark-antiquark systems for e^+e^- annihilations in terms of a local charge compensation function $G_c(\eta)$, which in fact, explains the short-range charge correlations observed in e^+e^- annihilations (Brandelik *et al* 1981).

The charge correlations will be generally caused by (i) resonance production (Hayot 1975), (ii) quark-cascade mechanism considered here, and, (iii) the recombination dynamics (Das and Hwa 1977; Hwa 1980a,b), which will take place after quantum chromodynamic 'fragmentations' (Altarelli and Parisi 1977) become dominant. We have separated here the effect due to (ii) only. Clearly what we have discussed will be inadequate in many kinematic regions, where processes (i) and (iii) are to be supplemented (Misra *et al* 1982). We have considered it useful to separate one effect which can in itself reasonably explain many phenomena of charge correlations in addition to the flow of charge quantum numbers (Berge *et al* 1981), and thus should not be ignored when we consider the other effects.

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